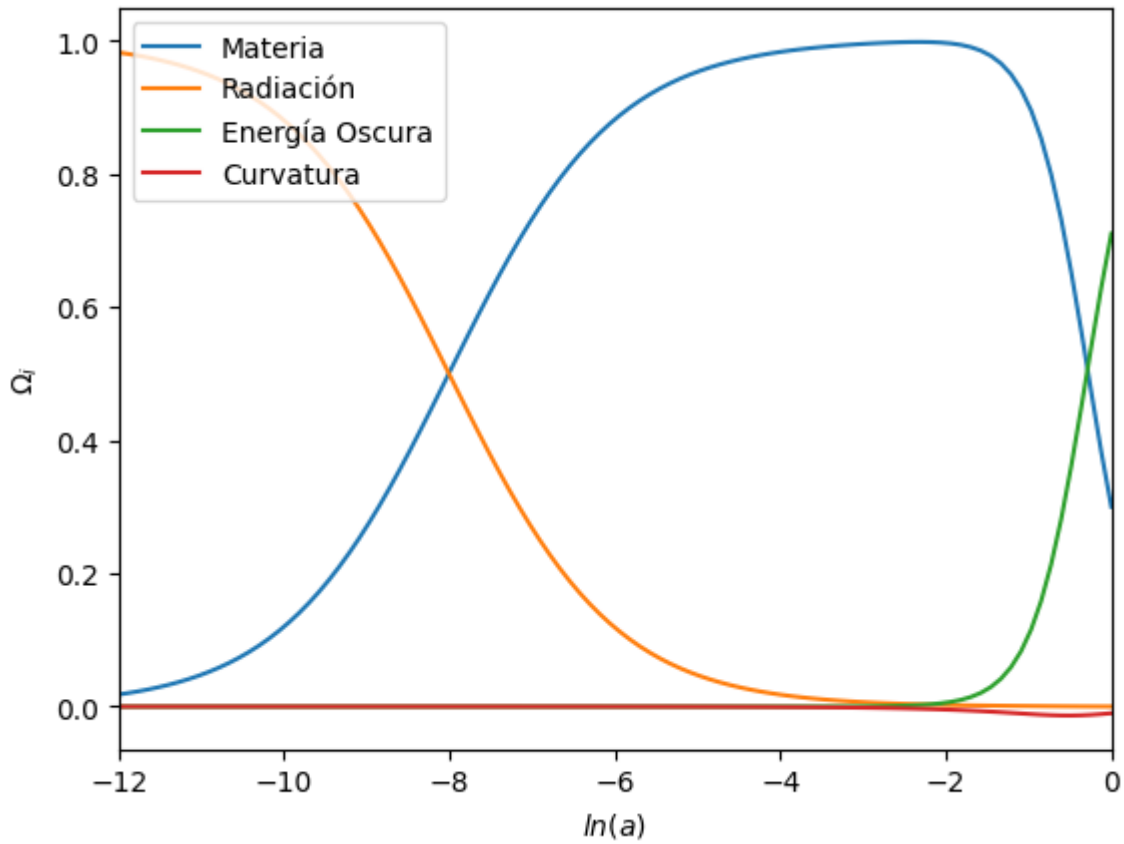
In [176]:
```
import numpy as np
from sympy import *
import matplotlib.pyplot as plt

from math import log, e
import math
from scipy.integrate import odeint

from scipy.interpolate import InterpolatedUnivariateSpline
from functools import partial
from scipy.integrate import quad

import pandas as pd
```

Densidades

In [19]:
```
OM, OR, OL, OK = 1, 4/3, 0, 2/3
M0, R0, L0, K0, H0= 0.3, 1e-4, 1-0.3-1e-4+0.01, -0.01, 68
c = 300000000
rn=np.linspace(0, -12, 100)
a = np.exp(rn)
z = 1/a-1
x0 = [M0, R0, L0, K0]
```

In [186]:
```
def DO(x, rn):
    M, R, L, K = x

    dMdt = 3*((OM*M+OR*R+OL*L+OK*K)-OM)*M
    dRdt = 3*((OM*M+OR*R+OL*L+OK*K)-OR)*R
    dLdt = 3*((OM*M+OR*R+OL*L+OK*K)-OL)*L
    dKdt = 3*((OM*M+OR*R+OL*L+OK*K)-OK)*K

    return [dMdt, dRdt, dLdt, dKdt]

s = odeint(DO,x0,rn)

M = s[:,0]
R = s[:,1]
L = s[:,2]
K = s[:,3]
```

