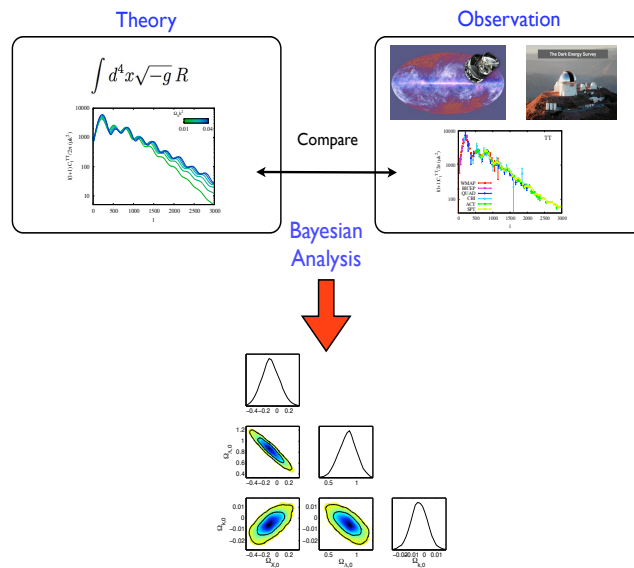


Updated Cosmology

with Python



José-Alberto Vázquez

ICF-UNAM / Kavli-Cambridge

In progress

August 12, 2017

Homework

Using the FRW metric.

1.- Show that adding Γ_{abc} to Γ_{bac} gives

$$g_{ab,c} = \Gamma_{abc} + \Gamma_{bac}. \quad (1)$$

2. Show that the derivative of the determinant g , of g_{ab} , is

$$\begin{aligned} g_{,c} &= gg^{ab}g_{ab,c}, \\ &= 2g\Gamma^a_{ac}. \end{aligned} \quad (2)$$

3. Show the validity of the **1st Bianchi identity**

$$R_{abcd} + R_{adbc} + R_{acdb} = 0, \quad (3)$$

and **2nd Bianchi identity**

$$R_{abcd;e} + R_{abde;c} + R_{abec;d} = 0. \quad (4)$$

4. Show that

$$R^a_{acd} = 0. \quad (5)$$

5. Show that $\nabla_a G^{ab} = 0$. with $G^{ab} \equiv R^{ab} - \frac{1}{2}g^{ab}R$.

6. Show the conservation of the energy-momentum tensor, $\nabla_\mu T^\mu_\nu = 0$, leads ($\nu = 0$) to the *continuity equation*:

$$\dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + \frac{p}{c^2}\right) = 0. \quad (6)$$

7. For a Schwarzschild black hole of mass M , show the Kretschmann scalar is

$$K = \frac{48G^2M^2}{c^4r^6}.$$