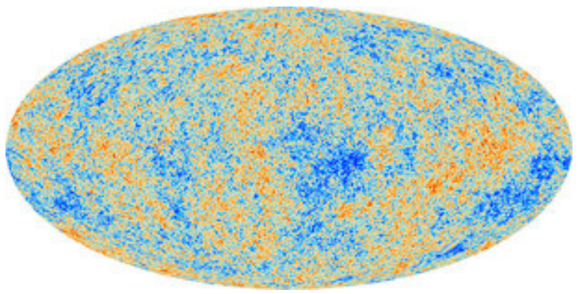
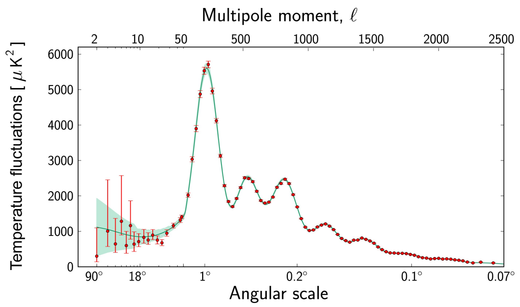
CMB Perturbations





J. Alberto Vázquez

ICF - UNAM

Perturbaciones

June 11-13, 2018

VI TALLER DE GRAVITACIÓN Y COSMOLOGÍA
ICF-UNAM, Cuernavaca, Morelos, 2 y 3 de agosto, 2018
Segunda Circular

26/05/2018

El objetivo del presente taller es servir como foro para discutir características y consecuencias de los trabajos recientes en los temas de gravitación, cosmología y áreas afines. Este taller también pretende establecer nuevas colaboraciones entre los participantes que trabajan en dichas áreas ampliando con ello las redes de investigación dentro de la comunidad científica mexicana.

PARTICIPANTES

La inscripción al evento sigue abierta para investigadores y estudiantes que contribuyan al taller. Por el momento tenemos confirmada la participación de los siguientes investigadores.

MIGUEL ASPEITIA
NORA BRETON
KAREN CABALLERO MORA
JOSÉ ANTONIO GONZÁLEZ CERVERA
FRANCISCO S. GUZMÁN
ALFREDO HERRERA AGUILAR
GERMAN IZQUIERDO
ANDRÉS PLAZAS

ESTUDIANTES

Los estudiantes interesados en participar deberán estar inscritos en un programa de posgrado, ser estudiantes regulares y enviar carta de apoyo (recomendación) de su supervisor.

Enviar resumen de plática (dado el caso) y carta de recomendación a más tardar el día viernes 8 de junio de 2018 a Juan Carlos Hidalgo al correo hidalgo@fis.unam.mx

HOSPEDAJE PARA ESTUDIANTES Y POSTDOCS

Habrá apoyo en forma de hospedaje y alimentos para estudiantes y postdocs. Se dará preferencia a quienes presenten plática corta.

SEDE DEL EVENTO

La sede del Taller será el auditorio del Instituto de Ciencias Físicas de la Universidad Nacional Autónoma de México, ubicado en el Campus de la Universidad Autónoma del Estado de Morelos, en la ciudad de Cuernavaca, Morelos.

Outline

Cosmic Microwave Background

The Hot Big Bang:

Recombination, Decoupling, Last Scattering

Black body radiation

Boltzmann equation

Temperature, Polarization,

Line of sight strategy

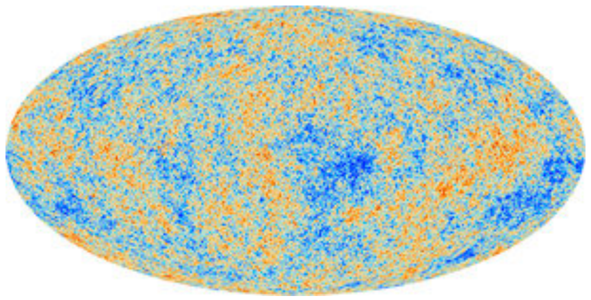
Perturbations — Talacha —

CMB Power Spectrum

Acoustic peaks
Codes
Observations

What else, Running, Non-gaussianity, Primordial Gravitational waves …

Motivation



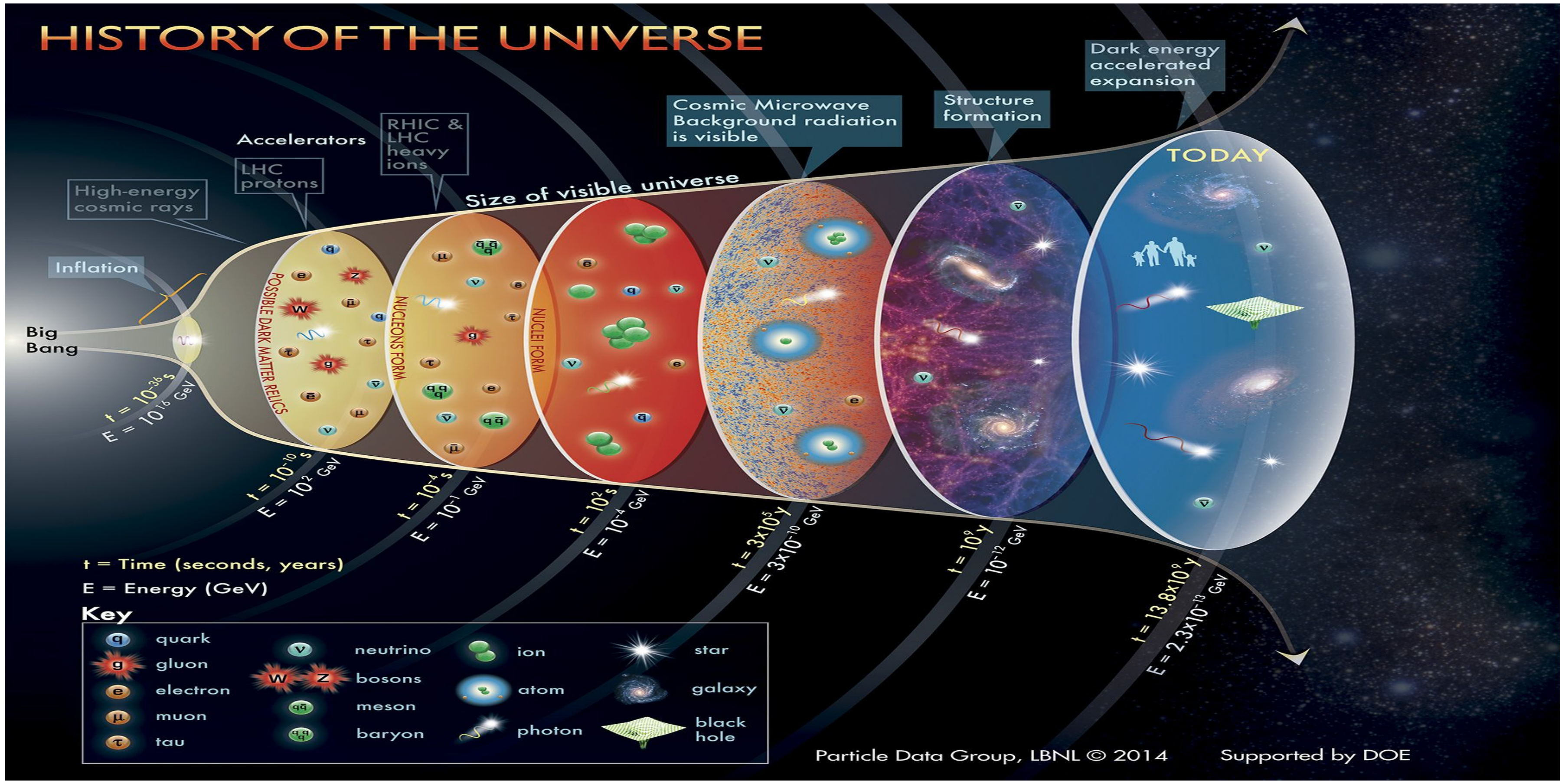
A must do!

- The cosmic microwave background (CMB) is the **thermal radiation left over** from the "Big Bang", also known as **"relic radiation"**.
- The CMB is a **snapshot of the oldest light** in our Universe, imprinted on the sky when the Universe was just **380,000 years old,** dating to the epoch of **recombination**.
- With a traditional **optical telescope**, the space between stars and galaxies is **completely dark**. However, a sufficiently sensitive radio telescope shows a faint background glow, almost exactly the **same in all directions**. This glow is strongest in the **microwave region**.

Motivation

It shows **tiny temperature fluctuations** that correspond to regions of slightly different densities, **representing the seeds of all future structure**:

the stars and galaxies of today.



The Hot Big Bang

The Hot Big Bang

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

The Hot Big Bang

- Once Big Bang Nucleosynthesis is over, at time t ~ 300s and temperature T ~ 8×10^8K, the **Universe is a thermal bath** of photons, protons, electrons, in addition to neutrinos and the unknown dark matter particle(s).

The key to understanding the thermal history of the universe is the comparison between the **rate of interactions Γ** and the **rate of expansion H**.

- $\Gamma \gg H$, Local thermal equilibrium is then reached **before** the effect of **the expansion becomes relevant.**
- As the universe cools, the **rate of interactions may decrease** faster than the expansion rate
- At $\Gamma \sim H$ the **particles decouple** from the thermal bath.

Different particle species may have different interaction rates and so may **decouple at different times**.

Fermi-Dirac (+) and Bose-Einstein (-)

$$f(p) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

For T<E?

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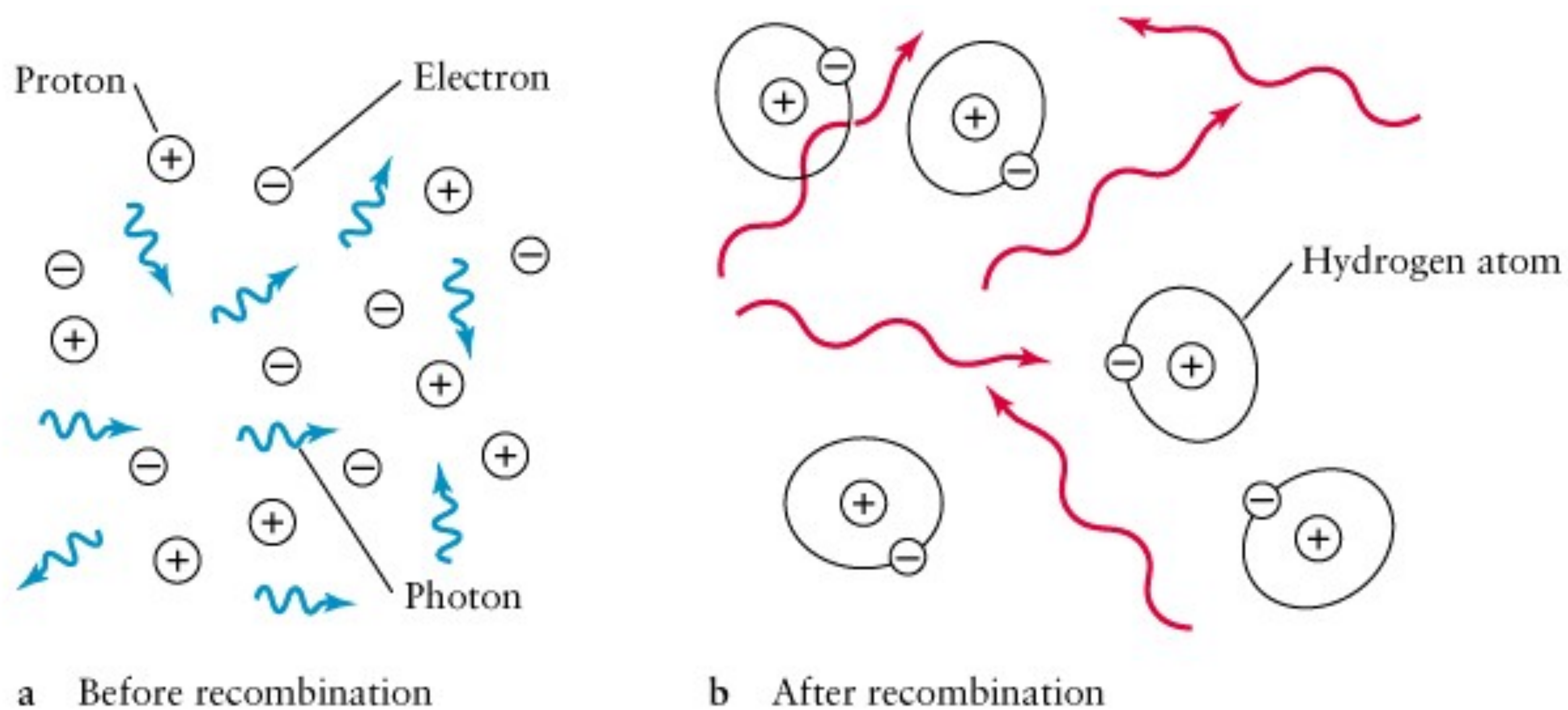
Recombination

Photons were tightly **coupled to the electrons** via Compton scattering, which in turn strongly interacted with protons via Coulomb scattering.

When the temperature became low enough, **the electrons and nuclei combined** to form neutral atoms (**recombination**), and the **density of free electrons fell sharply**.



Diff Compton Vs Thompson ?

Why recom?

Saha equation

T >1eV, when **baryons and photons were still in equilibrium** through electromagnetic reactions such as

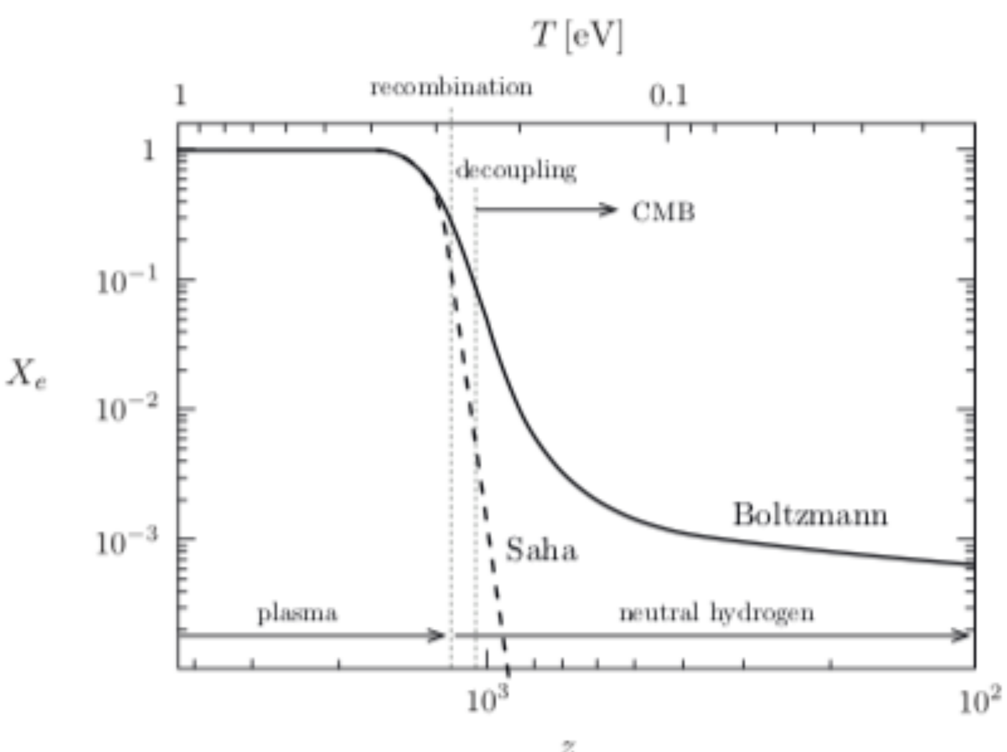
$$e^- + p^+ \rightleftharpoons H + \gamma$$

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right)$$

We wish to follow the **free electron fraction** defined as the ratio

$$X_e \equiv \frac{n_e}{n_b}$$

$$\left(\frac{1-X_e}{X_e^2}\right)_{\rm eq} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}$$



The Saha approximation **correctly identifies the onset of recombination**, but it is clearly **insufficient** if the aim is to determine the relic density of electrons after freeze-out.

m_i ? μ_γ ?

$$\left(\frac{1-X_e}{X_e^2}\right)_{\rm eq} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}$$

Recombination

Let us define the **recombination temperature T_{rec}** as the temperature where $X_e = 10^{-1}$, i.e. when **90% of the electrons have combined** with protons to form hydrogen.

$$T_{\rm rec} \approx 0.3eV \simeq 3600K.$$

Using $T_{\rm rec} = T_0(1+z_{\rm rec})$, with $T_0 = 2.7K$, gives the r**edshift of recombination**: $z_{\rm rec} \approx 1320$

Since matter-radiation equality is at $z_{eq} \simeq 3500$, then **recombination occurred in the matter-dominated era**. Using $a(t) = (t/t_0)^{2/3}$, the time of recombination

$$t_{\rm rec} = \frac{t_0}{(1+z_{\rm rec})^{3/2}} \sim 290\ 000yrs$$

zeq ?