In [187]:
```
plt.plot(rn,M,label='Materia')
plt.plot(rn,R,label='Radiación')
plt.plot(rn,L,label='Energía Oscura')
plt.plot(rn,K,label='Curvatura')
plt.ylabel('$\\Omega_i$')
plt.xlabel('$ln(a)$')
plt.legend(loc='upper left')
plt.xlim([-12, 0])
plt.show()
```

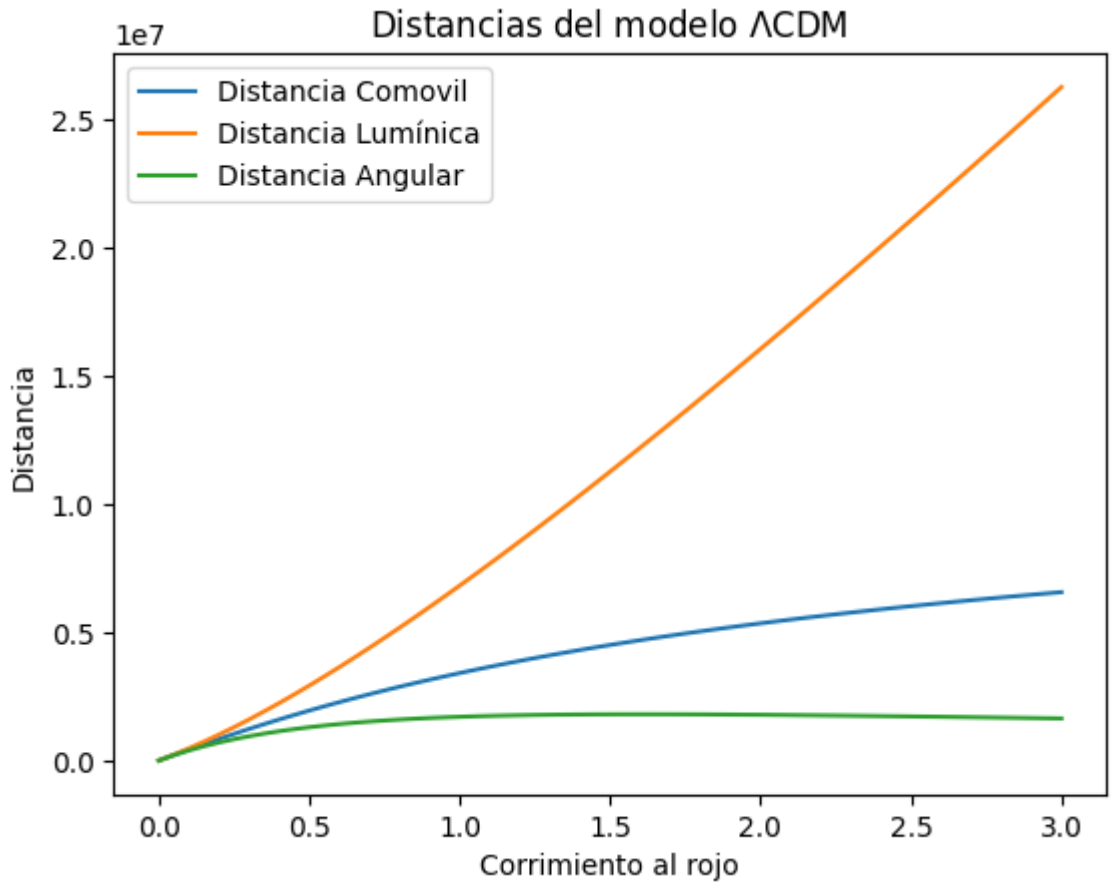


Distancias

Modelo ΛCDM

In [101]:
```
zz=np.linspace(0, 3, 60)

def HH1(z):
    return (M0*(z+1)**3 + K0*(z+1)**2 + L0)**-0.5

def X1(z):
    return c/H0*quad(HH1, 0, z)[0]

def DL1(z):
    return (z+1)*X1(z)

def DA1(z):
    return X1(z)/(1+z)

X1=np.vectorize(X1)
```

