International Journal of Modern Physics A
Vol. 23, No. 13 (2008) 1949–1962

AN ALTERNATIVE INTERPRETATION FOR THE MODULI FIELDS OF THE COSMOLOGY ASSOCIATED TO TYPE IIB SUPERGRAVITY WITH FLUXES

TONATIUH MATOS,[*,†,‡,‖] JOSÉ-RUBÉN LUÉVANO,[§,**]
HUGO GARCÍA-COMPEÁN[‡,¶,††] and J. ALBERTO VÁZQUEZ[*,‡,‡‡]

[†] *CIAR Cosmology and Gravity Program,*
Department of Physics and Astronomy, University of British Columbia,
Vancouver, British Columbia, Canada, V6T 1Z1

[‡] *Departamento de Física,*
Centro de Investigación y de Estudios Avanzados del IPN,
A.P. 14-740, 07000 México D.F., México

[§] *Departamento de Ciencias Básicas,*
Universidad Autónoma Metropolitana-Azcapotzalco,
C.P. 02200 México, D.F., México
and

[¶] *Centro de Investigación y de Estudios Avanzados del IPN, Unidad Monterrey,*
Cerro de las Mitras 2565, Col. Obispado, cp. 64060, Monterrey N.L., México

[‖] *tmatos@fis.cinvestav.mx*
[**] *jrle@azc.uam.mx*
[††] *compean@fis.cinvestav.mx*
[‡‡] *jvazquez@fis.cinvestav.mx*

Received 18 November 2007
Revised 12 February 2008

The aim of this work is to provide a basis to interpret the dilaton as the dark matter of the universe, in the context of a particular cosmological model derived from type IIB supergravity theory with fluxes. In this theory, the dilaton is usually interpreted as a quintessence field. But, with this alternative interpretation we find that (in this supergravity model) the model gives a similar evolution and structure formation of the universe compared with the ΛCDM model in the linear regime of fluctuations of the structure formation. Some free parameters of the theory are fixed using the present cosmological observations. In the nonlinear regime there are some differences between the type IIB supergravity theory with the traditional CDM paradigm. The supergravity theory predicts the formation of galaxies earlier than the CDM and there is no density cusp in the center of galaxies. These differences can distinguish both models and might give a distinctive feature to the phenomenology of the cosmology coming from superstring theory with fluxes.

Keywords: String cosmology; dark matter; dynamical system.

PACS numbers: 11.25.Wx, 95.35.+d, 98.80.-k

[*]Part of the Instituto Avanzado de Cosmologí́a (IAC) collaboration http://www.iac.edu.mx/.

1. Introduction

One of the main problems in physics now is to know the nature of the dark matter and the understanding of the accelerated expansion of the universe. These two phenomena have been observed in the last years and now there are a number of observations supporting the existence of the dark matter[1,2] and the accelerated expansion of the universe as well.[3–5] On the other side, one of the main problems of superstring theory is that there is not a real phenomenology which can support the theory. Usually, superstring theory is supported only by its mathematical and internal consistency, but not by real experiments or observations. For some people, like the authors, one way of how superstring theory can make contact with phenomenology is through the cosmology.[6] In the last years, a number of new observations have given rise to a new cosmology and to a new perception of the universe (see for example Ref. 7). In superstring theory there are six extra dimensions forming a compact internal Calabi–Yau manifold.[8,9] Size and shape of this manifold manifests, at the four-dimensional low energy effective field theory, a series of scalar fields (moduli of the theory) many of which apparently have not been seen in nature. In particular, two fields, the *dilaton* and the *axion*, are two very important components of the theory which cannot be easily fixed. In fact, one should find a physical interpretation for these fields or give an explanation of why we are not able to see them in nature. One interpretation is that there exist a mechanism for eliminating these fields during the evolution of the universe.[10] Recently, one of the most popular interpretations for the dilaton field is that it can be the dark energy of the universe, i.e. a quintessence field.[11–13] These last interpretations have been possible because after a nontrivial compactification, the dilaton field acquires an effective potential. This effective potential makes possible to compare the dilaton field with some other kinds of matter.[11–13] In this work we are giving the dilaton a *different* interpretation supposing that it is the dark matter instead of the dark energy.[14,15] Such attempts have been carried over in the past with other dilaton potentials.[16–20] Here, we will be very specific starting with an effective potential derived recently from the type IIB supergravity theory. The main goal of this work is to show that this interpretation could be closer to a realistic cosmology as the interpretation that the dilaton is the dark energy. We will see that the late cosmology is very similar to the ΛCDM one with this alternative interpretation. Nevertheless, we will also see that it is necessary to do something else in order to recover a realistic cosmology from superstring theory. On the other hand, a great deal of work has been done recently, in the context of string compactifications with three-form fluxes (R–R and NS–NS) on the internal six-dimensional space and the exploration on their consequences in the stabilization of the moduli fields including the dilaton Φ and axion C.[21–28] Moduli stabilization has also been used in string cosmology to fix other moduli fields than the volume modulus including dilaton+axion and Kahler moduli.[29] For a description of more realistic scenarios, see Ref. 30.

In the context of the type IIB supergravity theory on the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold with a self-dual three-form fluxes, it has been shown that after compactifying the effective dilaton–axion potential is given by[31]

$$V_{\rm dil} = \frac{M_P^4}{4(8\pi)^3} h^2 e^{-2\Sigma_i \sigma_i} \left[e^{-\Phi^{(0)}} \cosh(\Phi - \Phi^{(0)}) + \frac{1}{2}\, e^{\Phi} (C - C^{(0)})^2 - e^{-\Phi^{(0)}} \right], \quad (1)$$

where $h^2 = \frac{1}{6} h_{mnp} h_{qrs} \delta^{mq} \delta^{nr} \delta^{ps}$. Here h_{mnp} are the NS–NS integral fluxes, the superscript (0) in the fields stands for the fields in the vacuum configuration and finally σ_i with $i = 1, 2, 3$ are the overall size of each factor $\mathbf{T}^2$ of the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold (in Ref. 31 there is a misprint in the potential (1)). Here we will simplify the system supposing that the moduli fields σ_i are constant for the late universe.