Recombination **was not an instantaneous** process but proceeded relatively quickly nevertheless, with the fractional ionisation decreasing from X = 0.9 to X = 0.1 over a time interval **Δt ~ 70 000yrs.**

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Photon Decoupling

Photons are most strongly coupled to the primordial plasma through their interactions with electrons, through Thomson scattering

$$e^- + \gamma \rightleftharpoons e^- + \gamma$$

Thomson scattering is that it **introduces polarization** along the direction of motion of the electron

The **mean free path for photons** (the mean distance travelled between scatterings) is $\lambda = \frac{1}{n_e \sigma_T},$

and therefore the **interaction rate** at which a photon undergoes scattering $\Gamma_\gamma \approx n_e \sigma_T,$

Γ_γ decreases as the density of free electrons drops, and hence **photons and electrons decouple** when

$$\Gamma_\gamma(T_{\rm dec}) \sim H(T_{\rm dec}).$$

$$X_e(T_{\rm dec}) T_{\rm dec}^{3/2} \sim \frac{\pi^2}{2\zeta(3)} \frac{H_0 \sqrt{\Omega_m}}{\eta \sigma_T T_0^{3/2}}.$$

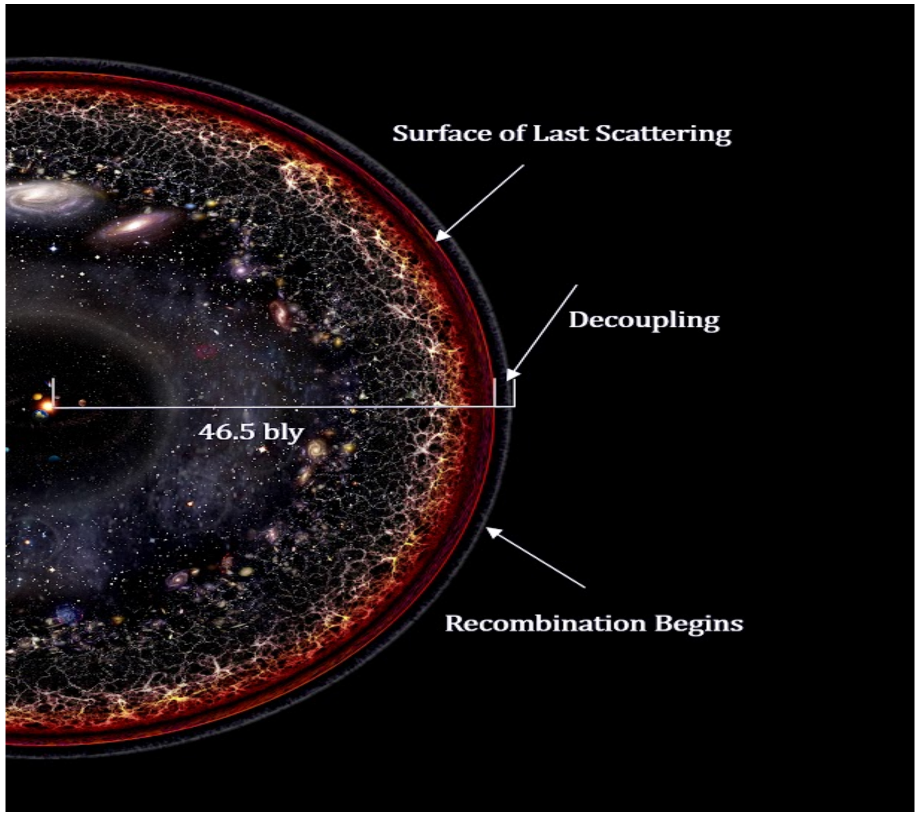
resultado?

Using the Saha equation for Xe(Tdec) $T_{\rm dec} \sim 0.27eV. \;\; z_{\rm dec} \sim 1100, \qquad t_{\rm dec} \sim 380\ 000$ yrs.

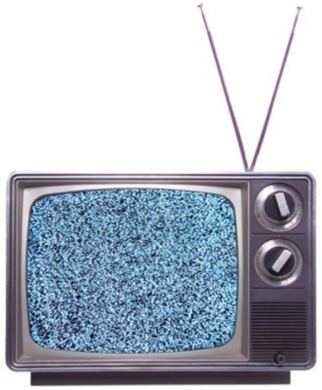
Last Scattering Surface

After their last scattering off an electron, **photons were able to travel unimpeded through the Universe**. These are the Cosmic Microwave Background **photons we receive today**, still with their blackbody distribution, now redshifted by a factor of 1100.

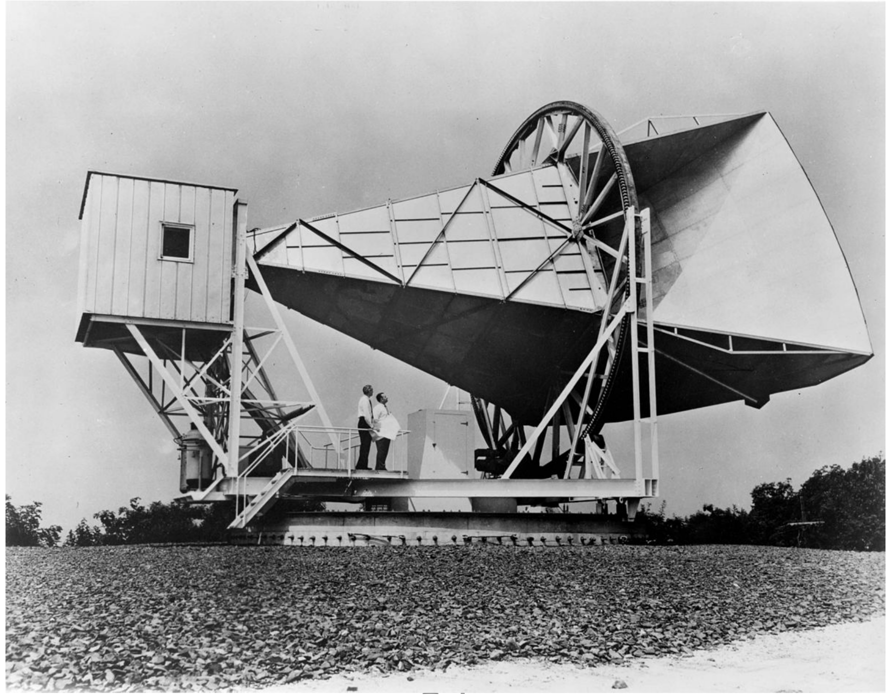
They constitute a **last scattering surface**, or more appropriately a **last scattering layer**



Isotropic CMB



- The CMB radiation was discovered in **1965** by **Arno Penzias and Robert Wilson**, while trying to identify **sources of noise in microwave** satellite communications.
- Their discovery was announced alongside the interpretation of the CMB as **relic thermal radiation** from the Big Bang by **Robert Dicke** and collaborators.
- Interestingly, the possibility of a cosmic thermal background were first entertained by **Gamow, Alpher and Herman** in **1948** as a consequence of **Big Bang nucleosynthesis**, but the idea was so beyond the experimental



nobel?

Timeline of Observations of the CMB

1941	Andrew McKellar was attempting to measure the average temperature of the interstellar medium, and reported the observation of an average bolometric temperature of 2.3 K based on the study of interstellar absorption lines.
1946	Robert Dicke predicts ".. radiation from cosmic matter" at <20 K but did not refer to background radiation[1]
1948	George Gamow calculates a temperature of 50 K (assuming a 3-billion-year old Universe), commenting it ".. is in reasonable agreement with the actual temperature of interstellar space", but does not mention background radiation.
1948	Ralph Alpher and Robert Herman estimate "the temperature in the Universe" at 5 K. Although they do not specifically mention microwave background radiation, it may be inferred.[2]
1950	Ralph Alpher and Robert Herman re-estimate the temperature at 28 K.
1953	George Gamow estimates 7 K.
1955	Émile Le Roux of the Nançay Radio Observatory, in a sky survey at λ=33 cm, reported a near-isotropic background radiation of 3 kelvins, plus or minus 2.
1956	George Gamow estimates 6 K.
1957	Tigran Shmaonov reports that "the absolute effective temperature of the radioemission background ... is 4±3K". It is noted that the "measurements showed that radiation intensity was independent of either time or direction of observation... it is now clear that Shmaonov did observe the cosmic microwave background at a wavelength of 3.2 cm"
1960s	Robert Dicke re-estimates a MBR (microwave background radiation) temperature of 40 K
1964	A. G. Doroshkevich and Igor Novikov publish a brief paper, where they name the CMB radiation phenomenon as detectable.
1964–65	Arno Penzias and Robert Woodrow Wilson measure the temperature to be approximately 3 K. Robert Dicke, P. J. E. Peebles, P. G. Roll, and D. T. Wilkinson interpret this radiation as a signature of the big bang.
1983	RELIKT-1 Soviet CMB anisotropy experiment was launched.
1990	FIRAS on COBE measures the black body form of the CMB spectrum with exquisite precision.
1992	Scientists who analyzed data from COBE DMR announce the discovery of the primary temperature anisotropy.
1999	First measurements of acoustic oscillations in the CMB anisotropy angular power spectrum from the TOCO, BOOMERANG, and Maxima Experiments.
2002	Polarization discovered by DASI.
2004	E-mode polarization spectrum obtained by the CBI.
2005	Ralph A. Alpher is awarded the National Medal of Science for his groundbreaking work in nucleosynthesis and prediction that the universe expansion leaves behind background radiation, thus providing a model for the Big Bang theory.
2006	Two of COBE's principal investigators, George Smoot and John Mather, received the Nobel Prize in Physics in 2006 for their work on precision measurement of the CMBR.

WMAP?

PLK?

JAVazquez

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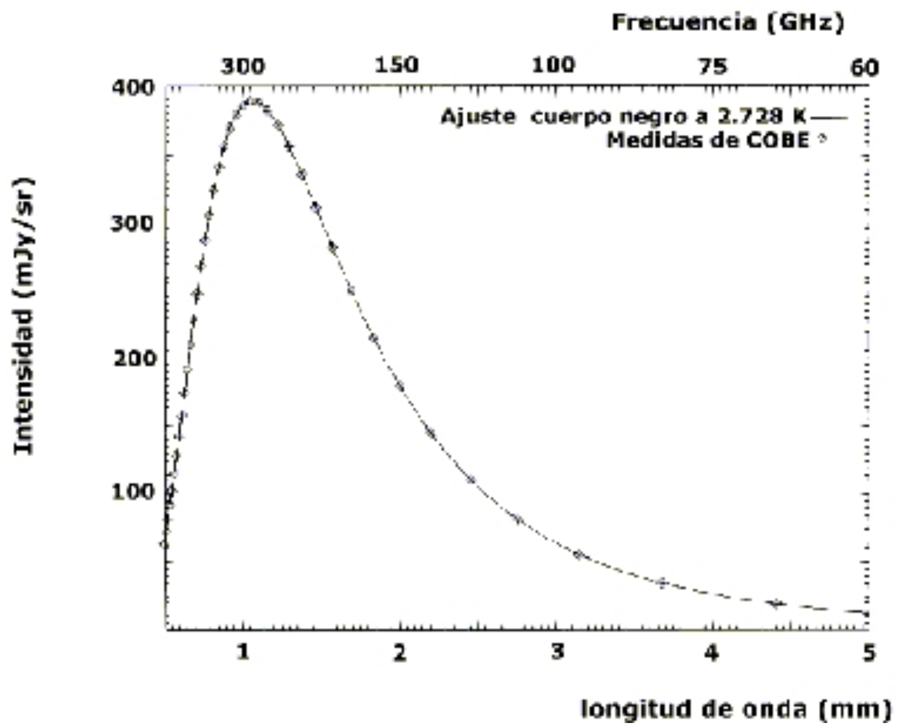


Property	Value
Temperature, $T_{\rm CMB}$	2.7255 K
Peak Wavelength, $\lambda_{\rm peak}$	0.106 cm
Number density of CMB photons, $n_{\gamma,0}$	$411\,{\rm cm}^{-3}$
Energy density of CMB photons, $u_{\gamma,0}$	$0.26\,{\rm eV\ cm}^{-3}$
Average photon energy, $\langle h\nu_{\rm CMB}\rangle$	$6.34 \times 10^{-4}\,{\rm eV}$
Photon/Baryon ratio, $1/\eta$	1.64×10^{9}

The **original detection** by Penzias and Wilson was at a wavelength of 73.5 mm, this being the wavelength of the telecommunication signals they were working with; this wavelength is **two orders of magnitude longer** than λpeak = 1.1mm of a T = 2.7255K blackbody.

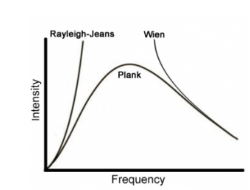
$$\langle T\rangle = \frac{1}{4\pi}\int T(\theta,\phi)\sin\theta d\theta d\phi = 2.7255 \pm 0.0006K$$

The **deviations from this mean temperature** from point to point on the sky are tiny.

$$\frac{\delta T}{T}(\theta,\phi) = \frac{T(\theta,\phi)-\langle T\rangle}{\langle T\rangle}$$

WMAP and Planck

$$\left\langle\left(\frac{\delta T}{T}\right)\right\rangle^{1/2} = 1.1\times 10^{-5}$$



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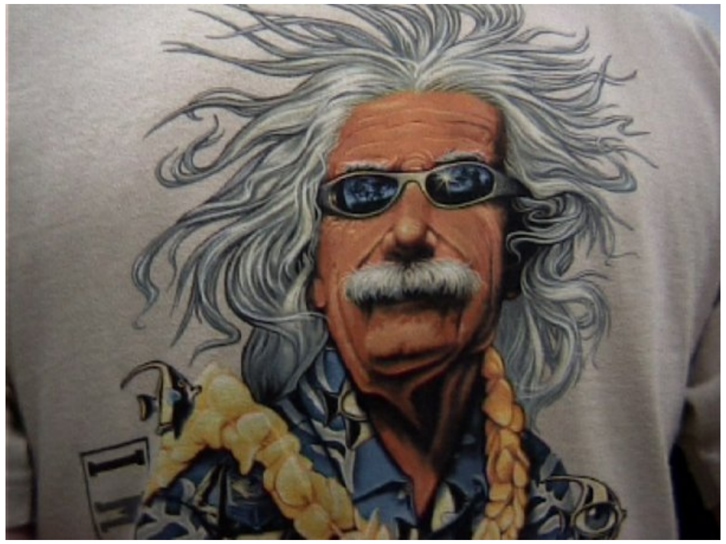
Linear Perturbations

Unperturbed



Perturbed

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Linear Perturbations

Metric perturbations $g_{\mu\nu} \longrightarrow \bar{g}_{\mu\nu} + a^2 h_{\mu\nu},$

The most general perturbation to the background metric is given by

$$h_{\mu\nu} dx^\mu dx^\nu = -2A d\eta^2 - 2B_i d\eta dx^i + 2H_{ij} dx^i dx^j.$$

Energy-momentum perturbations

$$\begin{aligned} T^0_0 &= -\bar{\rho}(1+\delta), \\ T^i_0 &= (\bar{\rho}+\bar{p})v^i \equiv q^i, \\ T^0_i &= -(\bar{\rho}+\bar{p})(v_i + B_i) \\ T^i_j &= \bar{p}[(1+\pi_L)\delta^i_j + \Pi^i_j]. \end{aligned}$$

SVT $H_{ij} = \underbrace{H_L \gamma_{ij} + \partial_{\langle i}\partial_{j\rangle} H_T}_{\text{scalar part}} + \underbrace{\partial_{(i} H^{(V)}_{j)}}_{\text{vector part}} + \underbrace{H^{(T)}_{ij}}_{\text{tensor part}},$

The Gauge Problem

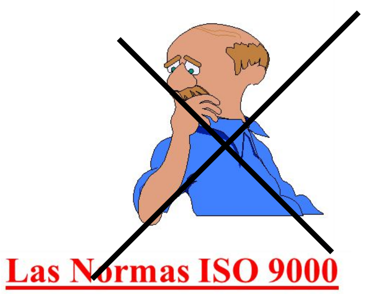
change of the time coordinate can introduce a fictitious density perturbation

$\rho(\eta) \to \rho(\eta + \xi^0(\eta, \mathbf{x})) \qquad \eta \to \eta + \xi^0(\eta, \mathbf{x})$

Gauge transformations

$$Q^{(1)} \to Q^{(1)} + \mathcal{L}_X \bar{Q},$$

$$\begin{aligned}
A &\to A - \frac{a'}{a}T - T', \\
B &\to B + L' + kT, \\
H_L &\to H_L - \frac{a'}{a}T - \frac{k}{3}L, \\
H_T &\to H_T + kL, \\
\delta &\to \delta + 3(1+w)\frac{a'}{a}T, \\
v &\to v + L', \\
\pi_L &\to \pi_L - \frac{\bar{p}'}{\bar{p}}T = \pi_L + 3(1+w)\frac{c_s^2}{w}\frac{a'}{a}T,
\end{aligned}$$



