BAYESIAN ANALYSIS FOR ROTATIONAL CURVES WITH ℓ-BOSON STARS AS A DARK MATTER COMPONENT

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INTRODUCTION

The study of the galaxies rotation curves is important because it allows us to infer the distribution of the matter contained in them, particularly that corresponding to dark matter.

Using Low Brightness Surface Galaxies (LBSG) rotational curves we inferred the free parameters of ℓ -boson stars as a dark matter component.

The ℓ -boson stars are numerical solutions to the non-relativistic limit of the Einstein-Klein-Gordon system, the Schrödinger-Poisson (SP) system. These solutions are parametrized by an angular momentum number ℓ = (n-1)/2, an excitation number n.

We perform a bayesian analysis by modifying SimpleMC, for the models with ℓ = 0ℓ = 1 and multistates of ℓ -boson stars. We used the Akaike information criterion (AIC), Bayesian information criterion (BIC) and the Bayes factor to compare the exited state and the multistate case with the ground state as the base model due to its simplicity.

MODEL

We take an spin 0 scalar field which is describe by a potential

$$V(\psi) = -\frac{1}{2} m_a^2 \psi^2$$

where m_a is the mass of this scalar field.

We use the non-relativistic limit of the Einstein-Klein Gordon system of equations for this scalar field as mentioned in [1], for the multistate spherical configurations the Schödinger-Poisson (SP) system

$$\frac{\hbar^2}{2m}\nabla^2_{r\ell}\psi_{n\ell 0} = \left(mV_{00} - \gamma_{n\ell 0}\right)\psi_{n\ell 0}$$

$$\nabla^2_{r\ell} V_{00} = 4\pi G m^2 \sum_{n_1,\ell_1} \left(2\ell_1 + 1\right) r^{2\ell_1} \psi^2_{n_1 \ell_1 0}$$

and by using the following expressions

$$\psi = \bar{\psi}\frac{\epsilon^2 c^2}{\hbar\sqrt{4\pi G}}, E = \bar{E}\epsilon^2 mc^2, \ r = \frac{\bar{r}}{\epsilon}\frac{\hbar}{mc}, V = \bar{V}\epsilon^2 c^2, M = \bar{M}\epsilon\frac{m_{pl}^2}{m},$$

we obtained a set of dimensionless equations.

To obtain the numerical solutions to the equation system, we add the following equation for the number of particles in each state

$$\frac{dN_\ell}{dr} = \psi^2_{n\ell 0} r^2,$$

and implemented the shooting method with a fourth order Runge Kutta.

Using the changes of variables for dimensionless solutions, we integrate to obtain the M(r) function,where

$$\rho(r) = \sum_{n,\ell} (2\ell + 1) m^2 \psi^2_{n\ell 0}.$$

Therefore we can use the expression for the circular speed with spherical symmetry to obtain the rotational curve of the system.

In particular, we choose three cases of states with zero nodes

- The ground state with ℓ = 0. Where the free parameters are m_a and

$$V_c^2(r) = 8.95 \times 10^{10} \frac{\epsilon^2}{r} \int_0^R r^2 \psi^2_{100} dr \left(\frac{km}{s}\right)^2$$

- An excited state with ℓ =1. With m_a and ϵ as free parameters.

$$V_c^2(r) = 8.95 \times 10^{10} \frac{\epsilon^2}{r} \int_0^R 3r^4 \psi^2_{210} dr \left(\frac{km}{s}\right)^2$$

- The multistate with ℓ = 0,1,2. Where the free parameters are m_a ,ϵ , $\psi_{100}(0), \psi_{210}(0)$ and $\psi_{320}(0)$

$$V_c^2(r) = 8.95 \times 10^{10} \frac{\epsilon^2}{r} \int_0^R r^2 \left(\psi^2_{100} + 3r^2\psi^2_{210} + 5r^4\psi^2_{320}\right) dr \left(\frac{km}{s}\right)^2.$$

ANALYSIS

We used flat priors for the nested sampler (NS) and modified SimpleMC that uses dynesty [2] as engine, this sampler allowed us to obtain the bayesian evidence which was used to obtain the Bayes factor allowing us to know which one of the three cases mentioned in the previous section the data favours as a dark matter component, we took the ground state as the base model due to it simplicity. In addition we calculated the Akaike and bayesian information criteria.