For the sake of simplicity in the derivation of the potential (1), some assumptions were made.[31] One of them is the assumption that the tensions of D-branes and orientifold planes cancel with the energy $V_{\rm dil}$ at $\Phi = \Phi_0$ and $C = C_0$. An assumption on initial conditions is that the dilaton is taken to deviate from equilibrium value, while the complex structure moduli are not. It is also assumed that (1) has a global minimum Φ_0, such that $V(\Phi_0) = 0$. Also that, the complex moduli are fixed and only the radial modulus σ feels a potential when the dilaton–axion system is excited. These assumptions make the model more simple, but still with the sufficient structure to be of interest in cosmological and astrophysical problems.

In order to study the cosmology of this model, it is convenient to define the following quantities: $\lambda\sqrt{\kappa}\phi = \Phi - \Phi^{(0)}$, $V_0 = \frac{M_P^4}{4(8\pi)^3} h^2 e^{-2\Sigma_i\sigma_i} e^{-\Phi^{(0)}}$, $C - C^{(0)} = \sqrt{\kappa}\psi$ and $\psi_0 = e^{\Phi^{(0)}}$, where λ is the string coupling $\lambda = e^{\langle\Phi\rangle}$ and $\lambda\sqrt{2\kappa}$ is the reduced Planck mass $M_p/\sqrt{8\pi}$. With this new variables, the dilaton potential transforms into

$$V_{\rm dil} = V_0(\cosh(\lambda\sqrt{\kappa}\phi) - 1) + \frac{1}{2}\, V_0 e^{\lambda\sqrt{\kappa}\phi} {\psi_0}^2 \kappa \psi^2 = V_\phi + e^{\lambda\sqrt{\kappa}\phi} V_\psi\,. \quad (2)$$

In some works the scalar field potential (2) is suggested to be the dark energy of the universe, that means, a quintessence field.[11–13,31] In this work we are not following this interpretation to the dilaton field. Instead of this, we will interpret the term $V_0(\cosh(\lambda\sqrt{\kappa}\phi) - 1)$ as the dark matter of the universe.[32–36] The remaining term in $V_{\rm dil}$ contains the contribution of the axion field C. This is what makes the difference between our work and previous ones. This interpretation allows us to compare the cosmology derived from the potential (2) with the Λ cold dark matter (ΛCDM) model. The rest of the fields coming from superstrings theory can be modeled as usual, assuming that this part of the matter is a perfect fluid. This perfect fluid has two epochs: radiation and matter dominated ones. In order to consider both epochs, we write the matter component as matter and radiation, with a state equation given by $\dot\rho_b + 3H\rho_b = 0$ and $\dot\rho_{\rm rad} + 4H\rho_{\rm rad} = 0$. For modeling the dark energy we can take the most general form supposing that it is also a perfect fluid with the equation of state given by $\dot\rho_L + 3\gamma_{\rm DE} H\rho_L = 0$, where $\gamma_{\rm DE}$ is smaller than 1/3 and can even be negative in the case it represents a phantom energy[37–40] field. It is just zero if ρ_L represents the cosmological constant $L = \rho_L$.

2. The Cosmology

Now, we proceed to describe the different epochs of the universe using this new interpretation. We can easily distinguish two behaviors of the scalar field potential: the exponential and the power laws. In the early universe the exponential behavior dominates the scalar fields potential. In this case we have the following analysis.

Inflation: In this epoch, the scalar field potential can be written as

$$V = V_0 \exp(\lambda\sqrt{\kappa}\phi)\left(1 + \frac{1}{2}\,\kappa{\psi_0}^2\psi^2\right), \tag{3}$$

because the exponential dominates completely the scenario of the evolution of the dilaton potential. The distinctive feature during this period is that the presence of the fluxes generate a quadratic term in the Friedman equation. The scalar field density $\rho = 1/2\dot{\phi}^2 + 1/2\dot{\psi}^2 e^{\tilde{\lambda}\sqrt{\kappa}\phi} + V$ appears quadratic in the field equations,

$$H^2 = \frac{\kappa}{3}\,\rho\left(1 + \frac{\rho}{\rho_0}\right). \tag{4}$$

Under this conditions it is known that these potentials are always inflationary in the presence of these fluxes.[41] Nevertheless, exponential potentials are inflationary without branes, in the traditional Friedman cosmology, only if $\lambda^2 < 2$ (see for example Ref. 42). Therefore, if we suppose that $\lambda^2 > 2$, the dilaton potential (2) is not inflationary without the quadratic density term. Thus, as the universe inflates, the quadratic term becomes much more smaller than the linear term and we recover the Friedman equation $H^2 = \frac{\kappa}{3}\rho$, where the exponential potential is not inflationary anymore. For these values of λ this gives a natural graceful-exit to this scalar field potential.[43] It remains to study which is the influence of the axion potential to this epoch.[44]

Densities evolution: The evolution of the densities is quite sensible to the initial conditions. Let us study the example of evolution shown in Fig. 1. As in the ΛCDM model, here also the recombination period starts around the redshift 10^3. The first difference we find between ΛCDM model and the IIB superstrings theory is just between the redshifts 10^3 and 10^2, where the interaction between the dilaton and matter gives rise to oscillations of the densities. It is just in this epoch where we have to look for observations that can distinguish between these two models. In this epoch the scalar field is already small $\lambda\sqrt{\kappa}|\phi| < 4$ and approaches the minimum of the potential in $\phi = 0$. Thus, potential (2) starts to behave as a power low potential, simulating a type ϕ^2 field. Therefore it is not surprising that this potential mimics very well the dark matter behavior. In the ΛCDM model, dark matter is modeled as dust and it is well known that power low potentials mimics dust fluids as they oscillate around the minimum of the potential.[45] In Figs. 1 and 2 this behavior is confirmed.