Where Ψ and Φ are **gauge-invariant quantities**, called Bardeen potentials

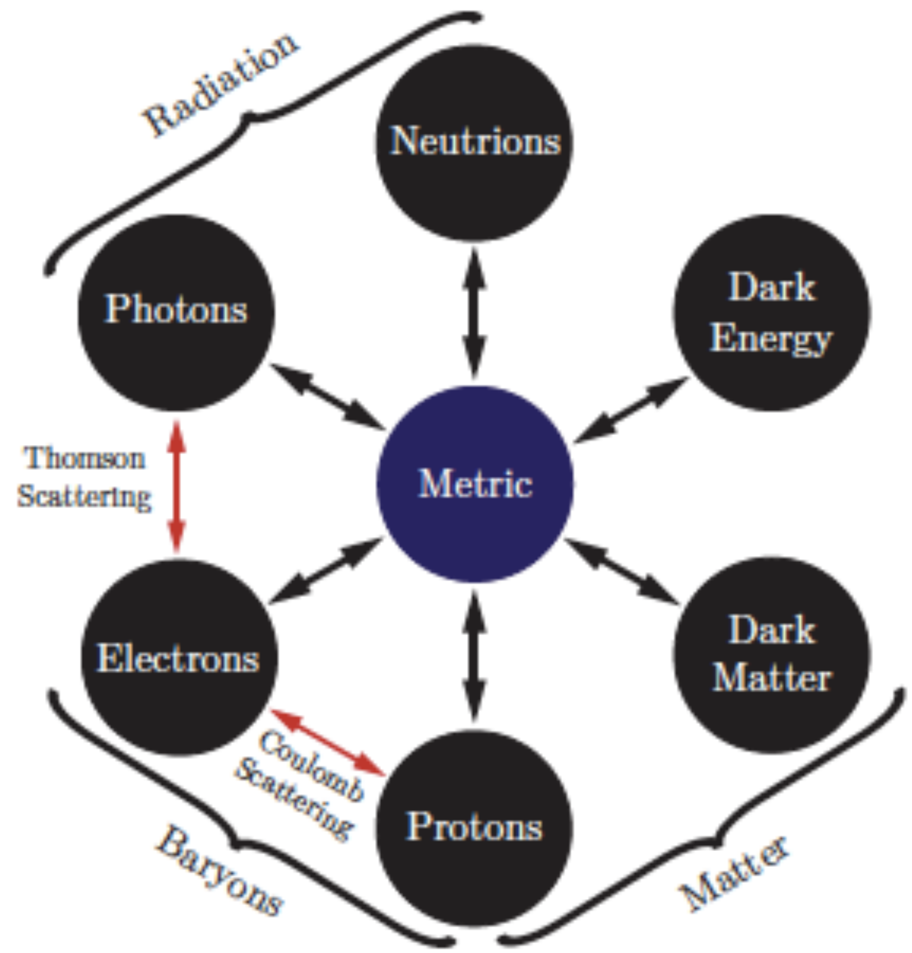
$$\Psi \equiv A - \frac{a'}{a}k^{-1}\sigma - k^{-1}\sigma', \quad \Phi \equiv H_L + \frac{1}{3}H_T - \frac{a'}{a}k^{-1}\sigma.$$

Perturbed Einstein's and conservation equation

$$\begin{aligned}
k^2\Phi + 3\frac{a'}{a}\left(\Phi' - \frac{a'}{a}\Psi\right) &= 4\pi G a^2 \bar{\rho}\delta, \\
k\left(\frac{a'}{a}\Psi - \Phi'\right) &= 4\pi G a^2 v(\bar{\rho} + \bar{p}), \\
-k^2(\Phi + \Psi) &= 8\pi G a^2 \bar{p}\Pi,
\end{aligned}$$

$$\begin{aligned}
-\delta' &= (1+w)[kv + 3\Phi'] + 3\frac{a'}{a}w\Gamma + 3\frac{a'}{a}\delta(c_s^2 - w), \\
v' &= \frac{a'}{a}(3c_s^2 - 1)v + k\Psi + \frac{kc_s^2}{1+w}\delta + \frac{kw}{1+w}\left[\Gamma - \frac{2}{3}\Pi\right],
\end{aligned}$$

The Boltzmann equation



Describes the statistical behaviour of a thermodynamic system not in a state of equilibrium

$$\frac{df}{d\eta} = C[f]$$

The Boltzmann equation

J.Santiago

$$\frac{df}{d\eta} = C[f]$$

The **distribution function** of the cosmic microwave background with temperature $\bar{T}$ is

$$\bar{f} = \left[\exp\left(\frac{E}{\bar{T}}\right) - 1\right]^{-1}.$$

We see that $\bar{f}$ depends just upon the energy E of a photon. Writing $T = T_0 a^{-1}$, we see that $\bar{f}$ is a function of aE only:

$$\bar{f}(aE) = \left[\exp\left(\frac{aE}{\bar{T}_0}\right) - 1\right]^{-1}. \qquad (3.2)$$

for observers in the unperturbed background at rest $E = -a\,p$, $\bar{f}$ depends solely of $P = a^2 p$.

Let us split the spatial momentum into its **magnitude p** and the **unit vector** of photon momentum n $\quad p^i \equiv pn^i$

$$f = f(\eta, \mathbf{x}, P, \mathbf{n})$$

The complete distribution function for each species can be split into background plus a perturbation part:

$$f(\eta, \mathbf{x}, P, \mathbf{n}) = \bar{f}(P) + F(\eta, \mathbf{x}, P, \mathbf{n}), \qquad (3.5)$$

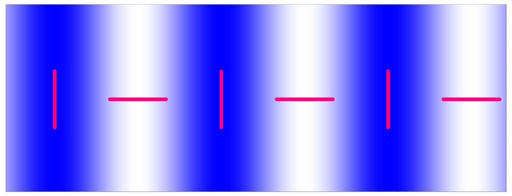
The Boltzmann equation

The **evolution of perturbations** in the universe is quantified by the Boltzmann equation:

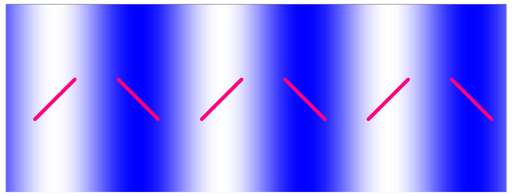
$$\left(\frac{\partial f}{\partial \eta}\right)_P + \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \eta} + \frac{\partial f}{\partial P}\frac{\partial P}{\partial \eta} + \frac{\partial f}{\partial n^i}\frac{\partial n^i}{\partial \eta} = C[f, G],$$

Relates the **effects of gravity** on the photon distribution function f to the **rate of interactions with other species**,

given by the collision term C[f,G].



To describe the **electromagnetic wave** $\boldsymbol{E} = \left(a_1 e^{i\delta_1}\boldsymbol{\epsilon}_1 + a_2 e^{i\delta_2}\boldsymbol{\epsilon}_2\right) e^{ip\boldsymbol{n}\,\boldsymbol{x} - i\omega t}.$



The **Stokes parameters** are then defined by

$$\begin{aligned} I &\equiv \langle \boldsymbol{E}\boldsymbol{E}^\star \rangle = a_1^2 + a_2^2, \\ Q &\equiv \langle \boldsymbol{E}_1\boldsymbol{E}_1^\star - \boldsymbol{E}_2\boldsymbol{E}_2^\star \rangle = a_1^2 - a_2^2, \\ U &\equiv \left\langle \left|\frac{\boldsymbol{E}_1 + \boldsymbol{E}_2}{\sqrt{2}}\right|^2 - \left|\frac{\boldsymbol{E}_1 - \boldsymbol{E}_2}{\sqrt{2}}\right|^2 \right\rangle \\ &= 2a_1 a_2 \cos(\delta_1 - \delta_2). \end{aligned}$$

spin?

The Stokes parameters can be express as **frequency-independent** fractional thermodynamic **equivalent temperatures**.

The previous distribution applies to polarization as well by simply replacing F → G (we use G to denote the linear polarization distribution function) and $f^- = f^{-\prime} \rightarrow 0$

The Boltzmann equation

$$\left(\frac{\partial f}{\partial \eta}\right)_P + \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \eta} + \frac{\partial f}{\partial P}\frac{\partial P}{\partial \eta} + \frac{\partial f}{\partial n^i}\frac{\partial n^i}{\partial \eta} = C[f, G],$$

The last term vanishes, because it is of second order in perturbation theory: $\bar{f}$ does **not depend on** n^i and hence $\partial f/\partial n^i$ is a perturbation. In addition $\partial n^i/\partial\eta$, is a **change in photon direction.**

effect?

The third term can be computed from the geodesic equation

$$\frac{\partial f}{\partial P}\frac{\partial P}{\partial \eta} = -P\bar{f}_{,P}\{i\mu k[\Phi + \Psi] + 2\Phi'\},$$

$$p^0\frac{dp^\mu}{d\eta} + \Gamma^\mu_{\alpha\beta}p^\alpha p^\mu = 0$$

The second term

$$\frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \eta} = i\mu k F(\eta, \mathbf{x}, P, \mathbf{n}).$$

Collecting the terms involving the **background only** $\left(\frac{\partial f}{\partial \eta}\right)_P = 0$

the preservation of the background black body spectrum

$$\left(\frac{\partial F}{\partial \eta}\right)_P + i\mu k F - P\bar{f}_{,P}\{i\mu k[\Phi + \Psi] + 2\Phi'\} = C[f, G]$$

Finally, making the substitution F → G, $\bar{f}'$ → 0, we get the simple evolution equation for the linear **polarization G**

$$\left(\frac{\partial G}{\partial \eta}\right)_P + i\mu k G = C_G[f, G]$$

Perturbed temperature

Writing the temperature function T in terms of the photon *brightness temperature perturbation* $\Delta \equiv \Delta T/\bar{T}$, we have

$$T(\eta, \mathbf{x}, \mathbf{n}) = \bar{T}(\eta)[1 + \Delta(\eta, \mathbf{x}, \mathbf{n})], \qquad (3.13)$$

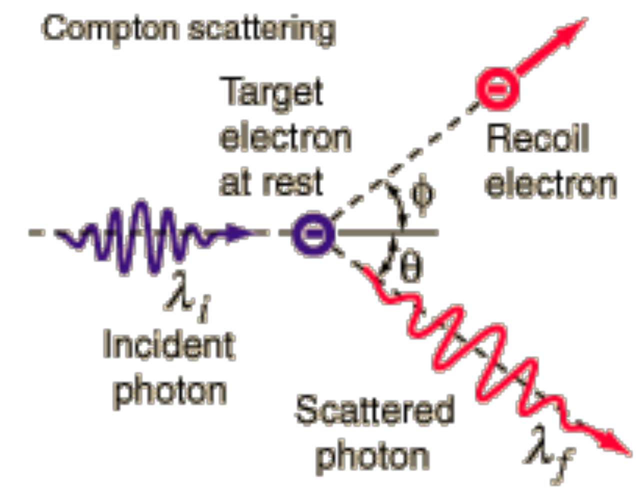
and therefore F and Δ are connected via

$$F(\eta, \mathbf{x}, P, \mathbf{n}) = -P\frac{\partial \bar{f}}{\partial P}\Delta(\eta, \mathbf{x}, \mathbf{n}). \qquad G(\eta, \mathbf{x}, P, \mathbf{n}) = -P\frac{\partial \bar{f}}{\partial P}\mathcal{Q}(\eta, \mathbf{x}, \mathbf{n}).$$

The simplify Boltzmann equation becomes

$$\Delta' + ik\mu\Delta = -i\mu k[\Phi + \Psi] - 2\Phi' + \hat{C}[f, G]$$

The Collision Term



The dominant term for the coupling of photons to the baryons is via inverse **Compton scattering**

$$e^-(\mathbf{q}) + \gamma(\mathbf{p}) \rightleftharpoons \mathbf{e}^-(\mathbf{q}') + \gamma(\mathbf{p}')$$

The amplitude can be calculated from the Feynman rules.

$$C[f,G] = a n_e \sigma_T \bar{f}_{,P} P \left\{ i\mu v_b + \Delta(\eta,\mathbf{x},\mathbf{n}) - \frac{1}{4}\int_{-1}^{1} \Delta(\eta,\mathbf{x},\mathbf{n}')[P_2(\lambda)P_2(\mu)+2]d\lambda \right.$$
$$\left. -\frac{1}{4}\int_{-1}^{1} \mathcal{Q}(\eta,\mathbf{x},\mathbf{n}')P_2(\mu)[-2\sqrt{6\pi 5_2}Y_2^0(\lambda)]d\lambda \right\}$$

The expansion of the temperature perturbation (Δ) and polarisations (Q and U), in terms of spherical harmonics $Y^{m}(n)$

$$\Delta(\eta,\mathbf{x},\mathbf{n}) = \sum_l (-i)^l \Delta_l(k,\eta) P_l(\hat{\mathbf{k}}\cdot\mathbf{n}), \qquad (Q \pm iU)(\eta,\mathbf{x},\mathbf{n}) = \sum_{l=2} (-i)^l (E_l^0 \pm iB_l^0)\sqrt{\frac{4\pi}{2l+1}}\, {}_{\mp 2}Y_l^0(\mathbf{n}),$$

$$C[f,G] = a n_e \sigma_T \bar{f}_{,P} P \left\{ i\mu v_b + \Delta(\eta,\mathbf{k},\mathbf{n}) + \frac{1}{10}\Delta_2 P_2(\mu) - \Delta_0 - \frac{\sqrt{6}}{10}[E_2 - \Delta_2] \right\}$$

The Boltzmann equation thus yields to the evolution equation of temperature perturbations

$$\Delta' + ik\mu\Delta + \kappa'\Delta = -i\mu k[\Phi + \Psi] - 2\Phi' + \kappa' \left\{ \frac{1}{4}\delta_\gamma - \Phi - i\mu v_b + \frac{1}{10}P_2(\mu)[\sqrt{6}E_2 - \Delta_2] \right\}$$

$$Q' + ik\mu Q + \kappa' Q = \frac{\kappa'}{10}\{P_2(\mu) - 1\}\left[\sqrt{6}E_2 - \Delta_2\right].$$

$\kappa' \equiv a n_e \sigma_T$ is the differential optical depth

$\mu = k^{-1}\mathbf{k}\cdot\mathbf{n}$ the direction cosine.

We have use the expressions for the first few moments of the distribution function

$$T^\mu_\nu = \int \sqrt{-g}\frac{p^\mu p_\nu}{|p_0|} f(p,x) d^3p \qquad \delta = 4\Phi + \frac{1}{\pi}\int \Delta(\mathbf{n})\mathbf{d\Omega}$$

We notice that is **not manifestly gauge-invariant**,

however by defining the **gauge invariant temperature perturbation**

$$\mathcal{M} = \Delta + 2\Phi$$

$$\mathcal{M}(\eta, \mathbf{x}, \mathbf{n}) = \sum_l (-i)^l \mathcal{M}_l(\eta, \mathbf{k}) P_l(\mathbf{n}),$$

$$\mathcal{M}' + ik\mu\mathcal{M} + \kappa'\mathcal{M} = i\mu k[\Phi - \Psi] + \kappa' \left\{ \frac{1}{4}D_g^\gamma - i\mu v_b + \frac{1}{10}P_2(\mu)\left[\sqrt{6}E_2 - \mathcal{M}_2\right] \right\}.$$

Solving ...

$$\mathcal{M}' + ik\mu\mathcal{M} + \kappa'\mathcal{M} = i\mu k[\Phi - \Psi] + \kappa'\left\{\frac{1}{4}D_g^{\gamma} - i\mu v_b + \frac{1}{10}P_2(\mu)\left[\sqrt{6}E_2 - \mathcal{M}_2\right]\right\}.$$

The procedure is as follows: For each Legendre polynomials P_l

- replace $\mathcal{M}(\eta, \mu)$ by its multipole expansion
- multiply by $P_l(\mu)$
- integrate both l.h.s. and r.h.s. of the new equation over μ : $\int_{-1}^{1} d\mu$
- use the orthogonality relation $\int_{-1}^{1} d\mu P_l(\mu) P_n(\mu) = 2\delta_{ln}/(2l+1)$

HW -0 ?

$$\begin{aligned}
\mathcal{M}_0' &= -\frac{k}{3}V_\gamma, \\
\mathcal{M}_1' &= \kappa'(V_b - V_\gamma) + k(\Psi - \Phi) + k\left(\mathcal{M}_0 - \frac{2}{5}\mathcal{M}_2\right), \\
\mathcal{M}_2' &= -\kappa'(\mathcal{M}_2 - \mathcal{C}) + k\left(\frac{2}{3}V_\gamma - \frac{3}{7}\mathcal{M}_3\right), \\
\mathcal{M}_l' &= -\kappa'\mathcal{M}_l + k\left(\frac{l}{2l-1}\mathcal{M}_{l-1} - \frac{l+1}{2l+3}\mathcal{M}_{l+1}\right), \quad l > 2,
\end{aligned}$$

$$\begin{aligned}
E_2' &= -\frac{k\sqrt{5}}{7}E_3 - \kappa'(E_2 + \sqrt{6}\mathcal{C}), \\
E_l' &= k\left(\frac{2\kappa_l}{2l-1}E_{l-1} - \frac{2\kappa_{l+1}}{2l+3}E_{l+1}\right) - \kappa' E_l, \quad l > 2.
\end{aligned}$$

Massless neutrinos follow the same multipole hierarchy as M,

however without polarisation

$$\begin{aligned}
\mathcal{N}_0' &= -\frac{k}{3}V_\nu, \\
\mathcal{N}_0' &= k(\Psi - \Phi) + k\left(\mathcal{N}_0 - \frac{2}{5}\mathcal{N}_2\right), \\
\mathcal{N}_l' &= k\left(\frac{l}{2l-1}\mathcal{N}_{l-1} - \frac{l+1}{2l+3}\mathcal{N}_{l+1}\right), \quad l > 1.
\end{aligned}$$