In [105]:
```
plt.plot(zz,X1(zz), label='Distancia Comovil')
plt.plot(zz,DL1(zz), label='Distancia Lumínica')
plt.plot(zz,DA1(zz), label='Distancia Angular')
plt.xlabel('Corrimiento al rojo')
plt.ylabel('Distancia')
plt.title("Distancias del modelo $\Lambda$CDM")
plt.legend(loc='upper left')
plt.show()
```



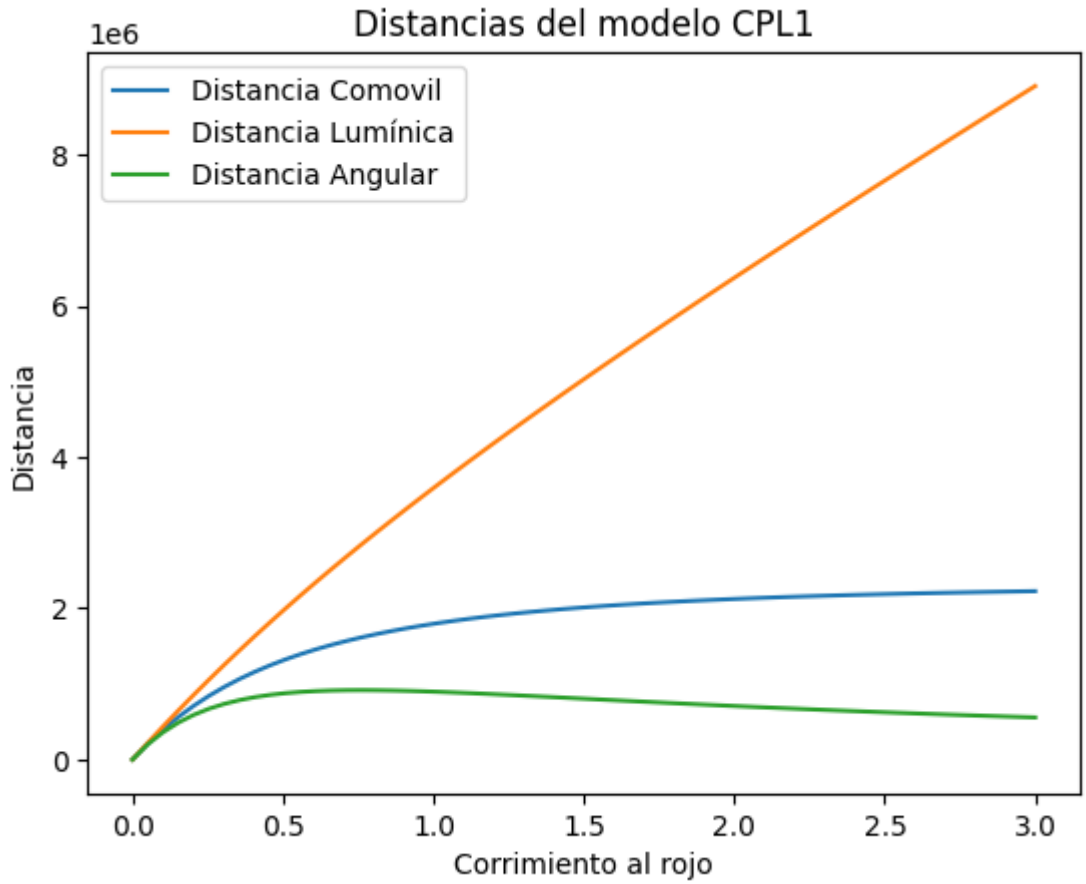
Modelos CPL

In [109]:
```
w0 = 0.9
wa = 0.5
```

Modelo 1

In [113]:
```
def HH2(z):
    return (M0*(z+1)**3 + K0*(z+1)**2 + L0*np.exp(-3*wa*z/(1 + z))*(z+1)**(3*(1 + w0

def X2(z):
    return c/H0*quad(HH2, 0, z)[0]

def DL2(z):
    return (z+1)*X2(z)

def DA2(z):
    return X2(z)/(1+z)

X2=np.vectorize(X2)
```