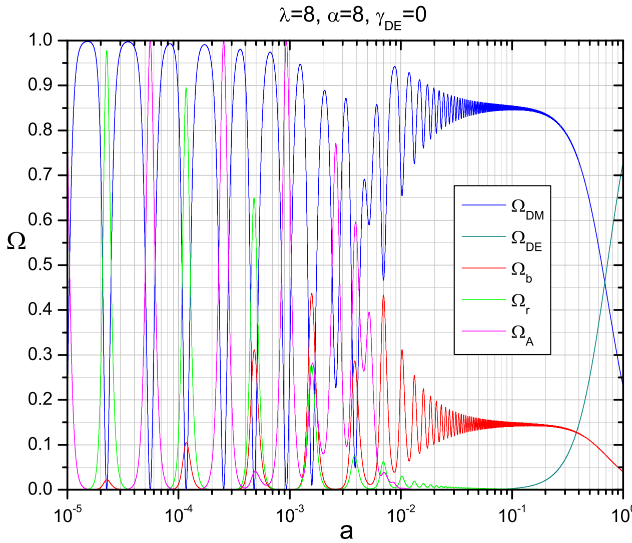

Fig. 1. Plot of the dynamics of the Ω's in the type IIB superstring theory with fluxes. Observe how this theory predicts a similar behavior of the matter content of the universe as the ΛCDM model. Here, the initial values of the dynamical variables at redshift $a = 1$ are: $x = 0$, $A = 0$, $u = \sqrt{0.23}$, $v = 1000$, $\Omega_{\rm DE} = 0.7299$, $\Omega_b = 0.04$, $\Omega_{\rm rad} = 4 \times 10^{-5}$, and w is determined by the Friedman restriction. The values for the constants are $\alpha = \lambda = 8$, $\tilde{\lambda} = 7$, $\Psi_0\sqrt{V_0/\rho_L} = 5000$. In all figures, the integration was made using the Adams–Badsforth–Moulton algorithm (variable step size). Each curve contains over 8×10^5 points.

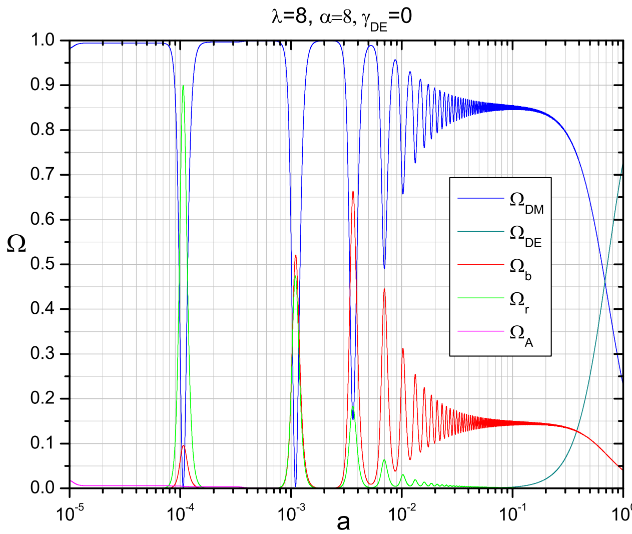

Fig. 2. Plot of the dynamics of the Ω's in the type IIB superstring theory with fluxes. Initial values of the dynamical variables at redshift $a = 1$ are the same as in Fig. 1. The values for the constants are $\Psi_0\sqrt{V_0/\rho_L} = 5000$, $\alpha = 8$, $\lambda = 8$, $\tilde{\lambda} = 0$. Each curve contains over 8×10^5 points.

Nevertheless, for redshifts bigger than $1/a - 1 = z \sim 10^3$, there are remarkable differences between the superstring model and the CDM one. The interaction of the dilaton field with matter provokes to be very difficult that radiation dominates the universe, thus big bang nucleosynthesis never takes place, at least in a similar way as in the CDM paradigm. Let us explain this point. The dilaton field interacts with matter through the factor $e^{\tilde{\alpha}(\Phi-\Phi^{(0)})}F^2 = e^{\alpha\sqrt{\kappa}\phi}F^2$, being F the field strength of the matter contents. Thus, Lagrangian for the superstrings system is

$$\mathcal{L} = \sqrt{-g}\big(R - \mathcal{L}_\phi - e^{\tilde{\lambda}\sqrt{\kappa}\phi}\mathcal{L}_\psi - e^{\alpha\sqrt{\kappa}\phi}\mathcal{L}_{\mathrm{matter}}\big)\,, \tag{5}$$

where we have differentiated the scalar field potential coupling constant λ from the axion–dilaton coupling constant $\tilde{\lambda}$ in order to generalized and clarify the cosmology of the system. In (1) both are the same $\lambda = \tilde{\lambda}$. The individual Lagrangians for the dilaton and axion fields respectively are,

$$\mathcal{L}_\phi = \frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V_\phi\,, \qquad \mathcal{L}_\psi = \frac{1}{2}\partial^\sigma\psi\partial_\sigma\psi + V_\psi\,. \tag{6}$$