The Line of Sight Strategy

So usually, we are interested in **M(η0,μ).**

Inspecting, one notices that the **l.h.s** can be written as

$$\mathcal{M}' + ik\mu\mathcal{M} + \kappa'\mathcal{M} =$$

$$e^{-i\mu k\eta}e^{-\kappa(\eta)}\dot{L} \qquad \text{where} \qquad L \equiv e^{i\mu k\eta}e^{\kappa(\eta)}\mathcal{M}$$

Hence, the Boltzmann equation translates into

$$\dot{L} = e^{i\mu k\eta}e^{\kappa(\eta)}\left[i\mu k(\Phi - \Psi) + \kappa'\left(\frac{1}{4}D_g^\gamma - i\mu V_b - \frac{1}{2}(3\mu^2 - 1)\mathcal{C}\right)\right]$$

and **integrated over conformal time**

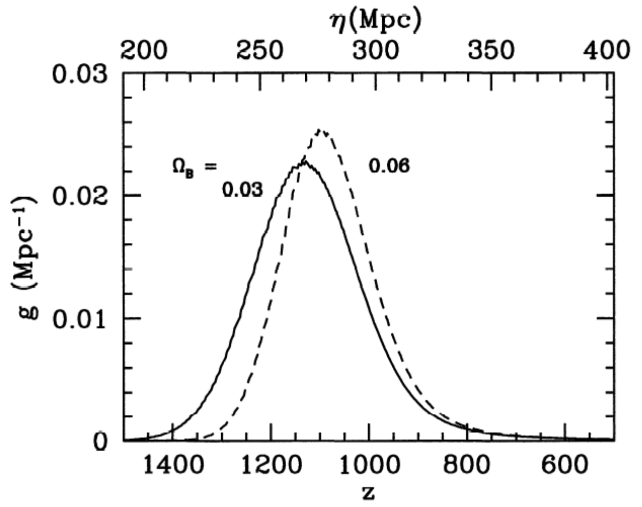
$$L(\eta_0) = \int_0^{\eta_0} d\eta e^{i\mu k\eta}e^{\kappa(\eta)}\left[i\mu k(\Phi - \Psi) + \kappa'\left(\frac{1}{4}D_g^\gamma - i\mu V_b - \frac{1}{2}(3\mu^2 - 1)\mathcal{C}\right)\right]$$

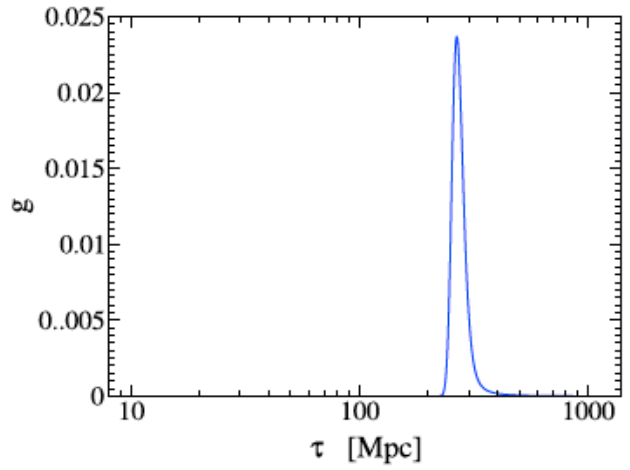
The **photon perturbation today** is given by

$$\mathcal{M}(\mu, \eta_0) = \int_0^{\eta_0} d\eta e^{i\mu k(\eta-\eta_0)}e^{\kappa(\eta)-\kappa(\eta_0)} \times \left[i\mu k(\Phi - \Psi) + \kappa'\left(\frac{1}{4}D_g^\gamma - i\mu V_b - \frac{1}{2}(3\mu^2 - 1)\mathcal{C}\right)\right] \tag{3.47}$$

The visibility function

The product $g \equiv \kappa' \exp(\kappa(\eta) - \kappa(\eta_0))$ plays an important role and is called the visibility function. Its peak defines the epoch of recombination.





$$\mathcal{M}(\mu,\eta_0) = \int_0^{\eta_0} d\eta e^{i\mu k(\eta-\eta_0)} e^{\kappa(\eta)-\kappa(\eta_0)} \times \left[i\mu k(\Phi - \Psi) + \kappa' \left(\frac{1}{4} D_g^\gamma - i\mu V_b - \frac{1}{2}(3\mu^2 - 1)\mathcal{C} \right) \right] \tag{3.47}$$

Each term in the above Equation containing factors of μ, can be integrated by parts, order to get rid of μ
Applying this procedure to all terms involving μ yields

$$\mathcal{M}(\mu,\eta_0) = \int_0^{\eta_0} e^{\imath\mu k(\eta-\eta_0)} S_T(k,\eta) d\eta$$

$$\begin{aligned} S_T &= -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi' - \Psi'] + g' \left[\frac{V_b}{k} + \frac{3}{k^2}\mathcal{C}' \right] + g'' \frac{3}{2k^2}\mathcal{C} \\ &+ g \left[\frac{1}{4} D_g^\gamma + \frac{V_b'}{k} - (\Phi - \Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}'' \right], \end{aligned}$$

$$S_T = -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi' - \Psi'] + g'\left[\frac{V_b}{k} + \frac{3}{k^2}\mathcal{C}'\right] + g''\frac{3}{2k^2}\mathcal{C} \;+\; g\left[\frac{1}{4}D_g^\gamma + \frac{V_b'}{k} - (\Phi - \Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}''\right],$$

The **density contrast $D_g{}^\gamma$** is the main contribution, driving the spectrum towards the **oscillatory behaviour**.

The **($\Phi - \Psi$) term** arises from the **gravitational redshift** when climbing out of the potential well at last scattering.

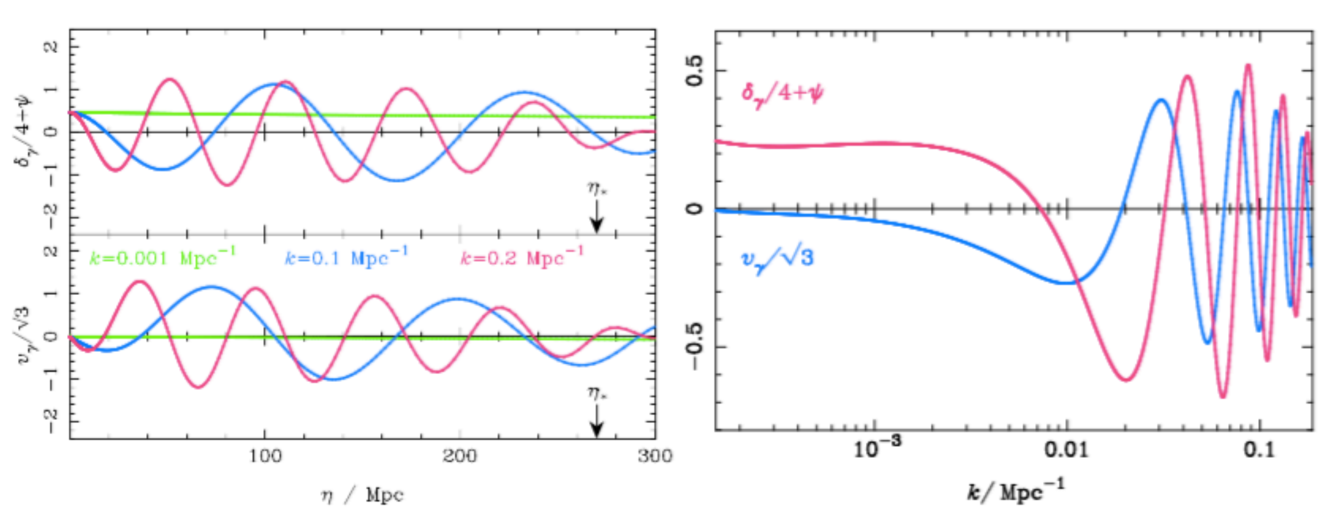
The **combination $D_g{}^\gamma/4 - (\Phi - \Psi)$** is known as the **ordinary Sachs-Wolfe effect (SW)**.

This gives the main contribution on scales that at decoupling were well outside the horizon

The **Doppler shift, V_b-term,** describes the blueshift caused by **last scattering electrons moving towards** the observer.

The term involving time derivatives of the potentials, **($\Phi' - \Psi'$)**, the **integrated Sachs-Wolfe effect (ISW)**.

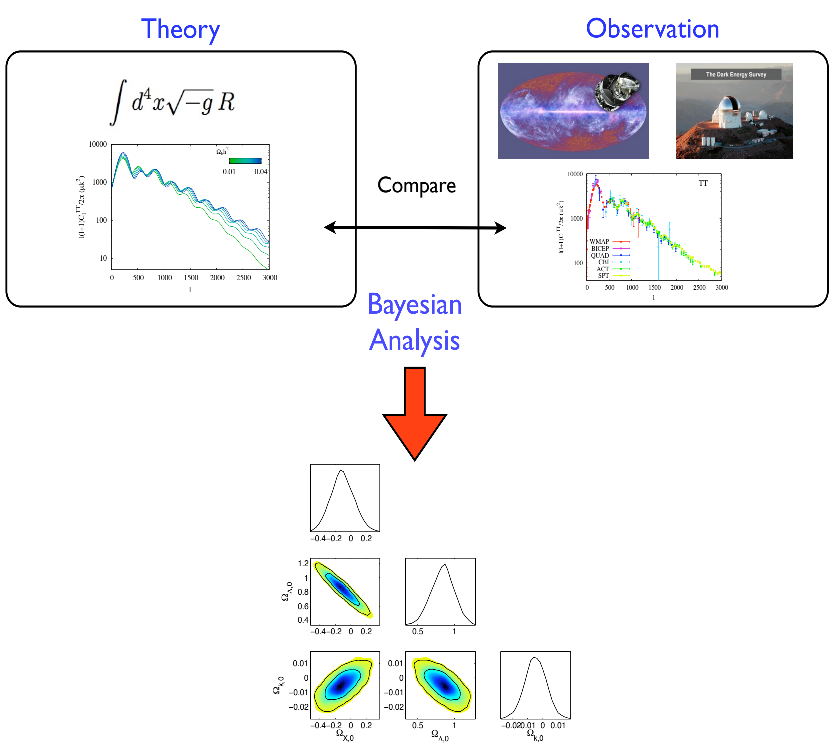
It describes the **change of the CMB photon energy** due to the **evolution of the potentials** along the line of sight.



CMB Spectrum

II & III

Updated Cosmology



José-Alberto Vázquez

ICF-UNAM / Kavli Institute for Cosmology

In progress

August 12, 2017

INSTITUTO DE CIENCIAS FÍSICAS

Outline

Cosmic Microwave Background

The Hot Big Bang:

Recombination, Decoupling, Last Scattering

Black body radiation

Boltzmann equation

Temperature, Polarization,

Line of sight strategy

Perturbations — Talacha —

CMB Power Spectrum

Acoustic peaks
Codes
Observations

What else, Running, Non-gaussianity, Primordial Gravitational waves …

VI TALLER DE GRAVITACIÓN Y COSMOLOGÍA
ICF-UNAM, Cuernavaca, Morelos, 2 y 3 de agosto, 2018
Segunda Circular

26/05/2018

El objetivo del presente taller es servir como foro para discutir características y consecuencias de los trabajos recientes en los temas de gravitación, cosmología y áreas afines. Este taller también pretende establecer nuevas colaboraciones entre los participantes que trabajan en dichas áreas ampliando con ello las redes de investigación dentro de la comunidad científica mexicana.

PARTICIPANTES

La inscripción al evento sigue abierta para investigadores y estudiantes que contribuyan al taller. Por el momento tenemos confirmada la participación de los siguientes investigadores.

MIGUEL ASPEITIA
NORA BRETON
KAREN CABALLERO MORA
JOSÉ ANTONIO GONZÁLEZ CERVERA
FRANCISCO S. GUZMÁN
ALFREDO HERRERA AGUILAR
GERMAN IZQUIERDO
ANDRÉS PLAZAS

ESTUDIANTES

Los estudiantes interesados en participar deberán estar inscritos en un programa de posgrado, ser estudiantes regulares y enviar carta de apoyo (recomendación) de su supervisor.

Enviar resumen de plática (dado el caso) y carta de recomendación a más tardar el día viernes 8 de junio de 2018 a Juan Carlos Hidalgo al correo hidalgo@fis.unam.mx

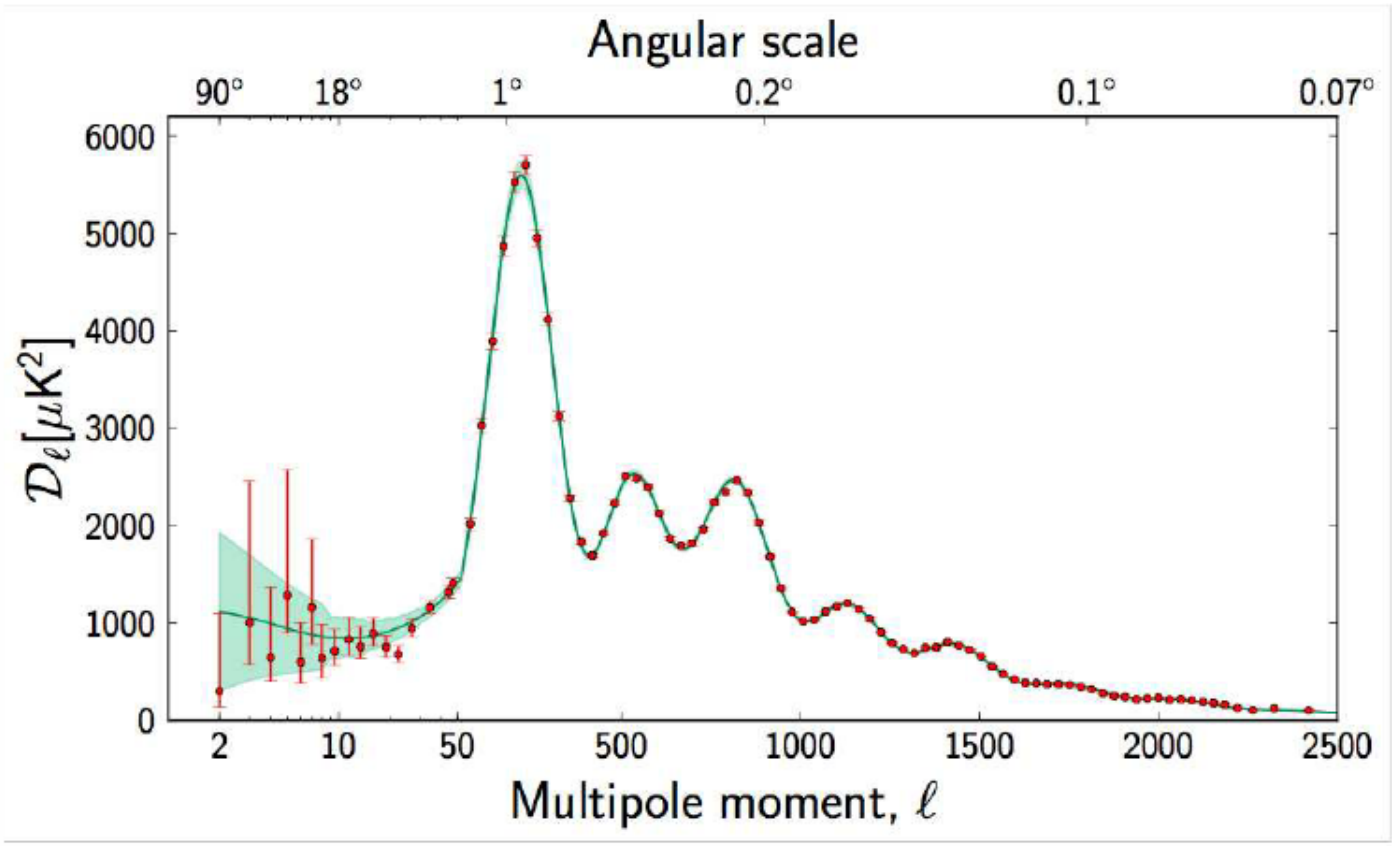
HOSPEDAJE PARA ESTUDIANTES Y POSTDOCS

Habrá apoyo en forma de hospedaje y alimentos para estudiantes y postdocs. Se dará preferencia a quienes presenten plática corta.

SEDE DEL EVENTO

La sede del Taller será el auditorio del Instituto de Ciencias Físicas de la Universidad Nacional Autónoma de México, ubicado en el Campus de la Universidad Autónoma del Estado de Morelos, en la ciudad de Cuernavaca, Morelos.

