A Simple Estimation of the Size of Carbon Atoms Using a Pencil Lead

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One of the main topics of elementary physics is the idea that every material is composed of "little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one other."[1] These particles could be atoms or molecules. Atoms are the smallest part into which any material can be divided, and when several atoms are joined together, molecules are formed. Some interesting experiments to estimate the size of such atoms or molecules have been done that do not involve sophisticated equipment. One of these early experiments was conducted by Lord Rayleigh (1842-1919), which consisted of a small drop of oil spread to form a circular patch on the surface of the water. With a few simple calculations it is possible to determine the size of the oil molecule composition and therefore to provide an estimate of the diameter of the carbon atom.[2,3] The main aim of this article is that students, from middle school to high school, even at the undergraduate level, gain a quantitative understanding of the size of atoms by performing a simple experiment easily designed within the classroom.

The only indispensable materials to carry out this experiment are a pencil lead and millimeter paper. If more complexity is required in the analysis of error propagation, measuring instruments such as a vernier or a ruler can be used. It is worth mentioning that all molecules are assumed to be approximately the same size.[4]

The pencil lead is composed of graphite (a crystalline form of carbon atoms,[5] the fourth most abundant chemical element in the universe[6]), which we can imagine as identical spherical particles. A lead has the shape of an elongated cylinder whose volume is given by

$$V_C = \pi R^2 H, \tag{1}$$

where R and H are the radius and the height of the pencil lead, respectively, as we can see in the left image of Fig. 1. The diameter $D = 2R$ and the height of the lead are taken to be known from the manufacturer. If you want to measure in the classroom, the diameter of the lead can be measured with the vernier, while the length can be measured with the help of millimeter paper or a ruler. If we draw a line on a sheet of millimeter paper, keeping the lead straight, i.e., completely vertical (to maintain the shape of the cylinder to avoid a calculation error), part of the lead material will have moved to the paper matrix, forming a very thin parallelepiped or tiny height box, whose volume is given by

$$V_B = 2RLh, \tag{2}$$

where L is the length of the line, $2R$ is the width of the line, and h is the height of the box, as we can see in Fig. 1, while the volume of carbon spent from the lead after drawing the line is

$$V'_C = \pi R^2 (H - H'), \tag{3}$$

where H' is the final height of the pencil after drawing the line. Under this assumption the material is deposited entirely on the surface of the paper without loss (some of the material is pulverized and dispersed during the tracing of the lines, but for practical purposes it is so sparse that it does not introduce substantial errors in the estimation of the volume deposited on the paper); we can match the volume removed from the pencil lead to the volume deposited on the paper (see Fig. 1).

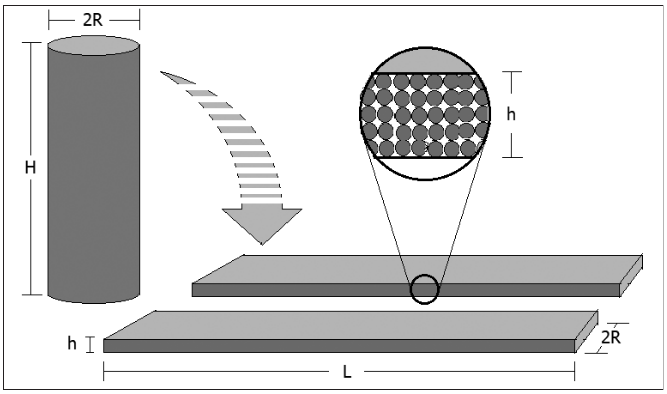

Fig. 1. It is assumed that the volume of the carbon cylinder is distributed in *n* identical boxes, whose height *h* gives us an estimate of the maximum size a molecule could have. In detail, it is considered that *h* is not the height of a single molecule but many of them.

