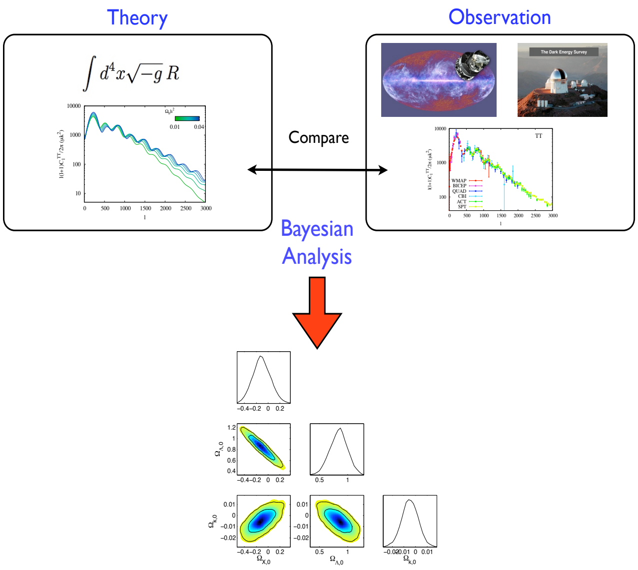
Updated Cosmology

with Python



José-Alberto Vázquez

ICF-UNAM / Kavli-Cambridge

In progress

August 12, 2017

Homework

1.- We start by assuming the components of the Universe behave as perfect fluids and hence described by a barotropic equation of state $p_i = (\gamma_i - 1)\rho_i c^2$, where γ_i describes each fluid: radiation ($\gamma_r = 4/3$), baryonic and dark matter ($\gamma_m = 1$), and dark energy in the form of cosmological constant ($\gamma_\Lambda = 0$). Once we introduce the dimensionless *density parameters*, defined as

$$\Omega_i = \frac{\kappa_0}{3H^2}\rho_i, \tag{1}$$

a) Show that the continuity equations (for all the fluids i) can be written as a dynamical system with the following form:

$$\Omega_i' = 3(\Pi - \gamma_i)\Omega_i, \tag{2}$$

with $\Pi = \sum_i \gamma_i \Omega_i$, and prime notation means derivative with respect to the e-fold parameter $N = \ln(a)$.

b) Also, show that the Friedmann equation becomes a constraint for the density parameters at all time $\sum_i \Omega_i = 1$.

c) Considering the initial conditions ($a = 1$) $\Omega_{r,0} = 10^{-4}$, $\Omega_{m,0} = 0.3$, $\Omega_{k,0} = -0.01$, $H_0 = 68\mathrm{kms}^{-1}\mathrm{Mpc}$, with cosmological constant, solve explicitly the dynamical system (2) [hint: use odeint], along with the Friedmann constraint to get the following plot [hint: use matplotlib].

d) The deceleration parameter is computed in terms of the contents of the universe, as $q = \frac{1}{2}\sum_i \Omega_i(1 + 3w_i)$. Use the solutions from above to plot $q(z)$, where $1 + z = 1/a$.

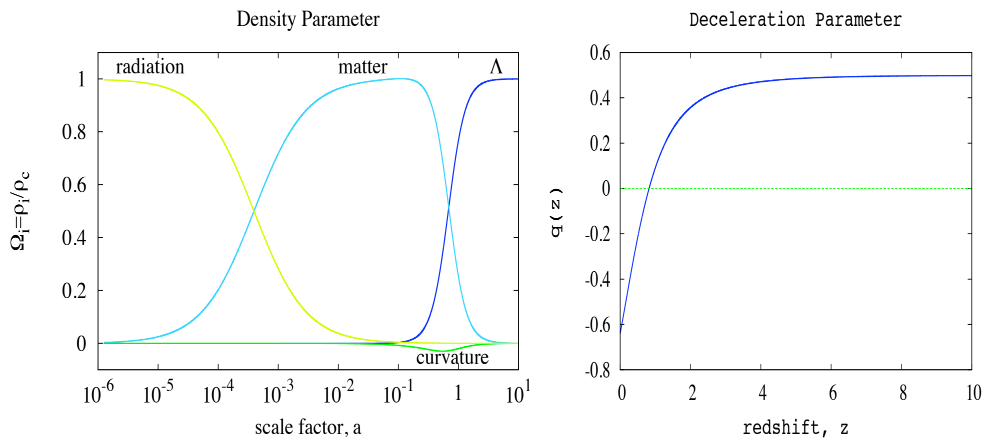


Figure 1: he evolution of the density parameters $\Omega_i(a)$. Deceleration parameter $q(z)$ as a function of redshift z for a multi-fluid universe. Notice that the universe is currently accelerating ($q(z = 0) < 0$).