Thus, in a flat Friedman–Robertson–Walker space–time the cosmological field equations are given by

$$H^2 = \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 e^{\tilde{\lambda}\sqrt{\kappa}\phi} + V_\phi + e^{\tilde{\lambda}\sqrt{\kappa}\phi}V_\psi + (\rho_b + \rho_{\mathrm{rad}})e^{\alpha\sqrt{\kappa}\phi} + \rho_L\right), \tag{7}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_\phi}{d\phi} = \tilde{\lambda}\sqrt{\kappa}e^{\tilde{\lambda}\sqrt{\kappa}\phi}\left(\frac{1}{2}\dot{\psi}^2 - V_\psi\right) - \alpha\sqrt{\kappa}e^{\alpha\sqrt{\kappa}\phi}(\rho_b + \rho_r)\,, \tag{8}$$

$$\ddot{\psi} + 3H\dot{\psi} + \frac{dV_\psi}{d\psi} = -\tilde{\lambda}\sqrt{\kappa}\dot{\phi}\dot{\psi}\,, \tag{9}$$

$$\dot{\rho}_b + 3H\rho_b = 0\,, \tag{10}$$

$$\dot{\rho}_{\mathrm{rad}} + 4H\rho_{\mathrm{rad}} = 0\,, \tag{11}$$

$$\dot{\rho}_L + 3\gamma_{\mathrm{DE}}H\rho_L = 0\,, \tag{12}$$

where the dots stand for the derivative with respect to the cosmological time and H is the Hubble parameter $H = \dot{a}/a$. In order to analyze the behavior of this cosmology, we transform Eqs. (7)–(12) using new variables defined by

$$x = \frac{\sqrt{\kappa}}{\sqrt{6}}\frac{\dot{\phi}}{H}\,, \qquad A = \frac{\sqrt{\kappa}}{\sqrt{6}}\frac{\dot{\psi}}{H}\,e^{\frac{1}{2}\tilde{\lambda}\sqrt{\kappa}\phi}\,, \tag{13}$$

$$y = \frac{\sqrt{\kappa}}{\sqrt{3}}\frac{\sqrt{\rho_b}}{H}\,e^{\frac{1}{2}\alpha\sqrt{\kappa}\phi}\,, \qquad z = \frac{\sqrt{\kappa}}{\sqrt{3}}\frac{\sqrt{\rho_{\mathrm{rad}}}}{H}\,e^{\frac{1}{2}\alpha\sqrt{\kappa}\phi}\,, \tag{14}$$

$$u = \frac{\sqrt{\kappa}}{\sqrt{3}}\frac{\sqrt{V_\phi}}{H}\,, \qquad v = \frac{\sqrt{\kappa}}{\sqrt{3}}\frac{\sqrt{V_2}}{H}\,, \tag{15}$$

$$l = \frac{\sqrt{\kappa}}{\sqrt{3}}\frac{\sqrt{\rho_L}}{H}\,, \qquad w = \frac{\sqrt{\kappa}}{\sqrt{3}}\frac{\sqrt{V_\psi}}{H}\,e^{\frac{1}{2}\tilde{\lambda}\sqrt{\kappa}\phi}\,, \tag{16}$$

where we have used the definition of the potentials $V_\phi = 2V_0 \sinh(1/2\sqrt{\kappa}\lambda\phi)^2$, $V_2 = 2V_0 \cosh(1/2\sqrt{\kappa}\lambda\phi)^2$ and $V_\psi = \frac{1}{2}V_0\kappa{\psi_0}^2\psi^2$ such that $V = V_\phi + V_\psi e^{\tilde{\lambda}\sqrt{\kappa}\phi}$ is the total scalar field potential. With these definitions Eqs. (7)–(12) transform into

$$x' = -3x - \sqrt{\frac{3}{2}}(\lambda uv + \alpha(y^2+z^2) + \tilde{\lambda}(w^2 - A^2)) + \frac{3}{2}\Pi x\,, \tag{17}$$

$$A' = -3A - \sqrt{3}\,\frac{\psi_0\sqrt{V_0}}{\sqrt{\rho_L}}\,wl - \sqrt{\frac{3}{2}}\tilde{\lambda}Ax + \frac{3}{2}\Pi A\,, \tag{18}$$

$$y' = \frac{3}{2}\left(\Pi - 1 + \alpha\sqrt{\frac{2}{3}}x\right)y\,, \tag{19}$$

$$z' = \frac{3}{2}\left(\Pi - \frac{4}{3} + \alpha\sqrt{\frac{2}{3}}x\right)z\,, \tag{20}$$

$$u' = \sqrt{\frac{3}{2}}\lambda vx + \frac{3}{2}\Pi u\,, \tag{21}$$

$$v' = \sqrt{\frac{3}{2}}\lambda ux + \frac{3}{2}\Pi v\,, \tag{22}$$

$$l' = \frac{3}{2}\,(\Pi - \gamma_{\mathrm{DE}})l\,, \tag{23}$$

$$w' = \sqrt{3}\,\frac{\psi_0\sqrt{V_0}}{\sqrt{\rho_L}}\,Al + \tilde{\lambda}\sqrt{\frac{3}{2}}\,wx + \frac{3}{2}\,\Pi w\,, \tag{24}$$

where now the primes stand for the derivative with respect to the N-foldings parameter $N = \ln(a)$. The quantity Π is defined as