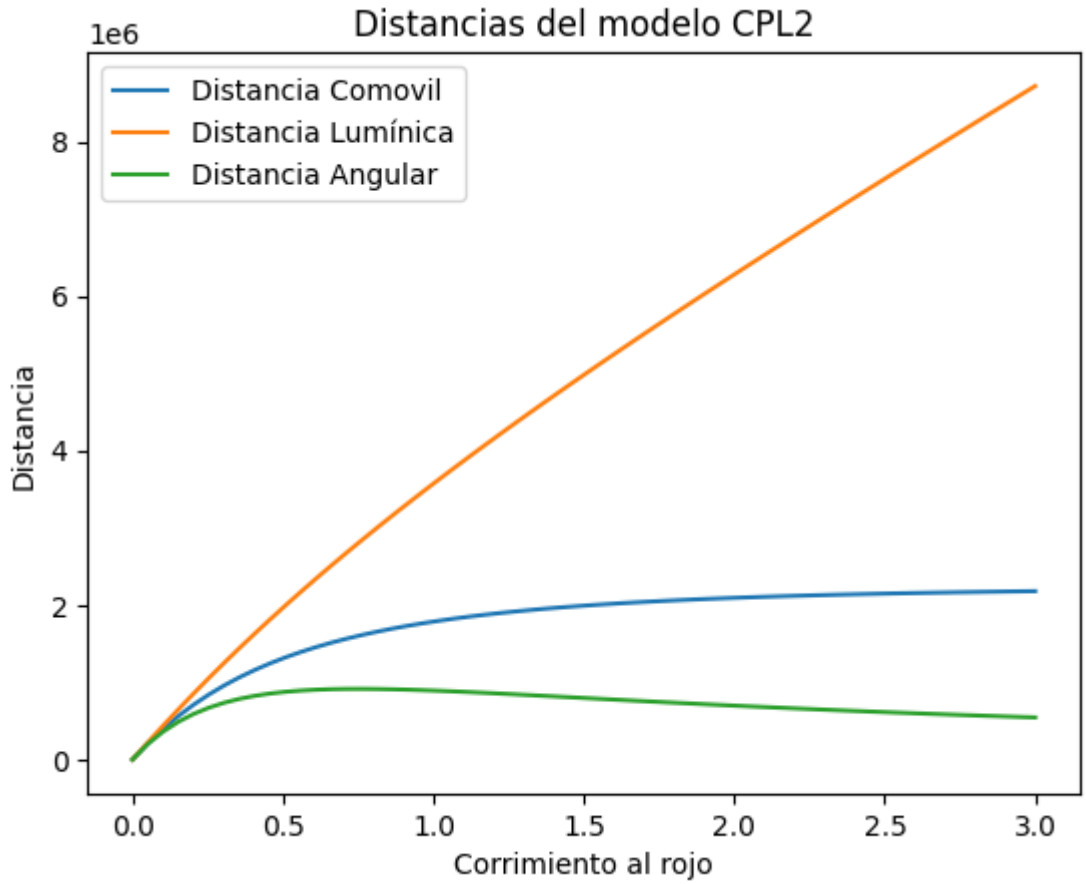
In [114]:
```
plt.plot(zz,X2(zz), label='Distancia Comovil')
plt.plot(zz,DL2(zz), label='Distancia Lumínica')
plt.plot(zz,DA2(zz), label='Distancia Angular')
plt.xlabel('Corrimiento al rojo')
plt.ylabel('Distancia')
plt.title("Distancias del modelo CPL1")
plt.legend(loc='upper left')
plt.show()
```



Modelo 2

In [119]:
```
def H3(z):
    return (M0*(z+1)**3 + K0*(z+1)**2 + L0*(z+1)**(3*(1 + w0 + wa*np.log(1 + z)/2)))*

def X3(z):
    return c/H0 * quad(H3, 0, z)[0]

def DL3(z):
    return (z+1)*X3(z)

def DA3(z):
    return X3(z)/(1+z)

X3=np.vectorize(X3)
```

In [120]:
```
plt.plot(zz,X3(zz), label='Distancia Comovil')
plt.plot(zz,DL3(zz), label='Distancia Lumínica')
plt.plot(zz,DA3(zz), label='Distancia Angular')
plt.xlabel('Corrimiento al rojo')
plt.ylabel('Distancia')
plt.title("Distancias del modelo CPL2")
plt.legend(loc='upper left')
plt.show()
```