2) A step further to the standard model is to consider the dark energy being dynamic, where the evolution of its EoS is usually parameterised. A commonly used form of $w(z)$ is to take into account the next contribution of a Taylor expansion in terms of the scale factor $w(a) = w_0 + (1-a)w_a$ or in terms of redshift $w(z) = w_0 + \frac{z}{1+z}w_a$; we refer to this model as CPL. The parameters w_0 and w_a are real numbers such that at the present epoch $w|_{z=0} = w_0$ and $dw/dz|_{z=0} = -w_a$; we recover ΛCDM when $w_0 = -1$ and $w_a = 0$.

a) Show the Friedmann equation for the CPL parameterisation turns out to be:

$$\frac{H(z)^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + (1-\Omega_{m,0}-\Omega_{k,0})(1+z)^{3(1+w_0+w_a)}e^{-\frac{3w_a z}{1+z}}.$$

b) Repeat the same process in a), but now use the equation of state $w(z) = w_0 + w_a ln(1+z)$

Consider the Universe from the previous exercise.

The *comoving distance* d_c is defined as

$$\chi_e = c\int_t^{t_0} \frac{dt}{R(t)} = \frac{c}{R_0}\int_0^z \frac{dz}{H(z)}. \tag{3}$$

The *luminosity distance* d_L is given by

$$d_L \equiv (1+z)R_0 S_k(\chi). \tag{4}$$

The angular distance is given by

$$d_{\rm A} \equiv \frac{R_0 S_k(\chi)}{(1+z)}. \tag{5}$$

where

$$R_0 = h_0^{-1}\sqrt{-k/\Omega_{k,0}} = \frac{H_0^{-1}}{\sqrt{|\Omega_{k,0}|}}. \tag{6}$$

3) By using the initial conditions from the previous homework (but k=0), plot the Comoving distance $d_{\rm c}$, luminosity distance d_L, and angular distance $d_{\rm A}$ for the ΛCDM model (see figure below).

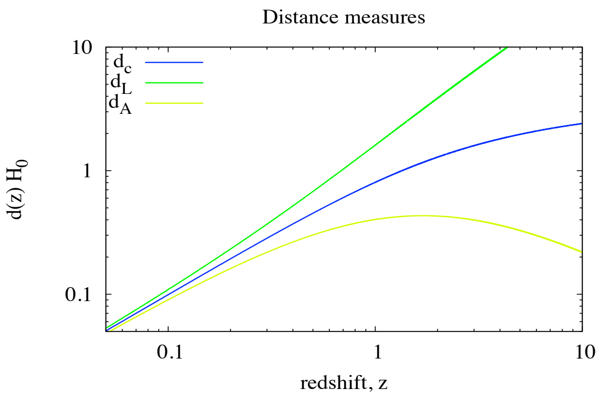


Figure 2: Comoving distance $d_{\rm c}$, luminosity distance d_L, and angular distance $d_{\rm A}$ for a universe filled with the same constituents.

b) Now using the CPL parameterisation, plot these three distances for $[w_0 = 0.9, w_a = 0.5]$ and $[w_0 = -1.1, w_a = -0.5]$.

c) Repeat the same process in b), but now use the equation of state $w(z) = w_0 + w_a ln(1+z)$ (with same combinations of w_0 and w_a).

4) As part of some models that allow deviations from ΛCDM we also use the polynomial-CDM model, that can be thought as a parameterisation of the Hubble function. This model has the following Friedmann equation:

$$\frac{H(z)^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + (\Omega_{1,0}+\Omega_{k,0})(1+z)^2 + \Omega_{2,0}(1+z)^1 + (1-\Omega_{m,0}-\Omega_{1,0}-\Omega_{2,0}-\Omega_{k,0}),$$

where $\Omega_{1,0}$ and $\Omega_{2,0}$ are two additional parameters, which within the ΛCDM both of them remain absent ($\Omega_{1,0} = 0$ and $\Omega_{2,0} = 0$). Nevertheless, $\Omega_{2,0}$ could be interpreted as a 'missing matter' component introduced to allow a symmetry that relates the big bang to the future conformal singularity [see JCAP09(2012)020].

5) By using the initial conditions from the previous homework (but k=0), plot the Comoving distance $d_{\rm c}$, luminosity distance d_L, and angular distance $d_{\rm A}$ for $[\Omega_{1,0} = 0.2, \Omega_{2,0} = -0.2]$ and $[\Omega_{1,0} = -0.2, \Omega_{2,0} = 0.2]$.

6) As a final step, use the exercise 3) b) (CPL, with the two combinations of parameters) to plot the data from Cosmic Chronometers ($H(z)$), Supernovae (d_L), BAO (d_H, d_M, d_V) along their distances.

Extra: Do the same with $f\sigma_8$ data.