$$\Pi = 2x^2 + 2A^2 + y^2 + \frac{4}{3}z^2\,. \tag{25}$$

The Friedman equation (7) becomes a constriction of the variables such that

$$x^2 + A^2 + y^2 + z^2 + u^2 + l^2 + w^2 = 1\,. \tag{26}$$

The density rate quantities $\Omega_x = \rho_x/\rho_{\mathrm{critic}}$ can be obtained using the variables (13)–(16), one arrives at

$$\begin{gathered}\Omega_{\mathrm{DM}} = x^2 + u^2\,, \quad \Omega_{\mathrm{DE}} = l^2\,, \quad \Omega_b = y^2\,,\\ \Omega_{\mathrm{rad}} = z^2\,, \quad \Omega_A = 1 - x^2 - u^2 - y^2 - z^2 - l^2\,,\end{gathered} \tag{27}$$

where Ω_{DM}, Ω_{DE}, Ω_b, Ω_{rad} and Ω_A respectively are the density rates for the dark matter (dilaton field), dark energy (cosmological constant), baryons, radiation and axion field. For the definition of this last one we have used the constriction (26). Equations (17)–(24) are now a dynamical system. The complete analysis of this system will be given elsewhere,[46] but the main results are the following. (1) The

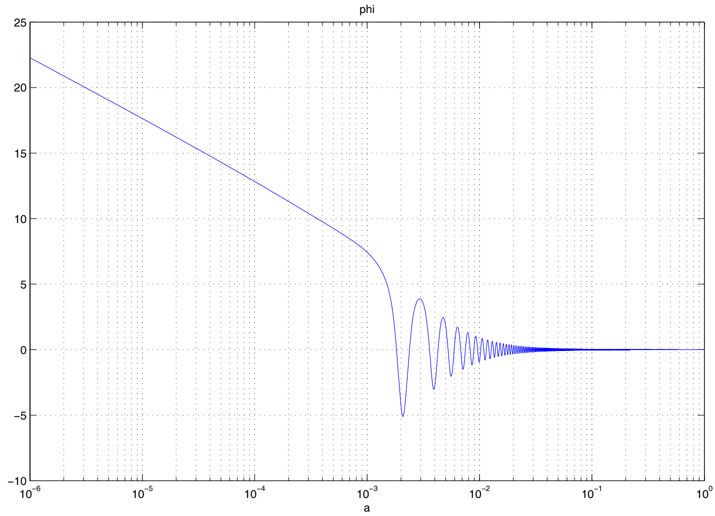

Fig. 3. Plot of the behavior of the scalar field ϕ. The scalar field starts from big values and riches very fast its minimum where it starts to oscillate. We plot $\lambda\sqrt{\kappa}\phi$, for $\lambda = 20$.

system contains many critical points, some of them are attractors with dark matter dominance, other with dark energy dominance. (2) The system depends strongly on the initial conditions. One example of the evolution of the densities is plotted in Figs. 1 and 2, where we show that the densities behave in a very similar way as the corresponding ones of the ΛCDM model before redshifts 10^2, which seem to be a generic behavior. The free constants λ, α and $\tilde{\lambda}$ are given in each figure. On the other side, we can see that after redshifts $z \sim 10^3$ one finds that $|\phi| < 0.04 m_{\rm Planck}$ and oscillating goes to zero, such that its exponential is bounded $0.01 < e^{-\lambda\sqrt{\kappa}\phi} < 1$ (see Fig. 3). In other words, it takes the exponential more than 13 Giga years to change from 0.01 to 1.

However, there is one fact that takes our attention in Fig. 1. We see from the behavior of the densities on the early universe after redshifts $\sim 10^3$, that radiation does not dominates the rest of the densities as it is required for big bang nucleosynthesis. This fact can also be seen as follows. In a radiation dominated universe we might set $l = y = u = v = w = A = 0$, in that case we can see by inspection of (17)–(24) that there is no way that radiation remains as a dominant component of the system. The situations radically change if we put $\tilde{\lambda} = 0$ in system (17)–(24), in this case radiation has no problems to be dominant somewhere. In order to show how the dilaton and axion interaction with matter work, we study the particular case $\tilde{\lambda} = 0$ and let us artificially drop out the matter interaction from the dilaton equation (8). In what follows we study this toy model. For this one it is convenient to change the variable w for $w = V_3$, with the definition of the potentials $V_3 = \sqrt{\kappa}\psi_0\psi$ such that $V = V_\phi^2 + 1/4(V_1 + V_2)^2 V_3^2$. Thus, Eqs. (17)–(24)

transform into the new system

$$
\begin{aligned}
x' &= -3x - \sqrt{\frac{3}{2}}\left(\lambda uv + \frac{\alpha}{4}(v+u)^2 w^2\right) + \frac{3}{2}\Pi x\,,\\
A' &= -3A - \sqrt{\frac{3}{2}}\psi_0\,\frac{1}{2}(v+u)^2 w + \frac{3}{2}\Pi A\,,\\
y' &= \frac{3}{2}\left(\Pi - 1 + \alpha\sqrt{\frac{2}{3}}x\right)y\,,\\
z' &= \frac{3}{2}\left(\Pi - \frac{4}{3} + \alpha\sqrt{\frac{2}{3}}x\right)z\,,\\
u' &= \sqrt{\frac{3}{2}}\lambda vx + \frac{3}{2}\Pi u\,,\\
v' &= \sqrt{\frac{3}{2}}\lambda ux + \frac{3}{2}\Pi v\,,\\
l' &= \frac{3}{2}\,(\Pi - \gamma_{\mathrm{DE}})l\,,\\
w' &= \sqrt{6}\psi_0 A\,.
\end{aligned}
\tag{28}
$$