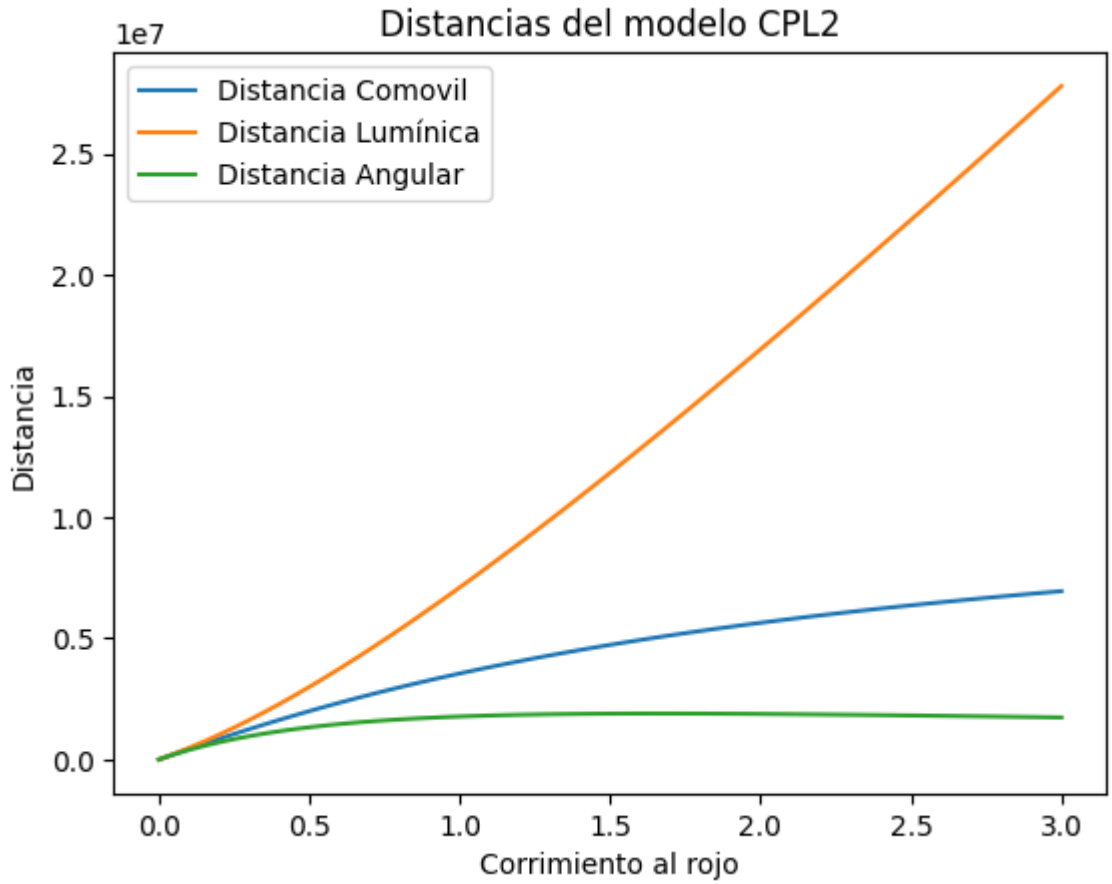


Modelo CDM Polinomial

In [121]:
```
OmP1 = -0.2
OmP2 = 0.2
```

In [122]:
```
def H4(z):
    return (M0*(z + 1)**3 + (OmP1 + K0)*(z + 1)**2 + OmP2*(z + 1) + (L0 - OmP1 - OmP2

def X4(z):
    return c/H0 * quad(H4, 0, z)[0]

def DL4(z):
    return (z+1)*X4(z)

def DA4(z):
    return X4(z)/(1+z)

X4=np.vectorize(X4)
```

In [123]:
```
plt.plot(zz,X4(zz), label='Distancia Comovil')
plt.plot(zz,DL4(zz), label='Distancia Lumínica')
plt.plot(zz,DA4(zz), label='Distancia Angular')
plt.xlabel('Corrimiento al rojo')
plt.ylabel('Distancia')
plt.title("Distancias del modelo CPL2")
plt.legend(loc='upper left')
plt.show()
```



Tiempos del universo

In [142]:
```
## Para este ejercicio se considera cero la densidad de radiación.
aa = 979.6825
H00=[50,70,90]
```

Modelos ΛCDM

Modelo 1 ($\Omega_{m,0} = 1, \Omega_{\Lambda,0} = 0$)

