Demostraciones faltantes:

$$\delta\tilde{g}_{ij} = \delta g_{ij} + \left(\frac{\partial x^\alpha}{\partial \tilde{x}^i}\frac{\partial x^\beta}{\partial \tilde{x}^j} - \delta^\alpha{}_i \delta^\beta{}_j\right)\bar{g}_{\alpha\beta}$$

$$+ T\dot{\bar{g}}_{ij} - L^i \partial_i \bar{g}_{ij} - a^2\{-2\tilde{\phi}\delta_{ij} + 2\tilde{E}_{ij}\}$$

$$= -a^2(-2\phi\delta_{ij} + 2E_{ij}) + \left(\frac{\partial x^0}{\partial \tilde{x}^i}\frac{\partial x^0}{\partial \tilde{x}^j}\right)\bar{g}_{00}$$

$$+ \left(\frac{\partial x^i}{\partial \tilde{x}^i}\frac{\partial x^j}{\partial \tilde{x}^j} - 1\right)\bar{g}_{ij} - T\partial_0(-a^2\delta_{ij})$$

$$- 2a^2(-\tilde{\phi}\delta_{ij} + \tilde{E}_{ij}) = -2a^2(-\phi\delta_{ij} + E_{ij}$$

$$+ a^2\cancel{(\partial_i T \partial_j T)}^{0} + (-2a^2)\delta_{ij}((1-\partial_i L^i)(1-\partial_j L^j) - 1)$$

$$+ 2aT\partial_0 a\delta_{ij} - \tilde{\phi}\delta_{ij} + \tilde{E}_{ij}$$

$$= -\phi\delta_{ij} + E_{ij} - \partial_i L^i\delta_{ij} - TH\delta_{ij} \overset{!}{=} \tilde{E}_{ij}$$

por lo que

$$\boxed{\tilde{E}_{ij} = E_{ij} - \partial_{\langle i} L_{j\rangle}}$$

2) Ver que Δ es invariante

Recuerde que $\bar{\rho}\Delta = \delta\rho + \mathring{\bar{\rho}}(v+B)$

$$= \delta\rho - 3\mathcal{H}(\bar{\rho}+\bar{p})(B+v)$$

Note que

$$\tilde{\bar{\rho}}\tilde{\Delta} = \delta\tilde{\rho} - 3\mathcal{H}(\tilde{\bar{\rho}}+\tilde{\bar{p}})(\tilde{B}+\tilde{v})$$

$$= \delta\tilde{\rho} - 3\mathcal{H}(\bar{\rho}+\bar{p})\tilde{B} - \tilde{q}\,3\mathcal{H}(\bar{p}+\bar{\rho})(\bar{p}+\bar{\rho})^{-1}.$$

$$= \delta\rho - 3\mathcal{H}(q + (\bar{\rho}+\bar{p})i) - T\mathring{\bar{\rho}}$$
$$- 3\mathcal{H}(\bar{\rho}+\bar{p})(B+T\cdot i))$$

$$= \delta\rho - 3\mathcal{H}(\bar{\rho}+\bar{p})(B - i + (\bar{\rho}+\bar{p})^{-1}q + i)$$
$$- T(\mathring{\bar{\rho}} + \cancel{3\mathcal{H}(\bar{\rho}+\mathring{\bar{p}})})T$$

$$= \delta\rho - 3\mathcal{H}(\bar{\rho}+\bar{p})(B+v) = \bar{\rho}\Delta$$

$\therefore$ Es invariante de Gauge.