The quantity Π is now defined as

$$
\Pi = 2x^2 + 2A^2 + y^2 + \frac{4}{3}\,z^2 + \gamma_{\mathrm{DE}} l^2 - \lambda\sqrt{\frac{2}{3}}(y^2 + z^2)x \tag{29}
$$

and the new Friedman constriction (7) reads

$$
x^2 + A^2 + y^2 + z^2 + u^2 + l^2 + \frac{1}{4}\,(u+v)^2 w^2 = 1\,. \tag{30}
$$

The density quantities Ω_x now are

$$
\begin{gathered}
\Omega_{\mathrm{DM}} = x^2 + u^2\,, \quad \Omega_{\mathrm{DE}} = l^2\,, \quad \Omega_b = y^2\,,\\
\Omega_{\mathrm{rad}} = z^2\,, \quad \Omega_A = 1 - x^2 - u^2 - y^2 - z^2 - l^2\,,
\end{gathered}
\tag{31}
$$

where we have used the constriction (30). Equations (28) are now a new dynamical system. The evolution of this one is shown in Fig. 4. From here we can see that now radiation dominates the early universe without problems and that the behavior of the densities is again very similar to the ΛCDM model but now for all redshifts. The only difference is at redshifts $10^2 < z < 10^3$, where the densities oscillate very hard. Unfortunately this time corresponds to the dark age, when the universe has no stars and there is nothing to observe which could give us some observational clue for this behavior.

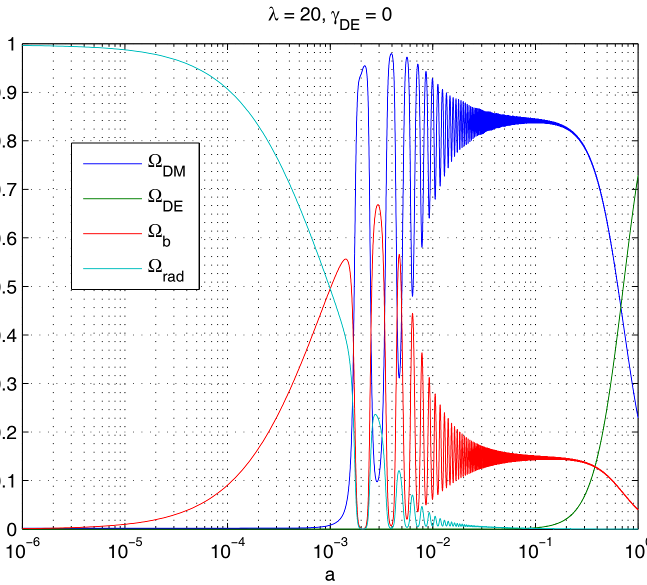

Fig. 4. Plot of the dynamics of the Ω's in the type IIB superstring theory with fluxes. Observe how this theory predicts a similar behavior of the matter content of the universe as the ΛCDM model, even for redshifts beyond 10^3. Here radiation dominates the universe for values less than $a \sim 10^{-3}$ and big bang nucleosynthesis takes place as in the CDM model. Here $\lambda = \alpha = 20$, the initial values of the dynamical variables at redshift $a = 1$ are: $x = 0$, $A = 0$, $u = \sqrt{0.23}$, $v = 1000$, $\Omega_{\rm DE} = 0.7299$, $\Omega_b = 0.04$, $\Omega_{\rm rad} = 4 \times 10^{-5}$, and w is determined by the Friedman restriction. Each curve contains over 3×10^5 points.

Finally, if the set both coupling constant $\alpha = \tilde{\lambda} = 0$, we recover a very similar behavior of the densities to the ΛCDM model, this behavior is shown in Fig. 5. Observe here that the densities have not oscillations any more, as in the ΛCDM model, supporting the idea that it is just the coupling between dilaton, axion and matter which makes difficult that the string theory reproduces the observed universe.

Structure formation: As shown in Figs. 1 and 4 the axion field can be completely subdominant, but it can dominates the universe at early times as in Fig. 2. At late times, $10^{-2} < a < 1$, the structure formation is determined by the dilaton field ϕ and its effective potential (2). In Ref. 47 it is shown that the scalar field fluctuations with a cosh potential follow the corresponding ones of the cold dark matter (CDM) model for the linear regime. There, it is shown that the field equations of the scalar field fluctuations can be written in terms of the ones of the ΛCDM model, in such a way that both models predict the same spectrum in the linear regime of fluctuations.

Galaxies formation: Other main difference between both models, the CDM and type IIB superstrings is just in the nonlinear regime of fluctuations. Here numerical simulations show that the scalar field virialize very early,[48,49] causing that in the superstring model galaxies form earlier than in the CDM paradigm. Furthermore, it

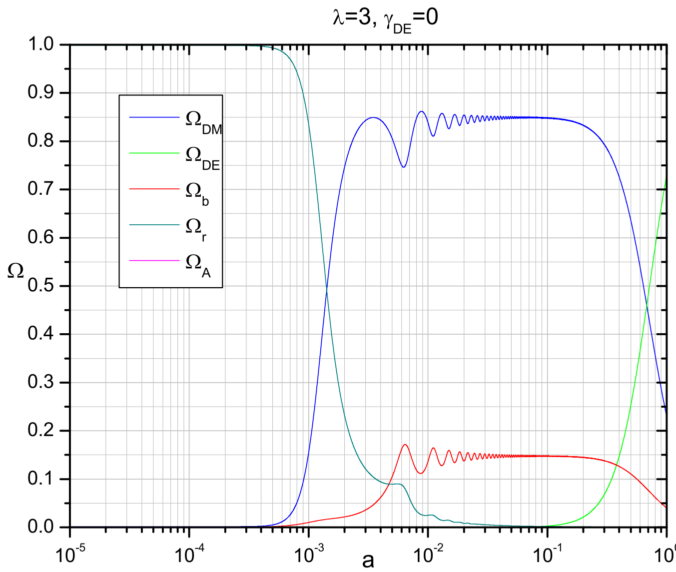

Fig. 5. Plot of the dynamics of the Ω's in the cosh model. Here all the coupling constants of the superstrings model $\alpha = \tilde{\lambda} = 0$, $\lambda = 3.0$, the initial values of the dynamical variables at redshift $a = 1$ are the same as in Fig. 1. Observe how this theory predicts an extremely similar behavior of the matter content of the universe as the ΛCDM model, for all redshifts. Each curve contains over 5×10^5 points.