RESULTS

we present our results based on the radial extension of the galaxies, they are divided in two sections: r<10 kpc and r>10 kpc; adding an additional restriction for the galaxies with a linear behavior. We noticed that must of the galaxies with r<10 kpc tended to have bigger masses while the galaxies with r>10 kpc tended to lighter masses.

The dashed line represents the needed to have a cut-off in the power spectrum [3]. The grey band represents the bounds for the mass found in [4]

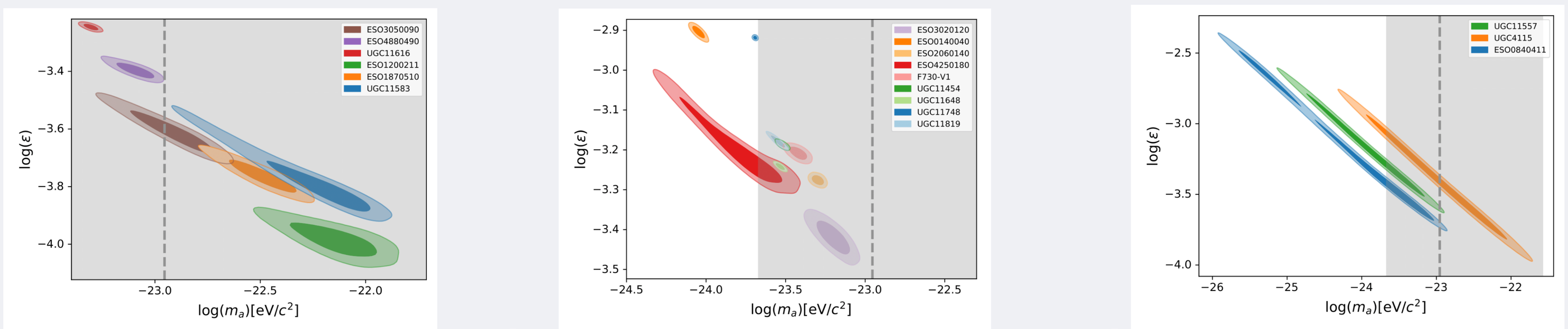

FIG 1. Contour plots for the parameter estimation of the ground state case in logarithmic scale.

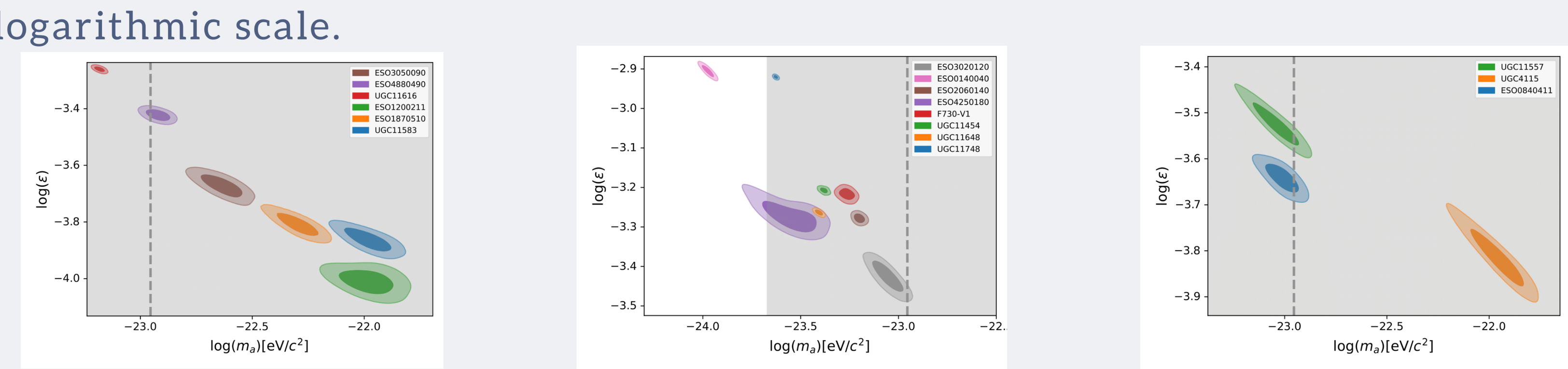

FIG 2. Contour plots for the parameter estimation of the excited case in logarithmic scale.