CMB Spectrum



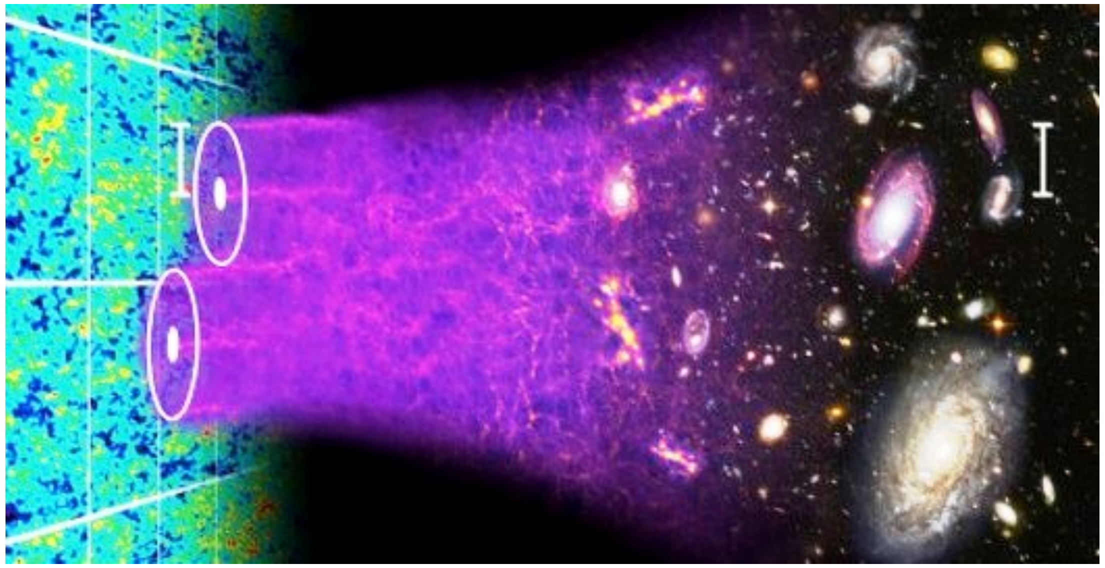
l=2?

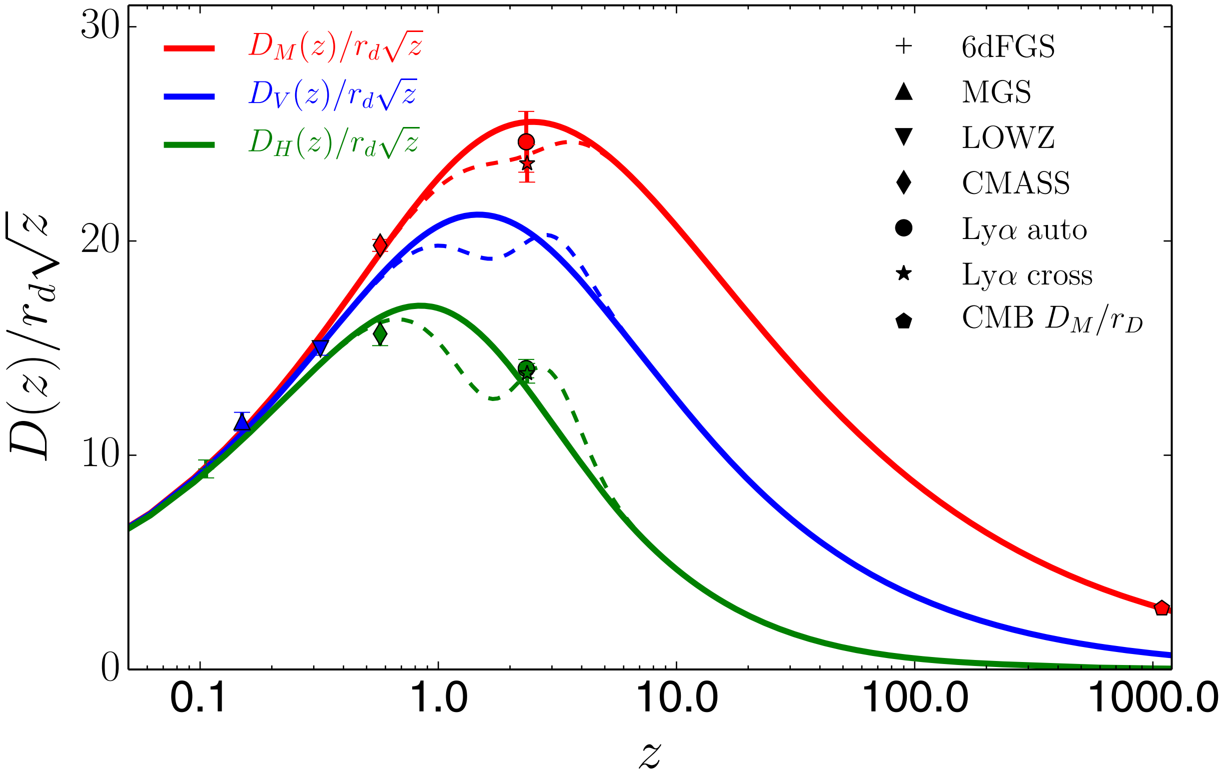
moon angle?

sun-angle?

$$S_T = -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi' - \Psi'] + g'\left[\frac{V_b}{k} + \frac{3}{k^2}\mathcal{C}'\right] + g''\frac{3}{2k^2}\mathcal{C} + g\left[\frac{1}{4}D_g^\gamma + \frac{V_b'}{k} - (\Phi - \Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}''\right],$$

CMB as a BAO





$$\mathbf{v} = \begin{pmatrix} \omega_b \\ \omega_{cb} \\ D_M(1090)/r_d \end{pmatrix}$$

Statistics of Random Fields

Consider a random field f(x) – i.e. at each point f(x) is some random number – with zero mean, ⟨f(x)⟩ = 0.

$$\langle T\rangle = \frac{1}{4\pi}\int T(\theta,\phi)\sin\theta d\theta d\phi = 2.7255 \pm 0.0006K$$

The **probability of realising some field configuration** is a functional Pr[f(x)].

The two point correlator is

$$\xi(\mathbf{x},\mathbf{y}) \equiv \langle f(\mathbf{x}) f(\mathbf{y})\rangle = \int \mathcal{D}f \, \mathrm{Pr}[f] f(\mathbf{x}) f(\mathbf{y}) \, ,$$

functional integral (or path integral) over field configurations

Statistical homogeneity means that the statistical properties of the translated field,

$\hat{T}_{\mathbf{a}} f(\mathbf{x}) \equiv f(\mathbf{x}-\mathbf{a})$, are the same as the original field Pr[f(x)] = Pr[$\hat{T}_a$f(x)]

$$\xi(\mathbf{x},\mathbf{y}) = \xi(\mathbf{x}-\mathbf{a},\mathbf{y}-\mathbf{a}) \quad \forall \mathbf{a}$$
$$\Rightarrow \quad \xi(\mathbf{x},\mathbf{y}) = \xi(\mathbf{x}-\mathbf{y}) \, ,$$

The two-point correlator only depends on the separation of the two points

Statistics of Random Fields

Statistical isotropy mean that the statistical properties of the **rotated field**

$$\hat{R} f(\mathbf{x}) \equiv f(\mathsf{R}^{-1}\mathbf{x}) \,,$$

are the same as the original field, i.e. Pr[f(x)] = Pr[R^f(x)].

$$\xi(\mathbf{x},\mathbf{y}) = \xi(\mathsf{R}^{-1}\mathbf{x}, \mathsf{R}^{-1}\mathbf{y}) \quad \forall \mathsf{R} \,.$$

Combining statistical homogeneity and isotropy gives

$$\begin{aligned} & \xi(\mathbf{x},\mathbf{y}) = \xi\left(\mathsf{R}^{-1}(\mathbf{x}-\mathbf{y})\right) \quad \forall \mathsf{R} \\ \Rightarrow \quad & \xi(\mathbf{x},\mathbf{y}) = \xi(|\mathbf{x}-\mathbf{y}|) \,, \end{aligned}$$

The two-point correlator depends only on the distance between the two points

Statistics of Random Fields

To constrain the form of the correlators in **Fourier space**

Note that for real fields, f(k) = f*(−k).

$$f(\mathbf{k}) = \int \frac{d^3\mathbf{x}}{(2\pi)^{3/2}} f(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \quad \text{and} \quad f(\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} f(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} .$$

Under translations

$$\begin{aligned} \hat{T}_{\mathbf{a}} f(\mathbf{k}) &= \int \frac{d^3\mathbf{x}}{(2\pi)^{3/2}} f(\mathbf{x}-\mathbf{a}) e^{-i\mathbf{k}\cdot\mathbf{x}} \\ &= \int \frac{d^3\mathbf{x}'}{(2\pi)^{3/2}} f(\mathbf{x}') e^{-i\mathbf{k}\cdot\mathbf{x}'} e^{-i\mathbf{k}\cdot\mathbf{a}} \\ &= f(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{a}} . \end{aligned}$$

$$\begin{aligned} & \langle f(\mathbf{k}) f^*(\mathbf{k}')\rangle = \langle f(\mathbf{k}) f^*(\mathbf{k}')\rangle e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{a}} \quad \forall \mathbf{a} \\ \Rightarrow \quad & \langle f(\mathbf{k}) f^*(\mathbf{k}')\rangle = F(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}') , \end{aligned}$$

For some (real) function F(k)

Different Fourier modes are uncorrelated

Under rotations

$$\begin{aligned} \hat{R} f(\mathbf{k}) &= \int \frac{d^3\mathbf{x}}{(2\pi)^{3/2}} f(\mathsf{R}^{-1}\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \\ &= \int \frac{d^3\mathbf{x}}{(2\pi)^{3/2}} f(\mathsf{R}^{-1}\mathbf{x}) e^{-i(\mathsf{R}^{-1}\mathbf{k})\cdot(\mathsf{R}^{-1}\mathbf{x})} \\ &= f(\mathsf{R}^{-1}\mathbf{k}) , \end{aligned}$$

R is a rotation matrix

$$\langle \hat{R} f(\mathbf{k}) [\hat{R} f(\mathbf{k}')]^* \rangle = \langle f(\mathsf{R}^{-1}\mathbf{k}) f^*(\mathsf{R}^{-1}\mathbf{k}') \rangle = F(\mathsf{R}^{-1}\mathbf{k})\delta(\mathbf{k}-\mathbf{k}') = F(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}')$$

This is only possible if $F(\mathbf{k}) = F(k)$

Statistics of Random Fields

Define the **power spectrum, Pf (k),** of a homogeneous and isotropic field,

$$\langle f(\mathbf{k}) f^*(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_f(k)\delta(\mathbf{k}-\mathbf{k}')\,.$$

$$\langle f(\mathbf{x}) f(\mathbf{y})\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\frac{d^3\mathbf{k}'}{(2\pi)^{3/2}} \underbrace{\langle f(\mathbf{k}) f^*(\mathbf{k}')\rangle}_{\frac{2\pi^2}{k^3}\mathcal{P}_f(k)\delta(\mathbf{k}-\mathbf{k}')} e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\mathbf{k}'\cdot\mathbf{y}}$$
$$= \frac{1}{4\pi}\int \frac{dk}{k}\mathcal{P}_f(k)\int d\Omega_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,.$$

$$\xi(\mathbf{x},\mathbf{y}) = \int \frac{dk}{k}\mathcal{P}_f(k) j_0(k|\mathbf{x}-\mathbf{y}|)\,.$$

Gaussian random fields

$$\Pr(\mathbf{f}) \propto \frac{e^{-f_i \xi_{ij}^{-1} f_j}}{\sqrt{\det(\xi_{ij})}}\,.$$

Since different Fourier modes are **uncorrelated**, they are **statistically independent** for Gaussian fields.

Random fields on the sphere

$$\langle f(\hat{\mathbf{n}}) f(\hat{\mathbf{n}}')\rangle = \sum_{lm}\sum_{l'm'} \underbrace{\langle f_{lm} f^*_{l'm'}\rangle}_{C_l\delta_{ll'}\delta_{mm'}} Y_{lm}(\hat{\mathbf{n}}) Y^*_{l'm'}(\hat{\mathbf{n}}')$$
$$= \sum_l C_l \underbrace{\sum_m Y_{lm}(\hat{\mathbf{n}}) Y^*_{lm}(\hat{\mathbf{n}}')}_{\frac{2l+1}{4\pi}P_l(\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')} = C(\theta)\,,$$

$$C_l = 2\pi\int_{-1}^{1} d\cos\theta\, C(\theta) P_l(\cos\theta)\,.$$

CMB power spectrum

The primary anisotropies **carried out by physical effects** before the **recombination** epoch, encoded in the fractional **temperature perturbation**, are expanded in terms of the **spherical harmonics** on the lls by

$$\frac{\Delta T}{\bar{T}}(\eta_0, \mathbf{x_0}, \mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}),$$

Assuming the $a_{l,m}$'s are **Gaussian random fields**, the two-point correlator gives $\langle a_{lm} a^*_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}.$

The angular **CMB power spectrum** C^{TT} is computed through the **two-point correlation function**

if not Gauss?

$$C(\theta) \equiv \left\langle \frac{\Delta T(\mathbf{n})}{\bar{T}} \frac{\Delta T(\mathbf{n}')}{\bar{T}} \right\rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\mathbf{n} \cdot \mathbf{n}').$$

where $n \cdot n' = \cos\theta$. The **initial conditions** $\Phi_{\rm ini} = R$. Because the evolution equations for Δ are independent of the direction k, we may write

$$\Delta_l(\eta_0, \mathbf{k}, \mathbf{n}) = \Phi_{\rm ini}(\mathbf{k}) \Delta_l(\eta_0, k, \mathbf{n}).$$

where **X and Y** represent temperature (T) and polarisations (E or B);

$$C_l^{XY} = \frac{4\pi}{(2l+1)^2} \int \frac{d^3k}{(2\pi)^3} \, \mathcal{P}_{\mathcal{R}}(k) \, \Delta_l^X(k) \Delta_l^Y(k),$$

Primordial power spectrum

$$C_l^{XY} = \frac{4\pi}{(2l+1)^2} \int \frac{d^3k}{(2\pi)^3}\, \mathcal{P}_{\mathcal{R}}(k)\, \Delta_l^X(k) \Delta_l^Y(k),$$

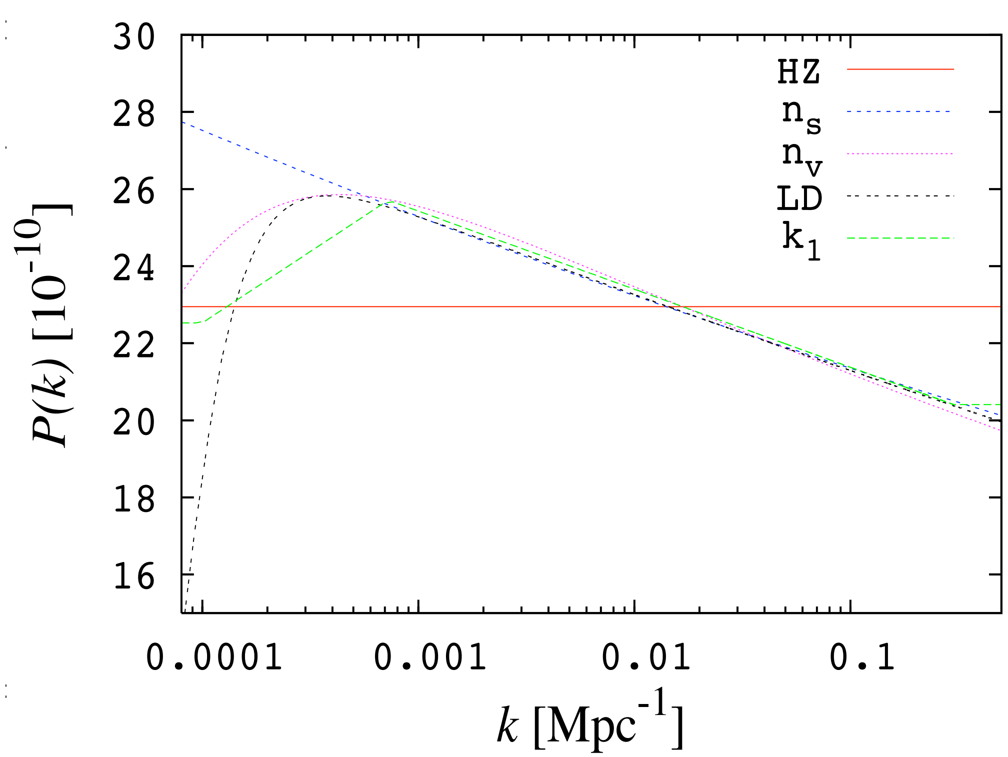
PR(k) is the **power spectrum of the initial curvature perturbations**