In [156]:
```
M00,L00,K00=1,0,0

def I1_1(x):
    t= x/(np.sqrt(M00*x + L00*x**4 + K00*x**2))
    return t
IE1_1= quad(I1_1, 0, 1)
t1_11 = aa/H00[0]*float(IE1_1[0])
t1_12 = aa/H00[1]*float(IE1_1[0])
t1_13 = aa/H00[2]*float(IE1_1[0])

print(t1_11,t1_12,t1_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

Modelo 2 ($\Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0$)

In [165]:
```
M00,L00,K00=0.3,0,0.7

def I1_2(x):
    t= x/(np.sqrt(M00*x + L00*x**4 + K00*x**2))
    return t
IE1_2= quad(I1_2, 0, 1)
t1_21 = aa/H00[0]*float(IE1_2[0])
t1_22 = aa/H00[1]*float(IE1_2[0])
t1_23 = aa/H00[2]*float(IE1_2[0])

print(t1_21,t1_22,t1_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

Modelo 3 ($\Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0.7$)

In [163]:
```
M00,L00,K00=0.3, 0.71, -0.01

def I1_3(x):
    t= x/(np.sqrt(M00*x + L00*x**4 + K00*x**2))
    return t
IE1_3= quad(I1_3, 0, 1)
t1_31 = aa/H00[0]*float(IE1_3[0])
t1_32 = aa/H00[1]*float(IE1_3[0])
t1_33 = aa/H00[2]*float(IE1_3[0])

print(t1_31,t1_32,t1_33)
```

```
18.950705520202725 13.536218228716232 10.528169733445958
```

In [177]:
```
pd1=pd.DataFrame()
pd1['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd1['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd1['H_0=50']=[t1_11,t1_21,t1_31]
pd1['70']=[t1_12,t1_22,t1_32]
pd1['90']=[t1_13,t1_23,t1_33]

pd1.head()
```

Out[177]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	18.950706	13.536218	10.528170

Modelos CLP

Modelo 1-1 ($w_0 = -1.5, w_a = -0.5$)

In [188]:
```
M00,L00,K00=1,0,0
w0,wa=-1.5,-0.5

def I2_1(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE2_1= quad(I2_1, 0, 1)
t2_11 = aa/H00[0]*float(IE2_1[0])
t2_12 = aa/H00[1]*float(IE2_1[0])
t2_13 = aa/H00[2]*float(IE2_1[0])

print(t2_11,t2_12,t2_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

In [166]:
```
M00,L00,K00=0.3,0,0.7
w0,wa=-1.5,-0.5

def I2_2(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE2_2= quad(I2_2, 0, 1)
t2_21 = aa/H00[0]*float(IE2_2[0])
t2_22 = aa/H00[1]*float(IE2_2[0])
t2_23 = aa/H00[2]*float(IE2_2[0])

print(t2_21,t2_22,t2_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

In [167]:
```
M00,L00,K00=0.3, 0.71, -0.01
w0,wa=-1.5,-0.5

def I2_3(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE2_3= quad(I2_3, 0, 1)
t2_31 = aa/H00[0]*float(IE2_3[0])
t2_32 = aa/H00[1]*float(IE2_3[0])
t2_33 = aa/H00[2]*float(IE2_3[0])

print(t2_31,t2_32,t2_33)
```

```
20.262097425540333 14.47292673252881 11.25672079196685
```

In [178]:
```
pd2=pd.DataFrame()
pd2['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd2['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd2['H_0=50']=[t2_11,t2_21,t2_31]
pd2['70']=[t2_12,t2_22,t2_32]
pd2['90']=[t2_13,t2_23,t2_33]

pd2.head()
```

Out[178]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	20.262097	14.472927	11.256721

Modelo 1-2 ($w_0 = -1, w_a = 0$)