has been shown that the scalar field does not have a cuspy central density profile.[50] Numerical and semianalytic simulations have shown that the density profiles of oscillations (collapsed scalar fields) are almost flat in the center.[51,52,48,49] It has also been possible to compare high and low surface brightness galaxies with the scalar field model and the comparison shows that there is a concordance between the model and the observations, provided that the values of the parameters are just $V_0 \sim (3 \times 10^{-27} m_{\rm Planck})^4$, $\lambda \sim 20$.[51] With this values of V_0 and λ, the critical mass for collapse of the scalar field is just $10^{12} M_\odot$,[52] as it is expected for the halos of galaxies. These two features of the scalar field collapse might give distinctive features to superstring theory. At the present time there is a controversy about the density profiles of the dark matter in the centers of the galaxies.[53–61] This model of the superstring theory predicts that the center of the galaxies contains an almost flat central density profile. We are aware that this result corresponds to the particular compactification $\mathbf{T}^6/\mathbb{Z}_2$, but it could be a general signature of string theory, in the sense that it could survive in a more realistic compactification (including branes and fluxes), that give rise to models that resemble the Standard Model. In this case, if the cuspy dark matter density profiles are observed or explained in some way, this model would be ruled out. But if these profiles are not observed, it would be an important astrophysical signature of string theory.

3. Conclusions

In this work we propose an alternative interpretation of the dilaton field in the type IIB supergravity on the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold model with fluxes.[31] This alternative interpretation allows us to compare this model with the ΛCDM one, which has been very successful in its predictions. The result is that, at least in this model, radiation seems to be subdominant everywhere, provoking difficulties to explain big bang nucleosynthesis. Even when we see that in this particular toy model, the behavior seems to be generic for all strings theories. If this is the case, it is possible that the dilaton and axion fields could not be interpreted as dark matter or dark energy, thus we should either seek other candidates and explain why we do not see the dilaton and axion scalar fields in our observations, or we have to explain big bang nucleosynthesis using the conditions given by superstrings theory we showed here, or we have to look for a mechanism to eliminate the coupling between dilaton and axion with matter at very early times. The last option maybe more realistic. Even if we solve the radiation dominance problem, there are some differences between ΛCDM and superstrings theory between $10^2 < z < 10^3$, because string theory predicts around 16 million years of densities oscillations during the dark age. Nevertheless, both models are very similar at late times, between $0 < z < 10^2$, maybe the only difference during this last period is their predictions on substructure formation and galaxies centers. While CDM predicts much more substructure in the universe and very sharp density profiles, scalar fields predict few substructure and almost constant density profiles in centers of galaxies. The confirmation of this observations could decide between these two models. We are aware that this orientifold model is still a toy model and it would be interesting to study more realistic compactifications (including brane and orientifold configurations) and see if our results, including that of the dark matter density profiles, survive and become a general feature of string theory. If this is the case, this alternative interpretation of the fields of the theory might permit a connection of string theory with the astrophysics phenomenology of dark matter, i.e. its connection with future astrophysical and cosmological observations. We conclude that this interpretation can give us a closer understanding of superstrings theory with cosmology.

Acknowledgments

The authors wish to thank Luis Ureña-López for the careful reading of the paper, Norma Quiroz and Cesar Terrero for many helpful discussions and for pointing out to us the Ref. 31. Correspondence with S. Sethi and A. Krause is appreciated. T. Matos wants to thank Matt Choptuik for his kind hospitality at the UBC. This work was partly supported by CONACyT México, under grants 32138-E, 42748, 47641 and 45713-F.