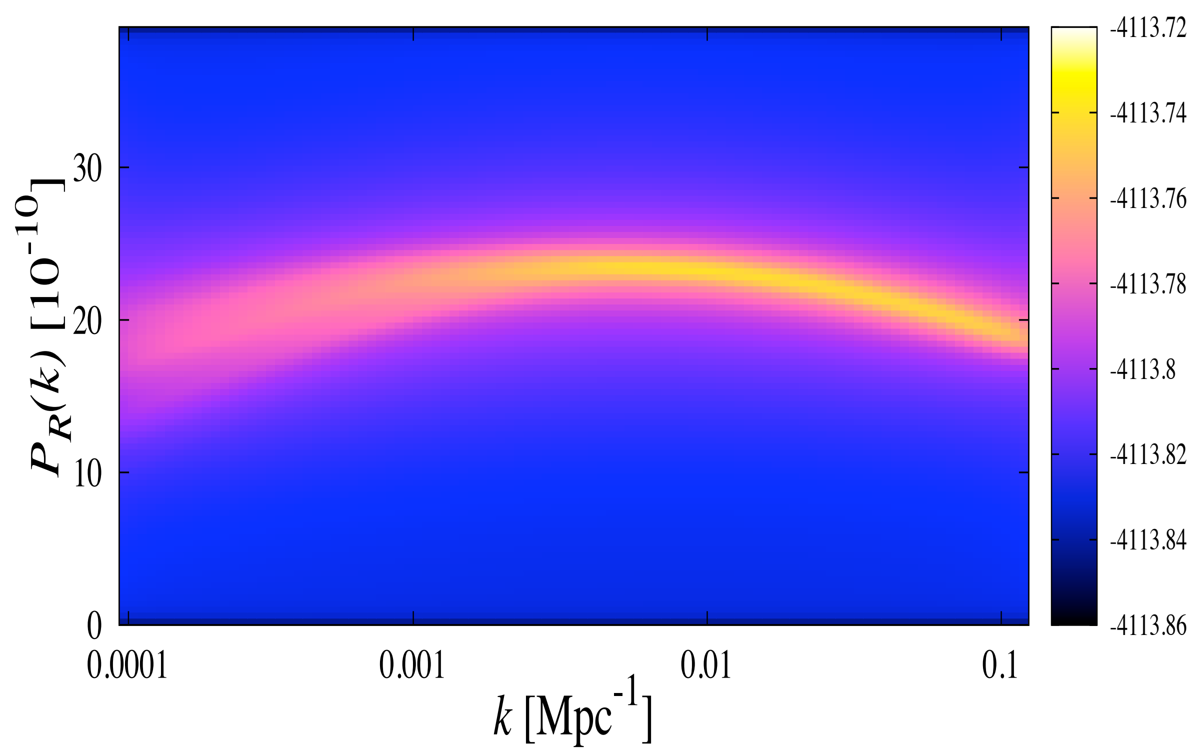
$$\mathcal{P}_{\mathcal{R}}(k) = \quad A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\, dn_s/d\ln k\, \ln(k/k_*) + \frac{1}{6}\, \frac{d^2 n_s}{d\ln k^2} (\ln(k/k_*))^2 + \ldots},$$

The spectrum is a featureless power law with scalar spectral index n_S.

scale-invariant?

$$\mathcal{B}_{n_{\rm run}, n_{\rm s}} = +2.0 \pm 0.3$$





Cl's Scalar

$$C_l^{XY} = \frac{4\pi}{(2l+1)^2}\int \frac{d^3k}{(2\pi)^3}\,\mathcal{P}_{\mathcal{R}}(k)\,\Delta_l^X(k)\Delta_l^Y(k),$$

The **moments** obtained from the **line of sight integration method**, in terms of the spherical Bessel functions

$$\Delta_l^T = (2l+1)\int d\eta j_l(k[\eta-\eta_0])S_T(k,\eta),$$

$$\Delta_l^E = (2l+1)\sqrt{\frac{(l-2)!}{(l+2)!}}\int_0^{\eta_0} d\eta S_E(k,\eta)j_l(x),$$

$$S_T = -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi'-\Psi'] + g'\left[\frac{V_b}{k}+\frac{3}{k^2}\mathcal{C}'\right] + g''\frac{3}{2k^2}\mathcal{C}$$
$$+\ g\left[\frac{1}{4}D_g^\gamma + \frac{V_b'}{k} - (\Phi-\Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}''\right],$$
$$S_E = \frac{3g\mathcal{C}}{4x^2},$$

The hierarchy for the **tensor multipoles**, temperature $\tilde{\Delta}^T_l$, polarisation $\tilde{\Delta}^P$ and cross-correlation $\tilde{\Delta}^{T,P}$

$$C^{\rm tens}_{XY;l} = \frac{4\pi}{(2l+1)^2}\int \frac{d^3k}{(2\pi)^3}\,\mathcal{P}_{\mathcal{T}}(k)\,\Delta^{\rm tens}_{X;l}(k)\Delta^{\rm tens}_{Y;l}(k),$$

where PT (k) is the **initial tensor pow er spectrum**

$$\mathcal{P}_{\mathcal{T}}(k) = A_{\rm t}\left(\frac{k}{k_0}\right)^{n_{\rm t}},$$

$$r(k) \equiv \frac{\mathcal{P}_{\mathcal{T}}(k)}{\mathcal{P}_{\mathcal{R}}(k)} = 64\pi\left(\frac{\dot\phi^2}{H^2}\right)_{k=aH}.$$

$$n_{\rm t} = -r_{\rm s}/8$$

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Cl's Tensor

$$C^{\rm tens}_{XY;l} = \frac{4\pi}{(2l+1)^2}\int \frac{d^3k}{(2\pi)^3}\,\mathcal{P}_{\mathcal{T}}(k)\,\Delta^{\rm tens}_{X;l}(k)\Delta^{\rm tens}_{Y;l}(k),$$

$$\begin{aligned}
\Delta^{\rm tens}_{T;l} &= \sqrt{\frac{(l+2)!}{(l-2)!}}\int_0^{\eta_0} d\eta S^{\rm tens}_T(k,\eta)\frac{j_l(x)}{x^2},\\
\Delta^{\rm tens}_{E,B;l} &= \int_0^{\eta_0} d\eta S^{\rm tens}_{E,B}(k,\eta) j_l(x),
\end{aligned}$$

where h is the longitudinal-scalar part of tensor decomposition

with the sources

$$\psi = \frac{1}{10}\tilde{\Delta}^T_0 + \frac{1}{7}\tilde{\Delta}^T_2 + \frac{3}{70}\tilde{\Delta}^T_4 - \frac{3}{5}\tilde{\Delta}^P_0 + \frac{6}{7}\tilde{\Delta}^P_2 - \frac{3}{70}\tilde{\Delta}^P_4.$$

$$\begin{aligned}
S^{\rm tens}_T(k,\eta) &= h'\exp(-\kappa) + g\psi,\\
S^{\rm tens}_E(k,\eta) &= g\left\{\psi - \frac{\psi''}{k^2} + \frac{2\psi}{x^2} - \frac{\psi'}{kx}\right\}\\
&\quad -g'\left\{\frac{2\psi'}{k^2} + \frac{4\psi}{kx}\right\} - 2g''\frac{\psi}{k^2},\\
S^{\rm tens}_B(k,\eta) &= g\left\{\frac{4\psi}{x} + \frac{2\psi'}{k}\right\} + 2g'\frac{\psi}{k}.
\end{aligned}$$

$$\begin{aligned}
\dot{\tilde{\Delta}}^T_0 &= -k\tilde{\Delta}^T_1 - \kappa'[\tilde{\Delta}^T_0 - \psi] - h',\\
\dot{\tilde{\Delta}}^P_0 &= -k\tilde{\Delta}^T_2 - \kappa'[\tilde{\Delta}^T_1 + \psi],\\
\dot{\tilde{\Delta}}^{T,P}_l &= \frac{k}{2l+1}\left[l\tilde{\Delta}^{T,P}_{l-1} - (l+1)\tilde{\Delta}^{T,P}_{l+1}\right] - \kappa'\tilde{\Delta}^{T,P}_l;
\end{aligned}$$

Solving ...

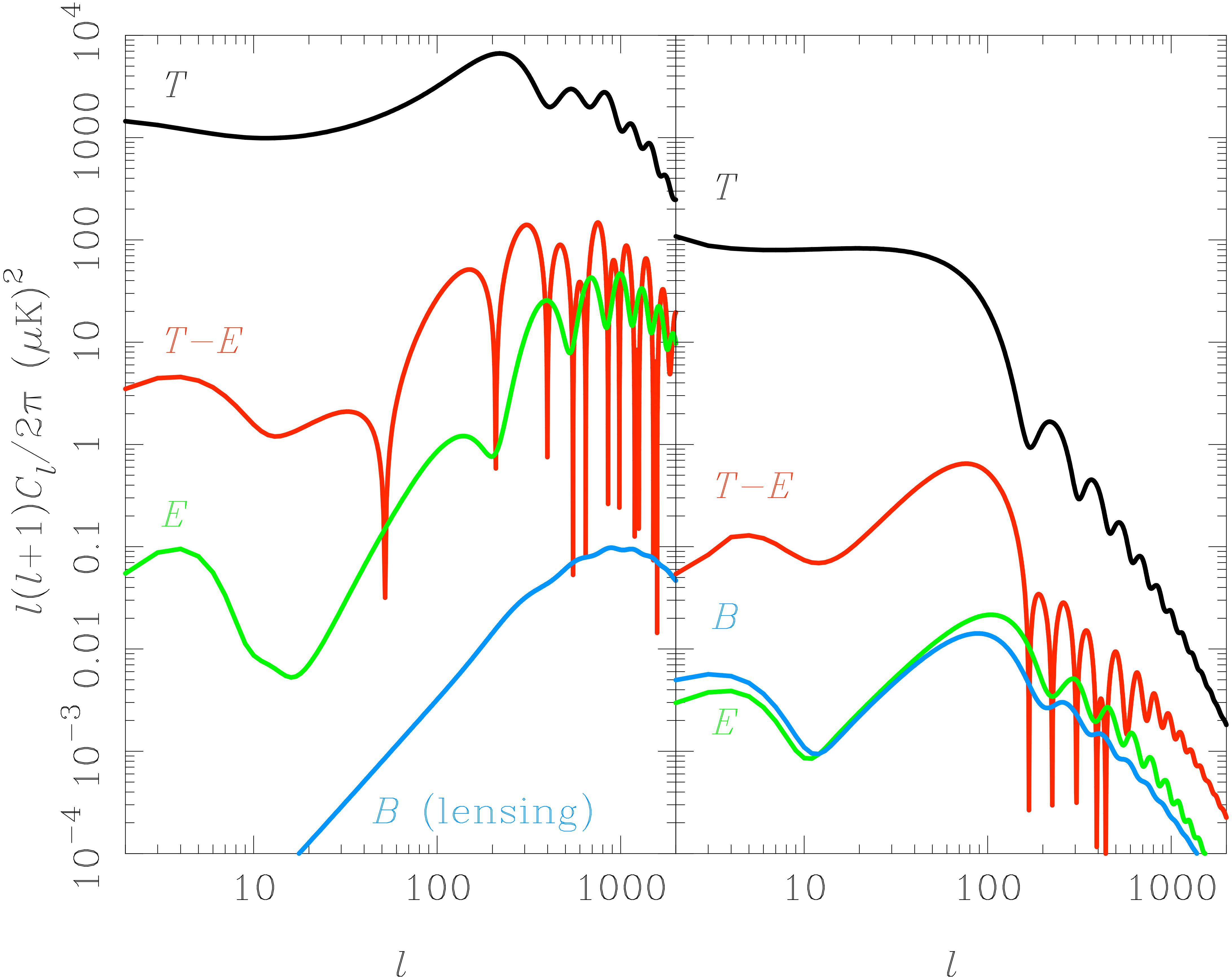
The **slow way** would be to get the C_l's **directly from the (vast) multipole hierarchy** of the photon distribution and the multipole hierarchy up to $l \equiv 3000$

In contrast, **the line of sight integration** gets the Δ_l's by folding the source term S with the spherical Bessel functions j_l.

While the **Bessel functions oscillate rapidly** in this convolution, the source term is most of the time rather **slowly changing**.

It thus suffices to calculate the sources at **few (cleverly chosen)** points and interpolate between

Voilà!



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Codes

The Boltzmann hierarchy is nowadays **solved numerically** with software packages such as

1995/ Fortran 77

- COSMICS: Cosmological initial conditions and microwave anisotropy
  * Chung-Pei Ma, Edmund Bertschinger. arXiv:astro-ph/9506072

  http://ascl.net/cosmics.html

1996/ Fortran 77

- CMBFAST: A microwave anisotropy code
  * Seljak Uros, Zaldarriaga Matias. arXiv:astro-ph/9603033, arXiv:astro-ph/9704265

  http://ascl.net/cmbfast.html

2000/ Fortran 90

- CAMB: Code for Anisotropies in the Microwave Background.
  * Antony Lewis, Anthony Challinor and Anthony Lasenby. arXiv:astro-ph/9911177

  http://camb.info/

2003/ C++

- CMBEASY: an Object Oriented Code for the Cosmic Microwave Background
  * Michael Doran arXiv:astro-ph/0302138v2

  http://www.thphys.uni-heidelberg.de/ robbers/cmbeasy/index.html

2001 Davis Anisotropy Shortcut (DASh)

DASh incorporates many analytic and semianalytic approximations that have been presented elsewhere in the literature and also some new ones. The Astrophysical Journal, 578:665-674, 2002

TABLE I. Comparison between CMB Codes [a]

	CAMB	CLASS	CMBEASY	CMBquick	CosmoLib [b]
Language	F90	C	C++	Mathematica	F90[c]
gauge [d]	syn.	syn./Newt. [e]	syn./gauge-inv.	Newt.	Newt.
open/close universe	Yes	No	No	No	No
massive neutrinos	Yes	Yes	Yes	Yes	No
tensor perturb.	Yes	Yes	Yes	Yes	Yes
CDM isocurvature mode	Yes	Yes	Yes	Yes	Yes
dark energy perturb.	Yes	Yes	Yes	No	Yes
nonzero $c_{s,b}^2$	Yes	Yes	Yes	No	Yes
dark energy EOS.	constant	$w_0 + w_a(1-a)$	arbitrary	-1	arbitrary
non-smooth primordial power	No	No	No	No	Yes
MCMC driver	Yes	No	Yes	No	Yes
periodic proposal density	No	No	No	No	Yes
data simulation	No	No	No	No	Yes
second-order perturb. [f]	No	No	No	Yes	No [g]

[a] Here we do not include CMBFast, which is no longer supported by its authors or available for download.

CAMB Web Interface

Supports the January 2011 Release

Most of the configuration documentation is provided in the sample parameter file provided with the application.

This form uses JavaScript to enable certain layout features, and it uses Cascading Style Sheets to control the layout of all the form components. I either of these features are not supported or enabled by your browser, this form will NOT display correctly.

Actions to Perform

- [x] Scalar C_l's
- [ ] Vector C_l's
- [ ] Tensor C_l's
- [x] Do Lensing
- [ ] Transfer Functions

- (•) Linear
- ( ) Non-linear Matter Power (HALOFIT)
- ( ) Non-linear CMB Lensing (HALOFIT)

Sky Map Output: None

Vector C_l's are incompatible with Scalar and Tensor C_l's. The Transfer functions require Scalar and/or Tensor C_l's.
The HEALpix synfast program is used to generate maps from the resultant spectra. The random number seed governs the phase of the a_{lm}'s generated by synfast
The default of zero causes synfast to generate a new see from the system time with each run. Specifying a fixed nonzero value will return fixed phases with successive runs.

*Maximum Multipoles and k*eta*

Scalar

2000	l_{max}
4000	$k*eta_{max}$

Tensor

1500	l_{max}
3000	$k*eta_{max}$

Tensor limits should be less than or equal to the scalar limits.