In [189]:
```
M00,L00,K00=1,0,0
w0,wa=-1,0

def I3_1(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE3_1= quad(I3_1, 0, 1)
t3_11 = aa/H00[0]*float(IE3_1[0])
t3_12 = aa/H00[1]*float(IE3_1[0])
t3_13 = aa/H00[2]*float(IE3_1[0])

print(t3_11,t3_12,t3_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

In [169]:
```
M00,L00,K00=0.3,0,0.7
w0,wa=-1,0

def I3_2(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE3_2= quad(I3_2, 0, 1)
t3_21 = aa/H00[0]*float(IE3_2[0])
t3_22 = aa/H00[1]*float(IE3_2[0])
t3_23 = aa/H00[2]*float(IE3_2[0])

print(t3_21,t3_22,t3_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

In [170]:
```
M00,L00,K00=0.3, 0.71, -0.01
w0,wa=-1,0

def I3_3(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE3_3= quad(I3_3, 0, 1)
t3_31 = aa/H00[0]*float(IE3_3[0])
t3_32 = aa/H00[1]*float(IE3_3[0])
t3_33 = aa/H00[2]*float(IE3_3[0])

print(t3_31,t3_32,t3_33)
```

```
18.950705520202728 13.536218228716235 10.52816973344596
```

In [179]:
```
pd3=pd.DataFrame()
pd3['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd3['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd3['H_0=50']=[t3_11,t3_21,t3_31]
pd3['70']=[t3_12,t3_22,t3_32]
pd3['90']=[t3_13,t3_23,t3_33]

pd3.head()
```

Out[179]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	18.950706	13.536218	10.528170

Modelo 1-3 ($w_0 = -0.5, w_a = 0.5$)

In [171]:
```
M00,L00,K00=1,0,0
w0,wa=-0.5, 0.5

def I4_1(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE4_1= quad(I4_1, 0, 1)
t4_11 = aa/H00[0]*float(IE4_1[0])
t4_12 = aa/H00[1]*float(IE4_1[0])
t4_13 = aa/H00[2]*float(IE4_1[0])

print(t4_11,t4_12,t4_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

In [172]:
```
M00,L00,K00=0.3,0,0.7
w0,wa=-0.5, 0.5

def I4_2(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE4_2= quad(I4_2, 0, 1)
t4_21 = aa/H00[0]*float(IE4_2[0])
t4_22 = aa/H00[1]*float(IE4_2[0])
t4_23 = aa/H00[2]*float(IE4_2[0])

print(t4_21,t4_22,t4_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

In [175]:
```
M00,L00,K00=0.3, 0.71, -0.01
w0,wa=-0.5, 0.5

def I4_3(x):
    t= x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0+wa) + 1))*np.exp(-3*wa*(1 - x)))
    return t
IE4_3= quad(I4_3, 0, 1)
t4_31 = aa/H00[0]*float(IE4_3[0])
t4_32 = aa/H00[1]*float(IE4_3[0])
t4_33 = aa/H00[2]*float(IE4_3[0])

print(t4_31,t4_32,t4_33)
```

```
15.733575393712469 11.238268138366049 8.74087521872915
```

In [183]:
```
pd4=pd.DataFrame()
pd4['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd4['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd4['H_0=50']=[t4_11,t4_21,t4_31]
pd4['70']=[t4_12,t4_22,t4_32]
pd4['90']=[t4_13,t4_23,t4_33]

pd4.head()
```

Out[183]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	15.733575	11.238268	8.740875

Modelo 2-1 ($w_0 = -1.5, w_a = -0.5$)

In [190]:
```
M00,L00,K00=1,0,0
w0,wa=-1.5,-0.5

def I5_1(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE5_1= quad(I5_1, 0, 1)
t5_11 = aa/H00[0]*float(IE5_1[0])
t5_12 = aa/H00[1]*float(IE5_1[0])
t5_13 = aa/H00[2]*float(IE5_1[0])

print(t5_11,t5_12,t5_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

In [198]:
```
M00,L00,K00=0.3,0,0.7
w0,wa=-1.5,-0.5


def I5_2(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE5_2= quad(I5_2, 0, 1)
t5_21 = aa/H00[0]*float(IE5_2[0])
t5_22 = aa/H00[1]*float(IE5_2[0])
t5_23 = aa/H00[2]*float(IE5_2[0])

print(t5_21,t5_22,t5_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

In [192]:
```
M00,L00,K00=0.3, 0.71, -0.01
w0,wa=-1.5,-0.5

def I5_3(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE5_3= quad(I5_3, 0, 1)
t5_31 = aa/H00[0]*float(IE5_3[0])
t5_32 = aa/H00[1]*float(IE5_3[0])
t5_33 = aa/H00[2]*float(IE5_3[0])

print(t5_31,t5_32,t5_33)
```

```
20.285701440695522 14.489786743353944 11.269834133719735
```