References

1. A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).

2. N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, *Science* **284**, 1481 (1999), arXiv:astro-ph/9906463.
3. B. P. Schmidt *et al.*, *Astrophys. J.* **507**, 46 (1998).
4. A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).
5. S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999).
6. G. Cleaver, *Adv. Space Res.* **35**, 106 (2005).
7. J. R. Primack, *Proc. 5th Int. UCLA Symp. on Sources and Detection of Dark Matter and Dark Energy in the Universe*, Marina del Rey, California, February 2002, ed. D. Cline (North-Holland, 2002).
8. M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies and Phenomenology* (Cambridge University Press, Cambridge, 1987).
9. J. Polchinski, *String Theory*, Vol. 2 (Cambridge University Press, Cambridge, 1998).
10. T. Damour and A. Vilenkin, *Phys. Rev. Lett.* **78**, 2288 (1997).
11. K. Choi, *Phys. Rev. D* **62**, 043509 (2000).
12. K. Choi, arXiv:hep-ph/9912218.
13. S. Hellerman, N. Kaloper and L. Susskind, *J. High Energy Phys.* **0106**, 003 (2001), arXiv:hep-th/0104180.
14. Y. M. Cho and Y. Y. Keum, *Mod. Phys. Lett. A* **13**, 109 (1998).
15. Y. M. Cho and Y. Y. Keum, *Class. Quantum Grav.* **15**, 907 (1998).
16. R. Dick, arXiv:hep-th/9609190.
17. R. Dick, *Mod. Phys. Lett. A* **12**, 47 (1997).
18. P. J. E. Peebles and A. Vilenkin, *Phys. Rev. D* **60**, 103506 (1999).
19. T. Matos and F. S. Guzman, *Class. Quantum Grav.* **17**, L9 (2000), arXiv:gr-qc/9810028.
20. T. Biswas, R. Brandenberger, A. Mazumdar and T. Multamaki, arXiv:hep-th/0507199.
21. K. Dasgupta, G. Rajesh and S. Sethi, *J. High Energy Phys.* **9908**, 023 (1999).
22. G. Curio and A. Krause, *Nucl. Phys. B* **602**, 172 (2001), arXiv:hep-th/0012152.
23. S. B. Giddings, S. Kachru and J. Polchinski, *Phys. Rev. D* **66**, 106006 (2002).
24. B. S. Acharya, A moduli fixing mechanism in M-theory, arXiv:hep-th/0212294.
25. S. Kachru, M. B. Schulz and S. Trivedi, *J. High Energy Phys.* **0310**, 007 (2003).
26. A. R. Frey and J. Polchinski, *Phys. Rev. D* **65**, 126009 (2002).
27. R. Blumenhagen, D. Lüst and T. R. Taylor, *Nucl. Phys. B* **663**, 319 (2003).
28. J. F. G. Cascales, M. P. García del Moral, F. Quevedo and A. M. Uranga, *J. High Energy Phys.* **0402**, 031 (2004).
29. R. Brandenberger, Y.-K. E. Cheung and S. Watson, Moduli stabilization with string gases and fluxes, arXiv:hep-th/0501032.
30. K. Becker, M. Becker and A. Krause, *Nucl. Phys. B* **715**, 349 (2005), arXiv:hep-th/0501130.
31. A. R. Frey and A. Mazumdar, *Phys. Rev. D* **67**, 046006 (2003), arXiv:hep-th/0210254.
32. T. Matos and L. A. Urena-Lopez, *Class. Quantum Grav.* **17**, L75 (2000), arXiv:astro-ph/0004332.
33. V. Sahni and L. Wang, *Phys. Rev. D* **63**, 103517 (2000).
34. A. Arbey, J. Lesgourgues and P. Salati, *Phys. Rev. D* **64**, 123528 (2001).
35. A. Arbey, J. Lesgourgues and P. Salati, *Phys. Rev. D* **65**, 083514 (2002).
36. T. Matos and L. A. Ureña-López, *Phys. Rev. D* **63**, 063506 (2001).
37. J.-G. Hao and X.-Z. Li, *Phys. Rev. D* **67**, 107303 (2003).
38. L. P. Chimento and R. Lazkoz, *Phys. Rev. Lett.* **91**, 211301 (2003).
39. G. Izquierdo and D. Pavon, astro-ph/0505601.

40. S. V. Sushkov, *Phys. Rev. D* **71**, 043520 (2005).
41. E. J. Copeland, A. R. Liddle and J. E. Lidsey, *Phys. Rev. D* **64**, 023509 (2001).
42. A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge University Press, Cambridge, 2000).
43. J. E. Lidsey, T. Matos and L. A. Urena-Lopez, *Phys. Rev. D* **66**, 023514 (2002), arXiv:astro-ph/0111292.
44. T. Matos and E. Ramirez, in preparation.
45. M. S. Turner, *Phys. Rev. D* **28**, 1243 (1983).
46. T. Matos, J. A. Vázquez and J.-R. Luévano, in preparation.
47. T. Matos and L. A. Urena-Lopez, *Phys. Rev. D* **63**, 063506 (2001), arXiv:astro-ph/0006024.
48. F. S. Guzmán and L. A. Ureña-López, *Phys. Rev. D* **68**, 024023 (2003), arXiv:astro-ph/0303440.
49. F. S. Guzmán and L. A. Ureña-López, *Phys. Rev. D* **69**, 124033 (2004).
50. W. Hu, R. Barkana and A. Gruzinov, *Phys. Rev. Lett.* **85**, 1158 (2000).
51. A. Bernal, T. Matos and D. Nuñez, Flat central density profiles from scalar field dark matter halo, arXiv:astro-ph/0303455.
52. M. Alcubierre, F. S. Guzman, T. Matos, D. Nunez, L. A. Urena-Lopez and P. Wiederhold, *Class. Quantum Grav.* **19**, 5017 (2002), arXiv:gr-qc/0110102.
53. J. Navarro, C. S. Frenk and S. S. M. White, *Astrophys. J.* **490**, 493 (1997).
54. W. J. G. de Blok, S. S. MacGaugh, A. Bosma and V. C. Rubin, *Astrophys. J.* **552**, L23 (2001).
55. S. S. MacGaugh, V. C. Rubin and W. J. G. de Blok, *Astron. J.* **122**, 2381 (2201).
56. W. J. G. de Blok, S. S. MacGaugh and V. C. Rubin, *Astron. J.* **122**, 2396 (2001).
57. P. Salucci, F. Walter and A. Borriello, arXiv:astro-ph/0206304.
58. A. Borriello, P. Salucci and L. Danese, *Mon. Not. R. Astron. Soc.* **341**, 1109 (2003).
59. G. Gentile, A. Burkert, P. Salucci, U. Klein and F. Walter, arXiv:astro-ph/0506538.
60. L. Monaco, M. Bellazzini, F. R. Ferraro and E. Pancino, *Mon. Not. R. Astron. Soc.* **356**, 1396 (2005).
61. S. Kristine, G. Riccardo and P. H. Martha, arXiv:astro-ph/0502166.