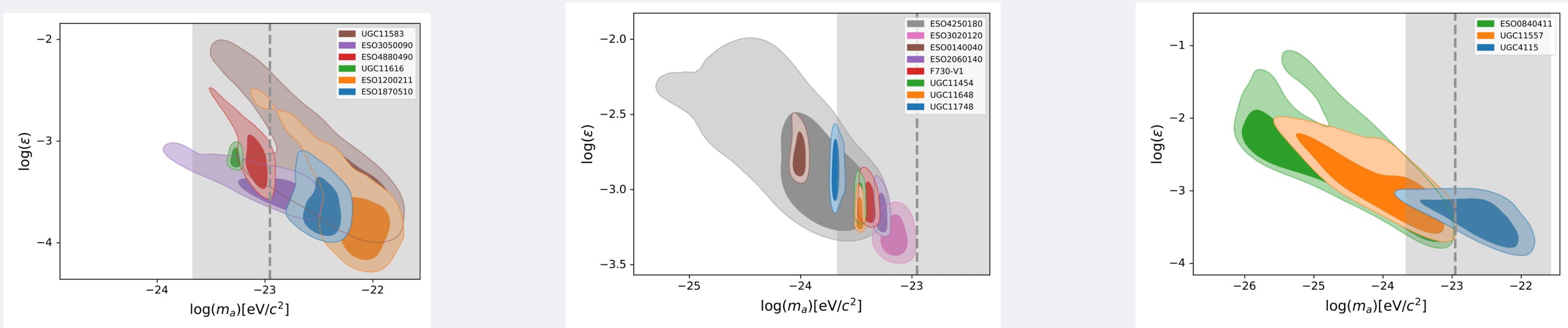

FIG 3. Contour plots for the parameter estimation of the excited case in logarithmic scale.

FIG 4. Left: Triangle plot for the galaxy ESO3020120. Right: Rotational curve for the same galaxy. The contour lines indicate the accuracy of the parameter estimation at 1σ and 2σ as the grey color bar shows.The blue dark line indicates the resulting rotational curve.

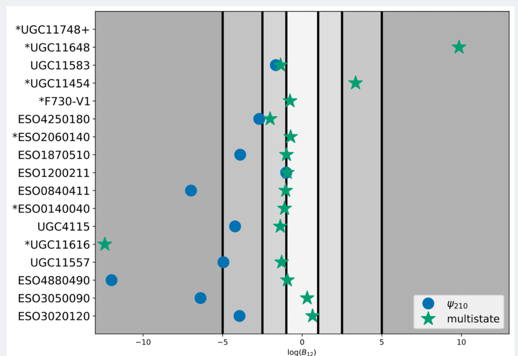

FIG 5. Bayes Factor for the comparisons between the ground state case as the base model and the other two cases. The shade regions indicates the evidence strength . Galaxies marked with a * have a smaller Bayes factor value and galaxies with a + have a bigger value.

CONCLUSIONS

This work analyzed the viability of ℓ -boson stars as a dark matter component in the rotational curves, using bayesian statistics tools such as the nested sampler and the Bayes factor, together with information criteria to know which case the data favours, being the multistate case.

Seventeen LBSG were analyzed taking into account three cases of boson stars, ground state, which was taken as the base model to calculate the Bayes factor, a single excited state and multistates. Where all galaxies prefer to be made up of multistates as FIG 5 suggests.

it is important to mention that by adding more states and therefore, increment the free parameters, the scalar field mass tends to become slightly bigger than the ground state.

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