

Tarea perturbaciones 2.

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Homework

$$g_{\mu\nu}(E_i)^\mu(E_j)^\nu = -\delta_{ij}$$

$$ds^2 = a^2(\eta) \{ (1+2\psi) d\eta^2 - 2B_i dx^i d\eta - [(1-2\phi)\delta_{ij} + 2E_{ij}] dx^i dx^j \}$$

$$(E_i)^\mu = a^{-1} [B_i \delta_0^\mu + (1+\phi)\delta_i^\mu - E_i^j \delta_j^\mu]$$

$$g_{\mu\nu}(E_i)^\mu(E_j)^\nu = g_{00}(E_i)^0(E_j)^0 + 2g_{0i}(E_i)^i(E_j)^0 + g_{kl}(E_i)^k(E_j)^l$$

$$= g_{00} [a^{-1}(B_i)] [a^{-1}(B_j)] + 2g_{0i} \{ a^{-1} [(1+\phi)\delta_i^k - E_i^k \delta_j^k] \} [a^{-1}(B_j)]$$

$$+ g_{kl} \{ a^{-1} [(1+\phi)\delta_i^k - E_i^k \delta_j^k] \} \{ a^{-1} [(1+\phi)\delta_j^l - E_j^l \delta_i^l] \}$$

$$= -a^2 [(1-2\phi)\delta_{kl} + 2E_{kl}] \{ a^{-1} [(1+\phi)\delta_i^k - E_i^k \delta_j^k] \} \{ a^{-1} [(1+\phi)\delta_j^l - E_j^l \delta_i^l] \}$$

$$= -[(1-2\phi)\delta_{kl} + 2E_{kl}] [(1+\phi)\delta_i^k - E_i^k \delta_j^k] [(1+\phi)\delta_j^l - E_j^l \delta_i^l]$$

$$= -[(1-\phi)\delta_{il} - E_i^j \delta_{jl} + 2E_{kl} \delta_i^k] [(1+\phi)\delta_j^l - E_j^l \delta_i^l]$$

$$= -[\delta_{il} \delta_j^l - E_i^j \delta_{il} \delta_j^l - E_i^j \delta_{jl} \delta_j^l + 2E_{kl} \delta_i^k \delta_j^l]$$

$$= -[\delta_{ij} - E_i^j - E_j^i + 2E_{ij}] = -\delta_{ij}$$

Homework:

$$T_0^i = q^i$$

$$T_j^i = -(\bar{P} + \delta P)\delta_j^i + \Pi_j^i$$

$$ds^2 = a^2(\eta) \{ (1+2\psi)d\eta^2 - 2B_i dx^i d\eta - [(1-2\phi)\delta_{ij} + 2E_{ij}]dx^i dx^j \}$$

$$T^{0i} = a^{-2}(q^i + \bar{P}B^i)$$

$$T^{ij} = a^{-2}[\bar{P}\delta^{ij} + (2\bar{P}\phi + \delta P)\delta^{ij} - 2\bar{P}E^{ij} - \Pi^{ij}]$$

$$T^i_0 = g_{00} T^{0i} = g_{00} T^{0i} + g_{j0} T^{ij}$$

$$= a^2(1+2\psi)a^{-2}(q^i + \bar{P}B^i) + \mathcal{O}(2)$$

$$= q^i$$

$$T^i_j = g_{jm} T^{im} = g_{j0} T^{i0} + g_{jk} T^{ik}$$

$$= -a^2[(1-2\phi)\delta_{jk} + 2E_{jk}]a^{-2}[\bar{P}\delta^{ik} + (2\bar{P}\phi + \delta P)\delta^{ik} - 2\bar{P}E^{ik} - \Pi^{ik}] + \mathcal{O}(2)$$

$$= -[\delta_{jk} - 2\phi\delta_{jk} + 2E_{jk}][\bar{P}\delta^{ik} + 2\bar{P}\phi\delta^{ik} + \delta P\delta^{ik} - 2\bar{P}E^{ik} - \Pi^{ik}]$$

$$= -[\bar{P}\delta_{jk}\delta^{ik} - 2\phi\bar{P}\delta_{jk}\delta^{ik} + 2E_{jk}\bar{P}\delta^{ik} + 2\bar{P}\phi\delta_{jk}\delta^{ik} + \delta P\delta_{jk}\delta^{ik} - 2\bar{P}E^{ik}\delta_{jk} - \Pi^{ik}\delta_{jk}]$$

$$= -(\bar{P} + \delta P)\delta^i_j + \Pi^i_j$$