Cosmological Parameters

Use Physical Parameters? Yes

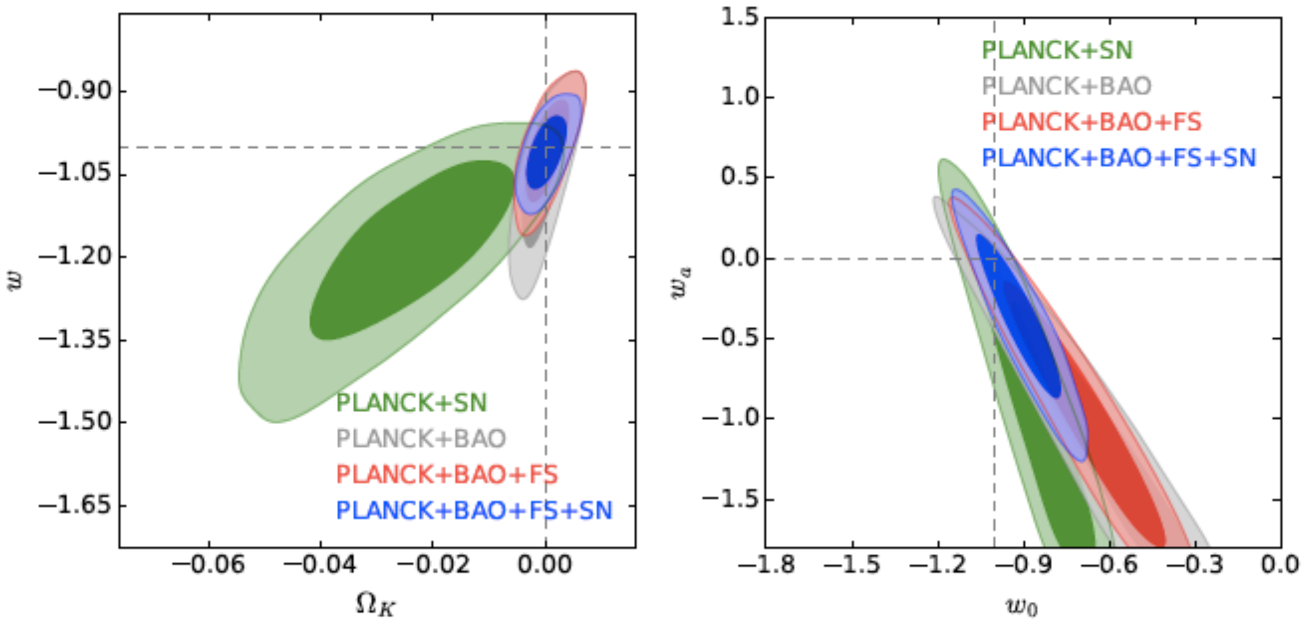
70	Hubble Constant
2.725	T_{cmb}

0.0226	$\Omega_b h^2$
0.114	$\Omega_c h^2$
0	$\Omega_\nu h^2$
0	Ω_k

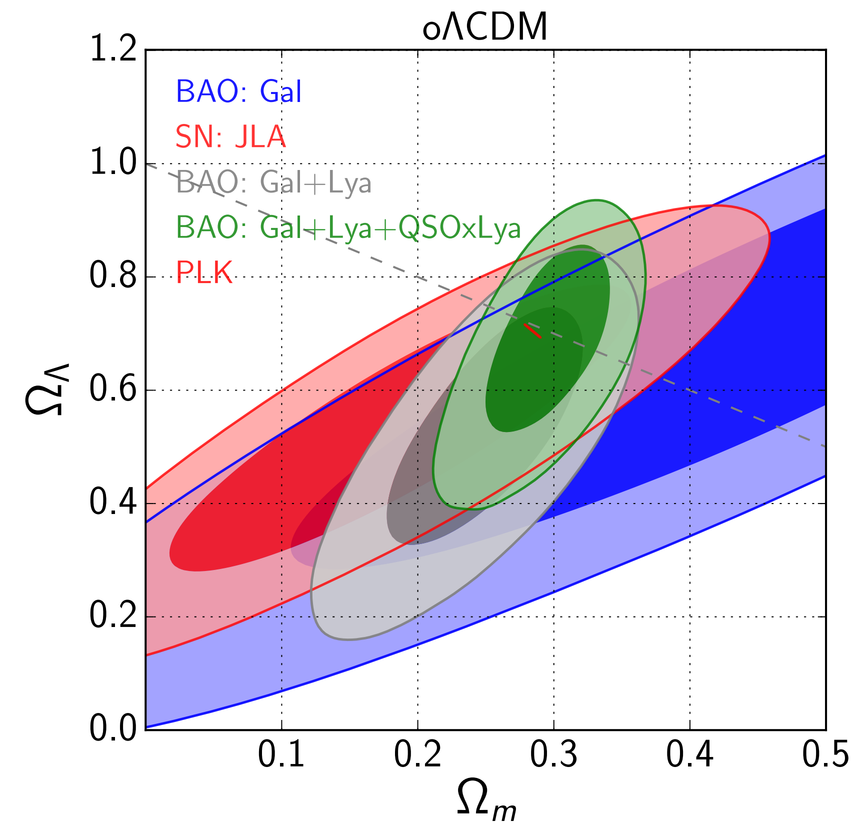
Neutrino mass splittings

0.24	Helium Fraction
3.04	Massless Neutrinos
0	Massive Neutrinos
-1	Eqn. of State
1	Comoving Sound Speed

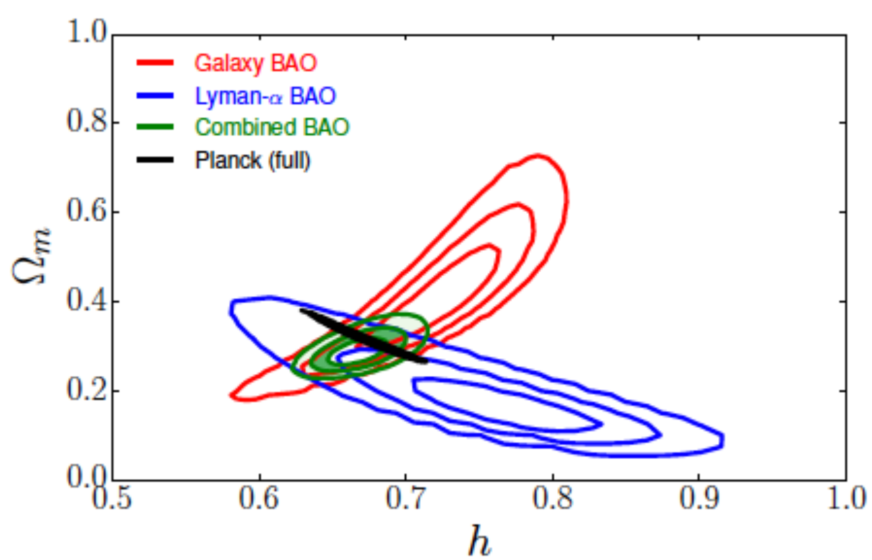
SimpleMC



DR12- Lyman-a



DR12 - arXiv:1607.03155



DR11 arXiv:1411.1074

To perform the analysis we built a simple and fast MCMC code: **Simple MC**

https://github.com/ja-vazquez/april

with A. Slosar

JAVazquez

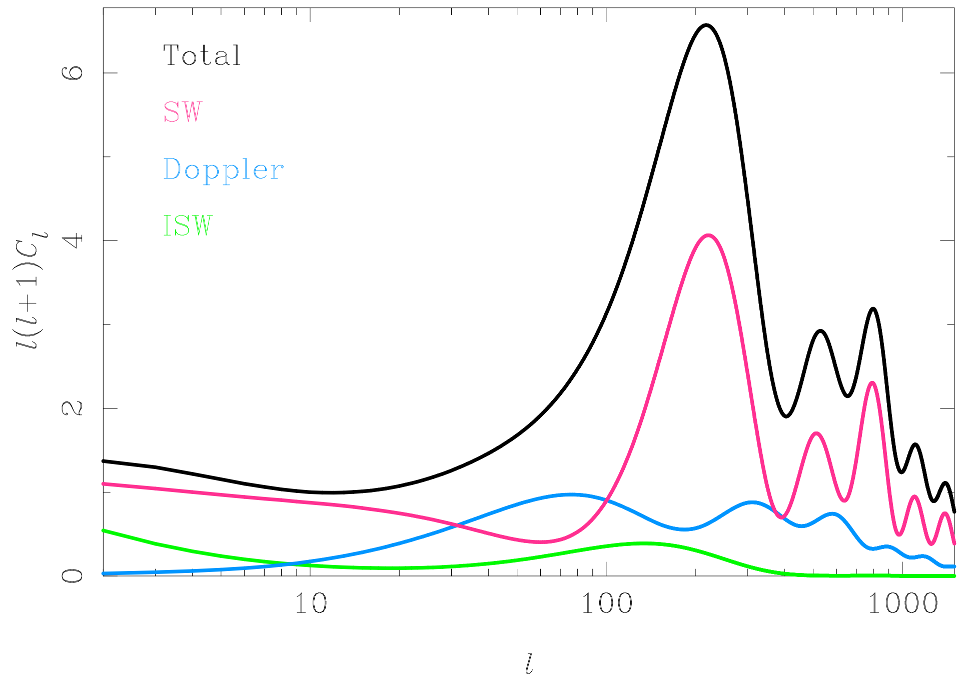


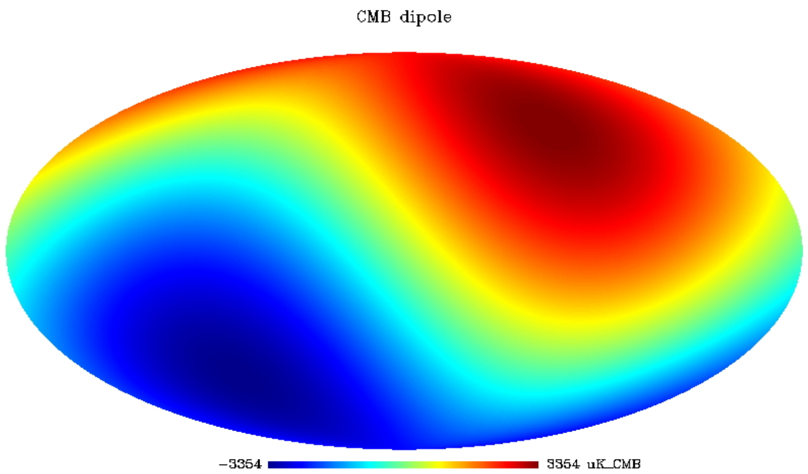
Figure 4.3: Total CMB temperature-spectrum and its different contributions: Sachs-Wolfe (SW) $D_g^\gamma/4 - (\Phi - \Psi)$; Doppler effect V_b^γ; and the integrated Sachs-Wolfe effect (ISW) coming from evolution of the potential along the line of sight. Figure from Challinor [**?**]

The l = 0 term of the correlation function **(the monopole)** vanishes if the mean temperature has been de defined correctly

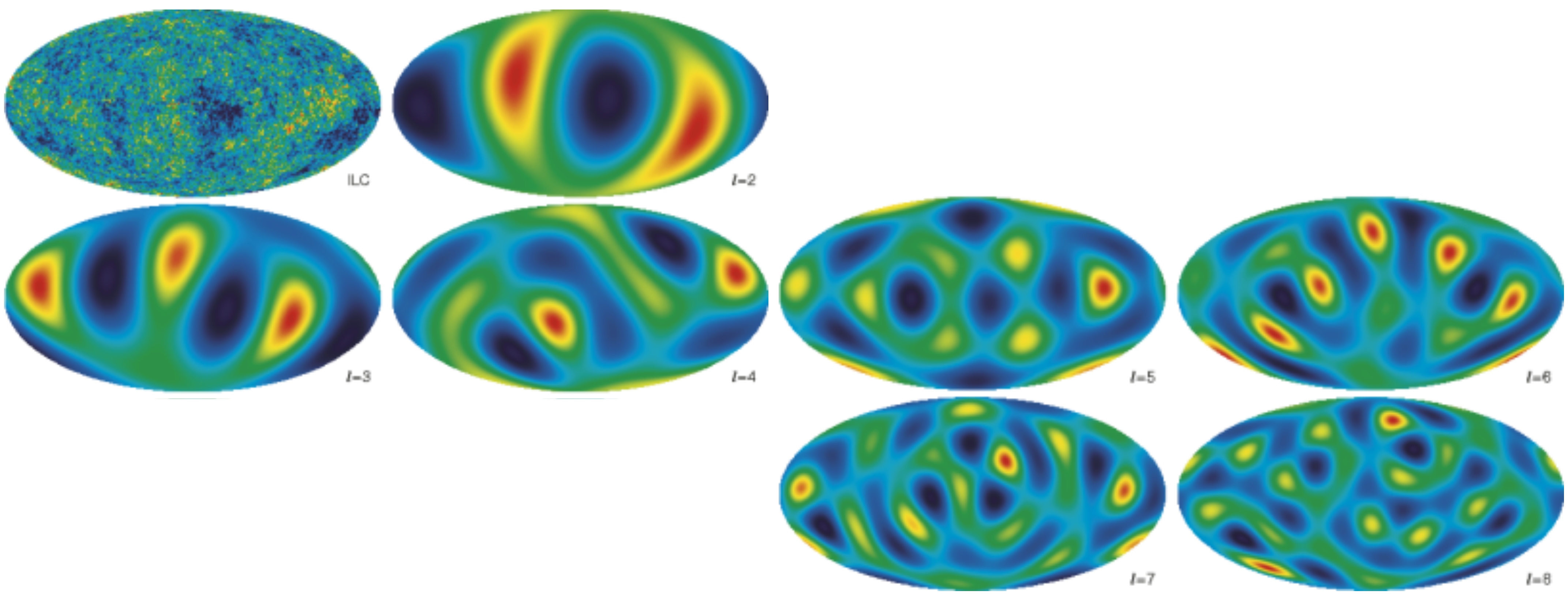
not l=1 ?

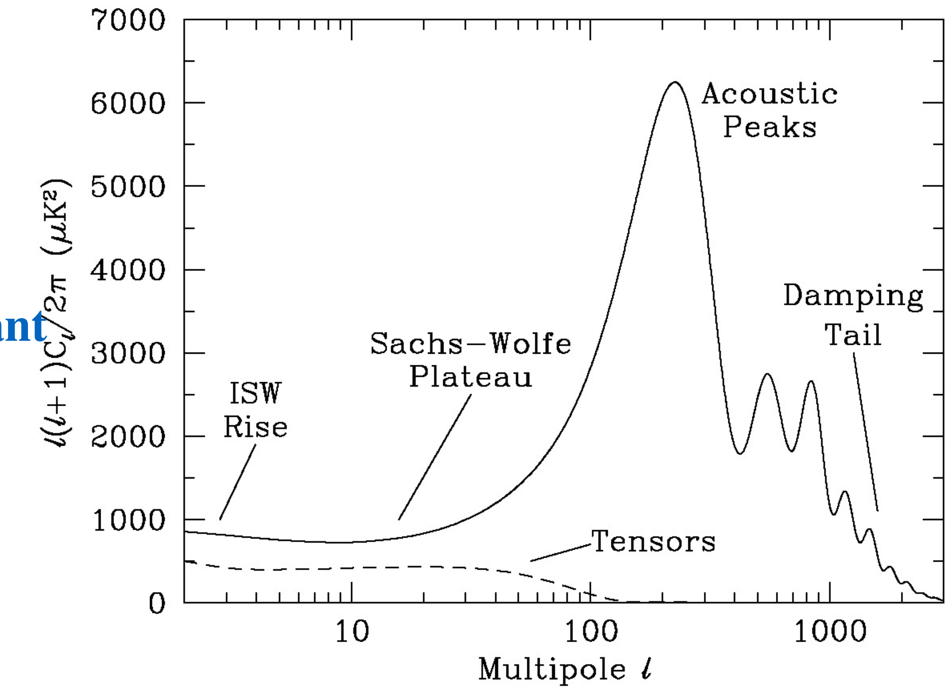
The l = 1 **(the dipole)** reflects the motion of the Earth through space. We are seeing the effect of the Earth's motion relative **to the local comoving frame of reference**.

The Earth is moving with a velocity $v = 369 kms^{-1}$ towards a point on the boundary of the **constellations of Crater and Leo.**



Earth vel and towards?





- **The Sachs-Wolfe effect (l < 100)** - The gravitational effects **are the dominant** contributions at large angular scales.

$$\int_0^\infty j_l^2(x)dx = \frac{1}{2l(l+1)}$$

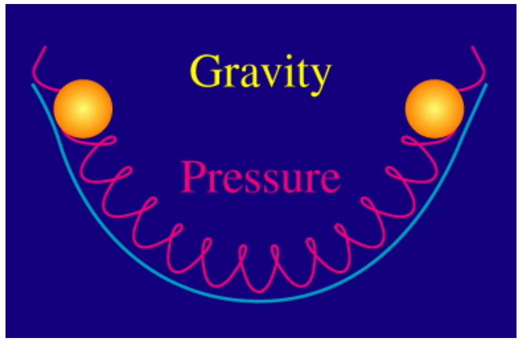
assume a **nearly scale-invariant** scalar spectrum $n_S \approx 1$, then is approximately constant, shown as a **flat plateau at low multipoles**.

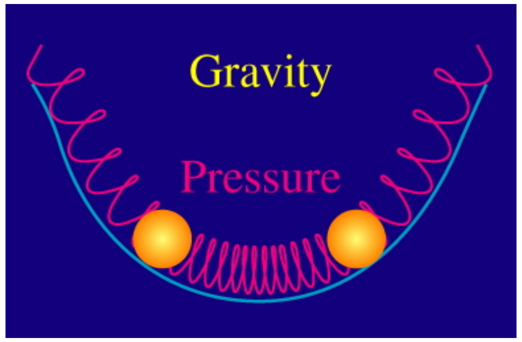
$$\frac{l(l+1)C_l}{2\pi} = \frac{1}{25}A_s$$

Primordial spectrum that varies as a **power-law in k** gives

$$C_l \sim \frac{\Gamma(l + n_s/2 - 1/2)}{\Gamma(l - n_s/2 + 5/2)}$$

- **Intermediate scales (100 < l < 1000)** - **Perturbations inside the horizon have evolved** causally and produced the anisotropy at the last scattering epoch ($l_{hor} \approx 200$). The **balance between the gravitational force and radiation pressure** is presented as series of characteristic peaks **called acoustic oscillations**.