And to make the effect more visible, it is possible to draw n lines of equal length l, so that the length L in Eq. (2) as a single line is given by $L = nl$. Thus, the following equality is satisfied: $V'_C = V_B$; therefore, from Eq. (2) and Eq. (3), we have

$$h = \pi \frac{R(H - H')}{2L}. \tag{4}$$

This height h can be considered as an upper bound for the size of the carbon atoms, since we assume that h is not the size of a single atom but many of them. Taking n large enough to significantly reduce the height of the pencil lead, it is possible to calculate this estimate numerically. According to the manufacturer's specifications for HB pencil lead, the height is H = 60 mm and the diameter is $2R$ = 0.5 mm (this is a popular standard measure; however, there are other presentations with different length and diameter).

We found that for this experiment it is easier to draw short

Table I. Results for the estimation of the carbon atom size are shown with different line numbers *n* and different line lengths *l*.

n	l (mm)	h (mm)
100	100	19
200	150	20
200	200	25
200	100	30
400	50	20

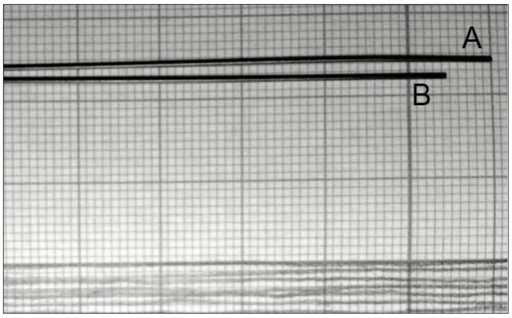

Fig. 2. The pencil lead (A) and (B), before and after drawing some lines in the millimeter paper, respectively.

lines, about $l = 10$ cm long, in the central columns of the millimeter paper, starting from above. Then we performed the test with $n = 50$ lines of $l = 100$ mm (Fig. 2), and we obtained that $H' = 59.5$ mm and therefore

$$h = \pi \frac{R(H - H')}{2nl} = \frac{(0.25\text{ mm})(0.5\text{ mm})}{2(50)(100\text{ mm})} \approx 0.000039$$
$$= 3.9 \times 10^{-8}\text{m} = 39\text{ nm}.$$

This result is reasonable as an upper limit for the size of carbon atoms; the individual size of the so-called graphene sheets has been systematically measured and varies from 2 to 20 nm (2×10^{-9} m to 2×10^{-8} m).[7]

The order of magnitude of the result does not change significantly when the number of lines drawn or their length is increased. This has been verified by changing both the number of lines n and their length l, the results of which are shown in Table I. Similar results can be found by performing analog experiments to the one presented here, although those are more sophisticated and require more equipment to carry them out.[8]

Acknowledgments

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References

1. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. 1 (Addison-Wesley, Reading, MA, 1963).
2. "Finding the size of atoms," IOPSpark, https://spark.iop.org/collections/finding-size-atoms.
3. https://thethoughtstash.wordpress.com/2012/11/17/estimating-the-size-of-an-atom. [Author: please check and update website]
4. C. Swartz, "All atoms are (about) the same size," *Phys. Teach.* **44**, 16–17 (Jan. 2006).
5. To be more precise, the graphite is a hexagonal structured crystal with a rhombohedral unit cell, and in bulk, arranged in a sheet-like structure, one could say that graphite is made up of overlapping sheets of graphene.
6. R. M. Hazen, R. T. Downs, L. Kah, and D. Sverjensky, "Carbon mineral evolution," *Rev. Mineral. Geochem.* **75** (1), 79–107 (2013).
7. Z. Shen, H. Ye, C. Zhou, M. Kröger, and Y. Li, "Size of graphene sheets determines the structural and mechanical properties of 3D graphene foams," *Nanotechnology* **29** (10), 104001 (2018).
8. B. Schaefer, E. Collett, A. Tabor-Morris, and J. Croman, "An estimation of the number and size of atoms in a printed period," *Phys. Teach.* **49**, 438–439 (Oct. 2011).

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