In [193]:
```
pd5=pd.DataFrame()
pd5['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd5['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd5['H_0=50']=[t5_11,t5_21,t5_31]
pd5['70']=[t5_12,t5_22,t5_32]
pd5['90']=[t5_13,t5_23,t5_33]

pd5.head()
```

Out[193]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	20.285701	14.489787	11.269834

Modelo 2-2 ($w_0 = -1$, $w_a = 0$)

In [194]:
```
M00,L00,K00=1,0,0
w0,wa=-1,0

def I6_1(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE6_1= quad(I6_1, 0, 1)
t6_11 = aa/H00[0]*float(IE6_1[0])
t6_12 = aa/H00[1]*float(IE6_1[0])
t6_13 = aa/H00[2]*float(IE6_1[0])

print(t6_11,t6_12,t6_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

In [195]:
```
M00,L00,K00=0.3,0,0.7
w0,wa=-1,0


def I6_2(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE6_2= quad(I6_2, 0, 1)
t6_21 = aa/H00[0]*float(IE6_2[0])
t6_22 = aa/H00[1]*float(IE6_2[0])
t6_23 = aa/H00[2]*float(IE6_2[0])

print(t6_21,t6_22,t6_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

In [196]:
```
M00,L00,K00=0.3, 0.71, -0.01
w0,wa=-1,0

def I6_3(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE6_3= quad(I6_3, 0, 1)
t6_31 = aa/H00[0]*float(IE6_3[0])
t6_32 = aa/H00[1]*float(IE6_3[0])
t6_33 = aa/H00[2]*float(IE6_3[0])

print(t6_31,t6_32,t6_33)
```

```
18.950705520202728 13.536218228716235 10.52816973344596
```

In [197]:
```
pd6=pd.DataFrame()
pd6['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd6['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd6['H_0=50']=[t6_11,t6_21,t6_31]
pd6['70']=[t6_12,t6_22,t6_32]
pd6['90']=[t6_13,t6_23,t6_33]

pd6.head()
```

Out[197]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	18.950706	13.536218	10.528170

Modelo 2-3 ($w_0 = -0.5$, $w_a = 0.5$)

In [199]:
```
M00,L00,K00=1,0,0
w0,wa=-0.5,0.5

def I7_1(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE7_1= quad(I7_1, 0, 1)
t7_11 = aa/H00[0]*float(IE7_1[0])
t7_12 = aa/H00[1]*float(IE7_1[0])
t7_13 = aa/H00[2]*float(IE7_1[0])

print(t7_11,t7_12,t7_13)
```

```
13.062433333333333 9.330309523809523 7.256907407407407
```

In [200]:
```
M00,L00,K00=0.3,0,0.7
w0,wa=-0.5,0.5


def I7_2(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE7_2= quad(I7_2, 0, 1)
t7_21 = aa/H00[0]*float(IE7_2[0])
t7_22 = aa/H00[1]*float(IE7_2[0])
t7_23 = aa/H00[2]*float(IE7_2[0])

print(t7_21,t7_22,t7_23)
```

```
15.84721195417544 11.319437110125316 8.804006641208579
```

In [201]:
```
M00,L00,K00=0.3, 0.71, -0.01
w0,wa=-0.5,0.5

def I7_3(x):
    t=x/(np.sqrt(M00*x + K00*x**2 + (L00*x**(-3*(w0 - 0.5*wa*np.log(x)) + 1))))
    return t
IE7_3= quad(I7_3, 0, 1)
t7_31 = aa/H00[0]*float(IE7_3[0])
t7_32 = aa/H00[1]*float(IE7_3[0])
t7_33 = aa/H00[2]*float(IE7_3[0])

print(t7_31,t7_32,t7_33)
```

```
14.713166734634775 10.509404810453411 8.173981519241542
```

In [202]:
```
pd7=pd.DataFrame()
pd7['$\Omega_{m,0}$']= [1.0,0.3,0.3]
pd7['$\Omega_{\Lambda,0}$'] = [0.0,0.0,0.7]
pd7['H_0=50']=[t7_11,t7_21,t7_31]
pd7['70']=[t7_12,t7_22,t7_32]
pd7['90']=[t7_13,t7_23,t7_33]

pd7.head()
```

Out[202]:

	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
0	1.0	0.0	13.062433	9.330310	7.256907
1	0.3	0.0	15.847212	11.319437	8.804007
2	0.3	0.7	14.713167	10.509405	8.173982