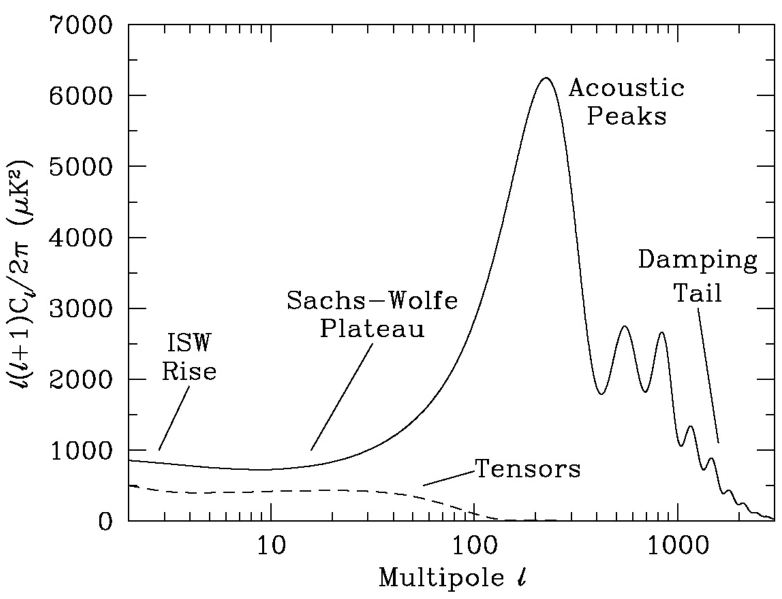


$$C_l^{XY} = \frac{4\pi}{(2l+1)^2}\int \frac{d^3k}{(2\pi)^3}\, \mathcal{P}_{\mathcal{R}}(k)\, \Delta_l^X(k)\Delta_l^Y(k),$$

- **Small scales (l > 1000)** - The **thickness of the last scattering surface** leads to a damping of $C_l^T \sim l^{-4}$ at the highest multipoles, commonly called the **Silk effect.**

The total mean-squared distance that a photon will have moved by such a random walk by the time η_* is therefore

which defines a **damping scale** $$\int_0^{\eta_*} \frac{d\eta'}{a n_e \sigma_T} \sim \frac{1}{k_D^2}$$

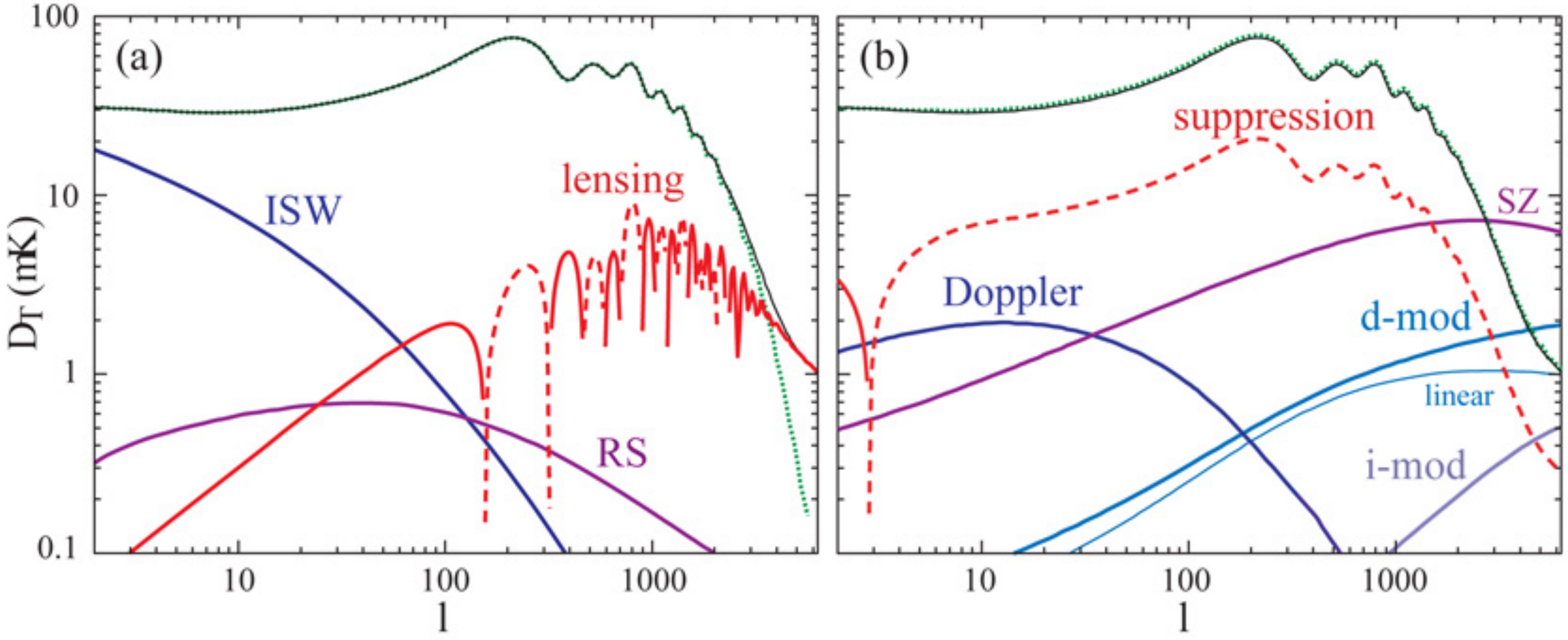


- At these scales, important contributions are also provided by secondary anisotropies:

gravitational lensing, Rees-Sciama effect (RS), Sunyaev-Zel'dovich effect (SZ), kinetic Sunyaev-Zel'dovich effect, Ostriker-Vishniac effect (OV), foregrounds from discrete sources

Inverse Compton scattering by energetic electrons in the intracluster medium of massive galaxy clusters alters the blackbody spectrum of CMB photons travelling through the cluster

Caused by a time dependent gravitational potential during the nonlinear stages of evolution.

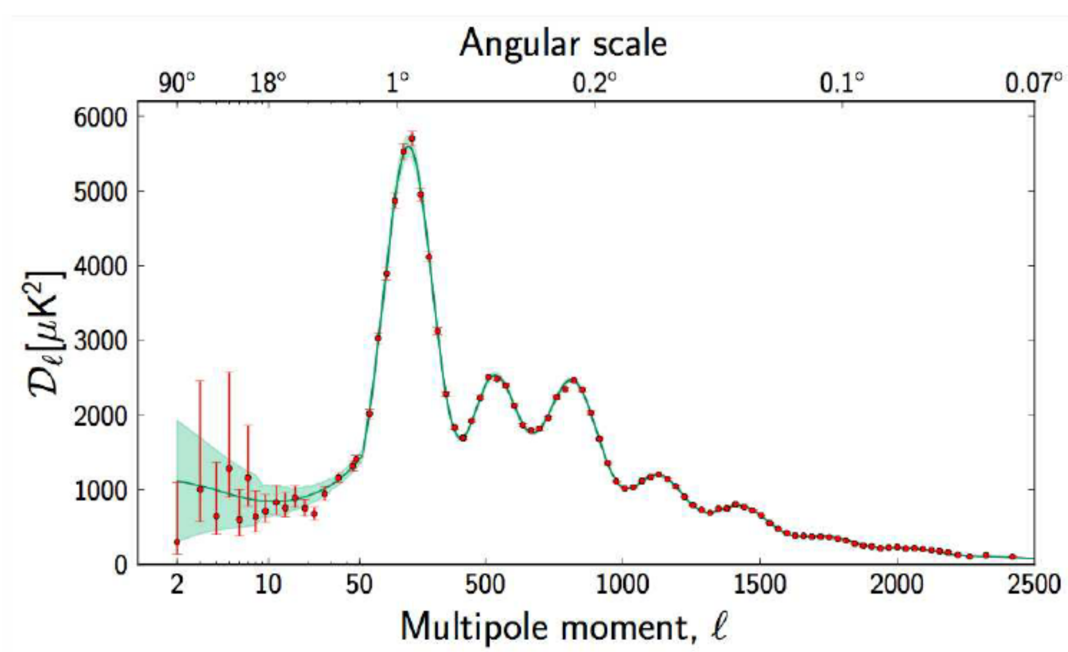


COSMOLOGICAL PARAMETERS

The **whole structure of the CMB** depends strongly on the initial conditions emerging from the inflationary era (P_{R},T), on the matter-energy content ($\Omega_{i,0}$), and on the expansion rate history (H_0).

These parameters, commonly **called standard parameters**, are considered as the principal quantities used **describe the universe**.

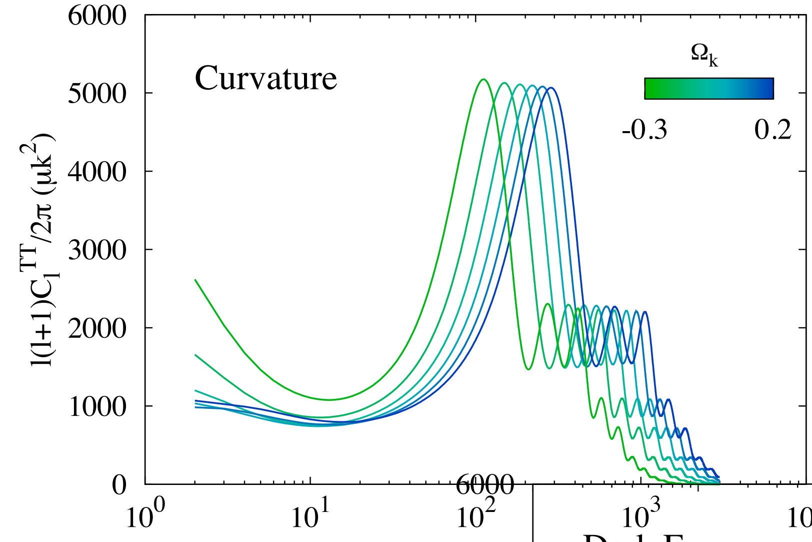
They are not, however, **predicted** by any fundamental theory, rather **we have to fit them by hand** in order to determine which **combination** best describes the current astrophysical observations

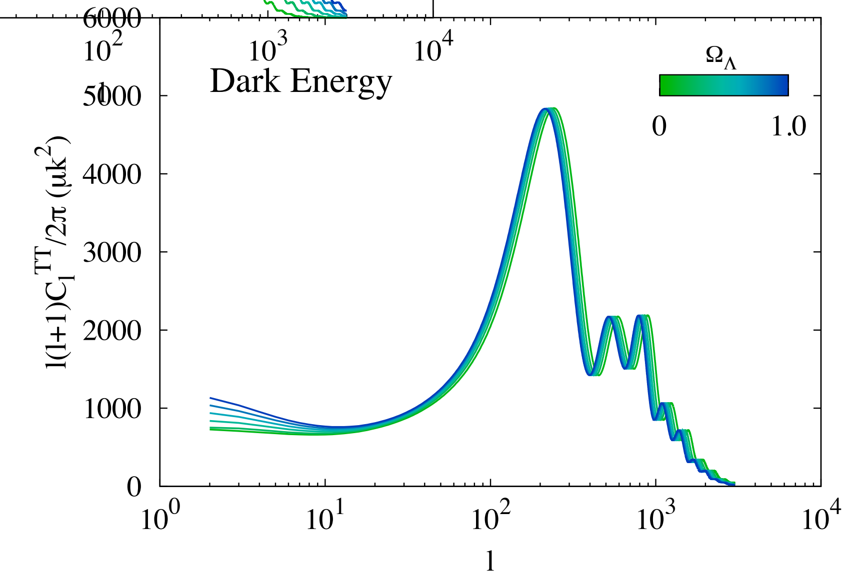


PARAMETERS

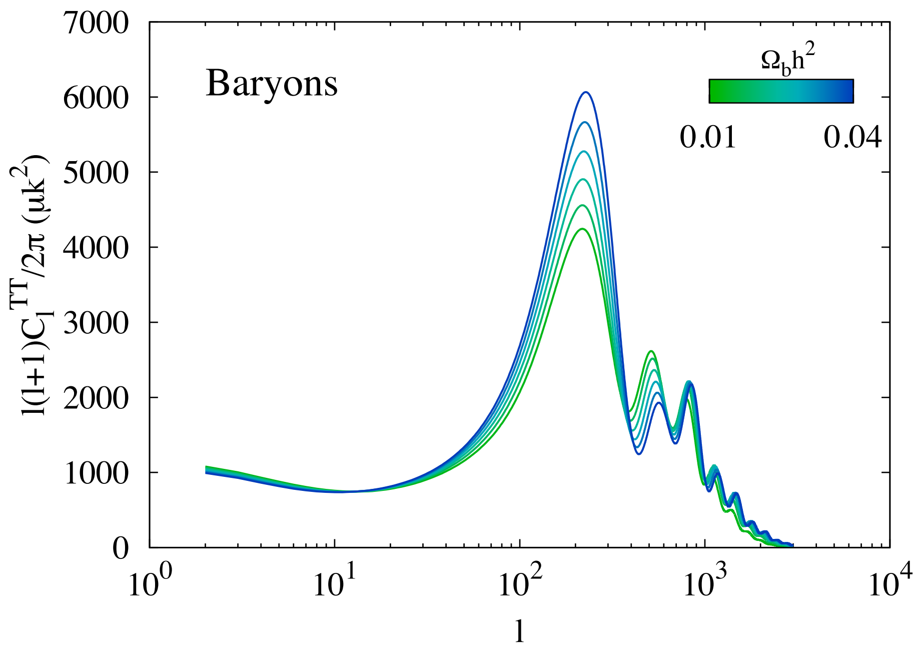
$$d_A(z) \approx 2\frac{c}{H_0}\frac{1}{\Omega_{m,0}\, z}\,.$$

$$\theta_{hor,s} \simeq \frac{1}{\sqrt{3}}\left(\frac{(1-\Omega_{k,0})}{z_{dec}}\right)^{1/2} = 0.017\,\text{radians} \sim 1^\circ$$





Both parameters principally affect the anisotropies **through dA** and so simply shift the peaks.



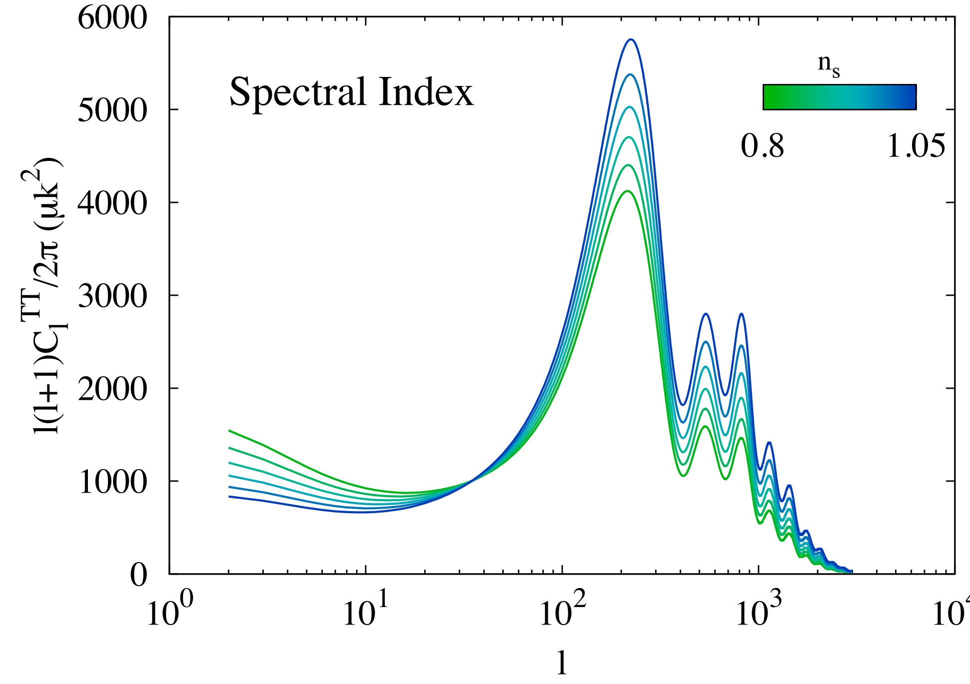
The **increase in baryon** inertia **reduces the sound speed**, **shifting the acoustic peaks** in temperature and polarization to smaller scales (larger l).

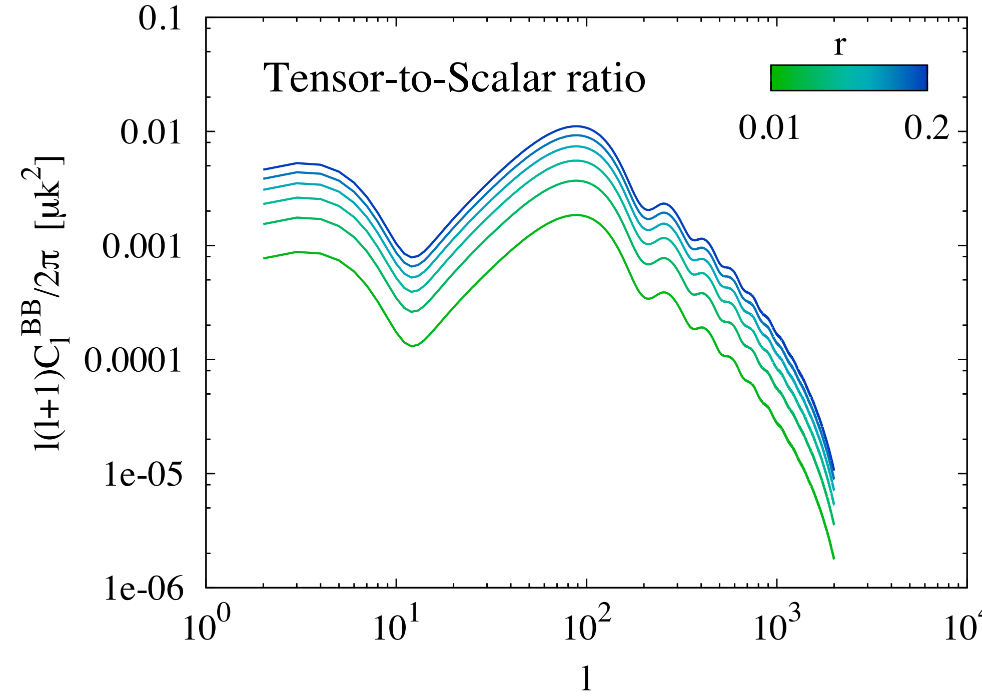
The **increase** in the number density of electrons in the plasma **reduces the photon mean-free path**, lp, reducing the amount of **diffusion damping** and so increasing power on small scales.

Inflation

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}-1},$$
$$\mathcal{P}_{\mathcal{T}}(k) = A_{\rm t}\left(\frac{k}{k_0}\right)^{n_{\rm t}}.$$

$$n_{\rm s}-1 \simeq -6\,\epsilon_{\rm v}(\phi)+2\,\eta_{\rm v}(\phi),$$
$$n_{\rm t} \simeq -2\,\epsilon_{\rm v}(\phi),$$
$$r \simeq 16\,\epsilon_{\rm v}(\phi).$$



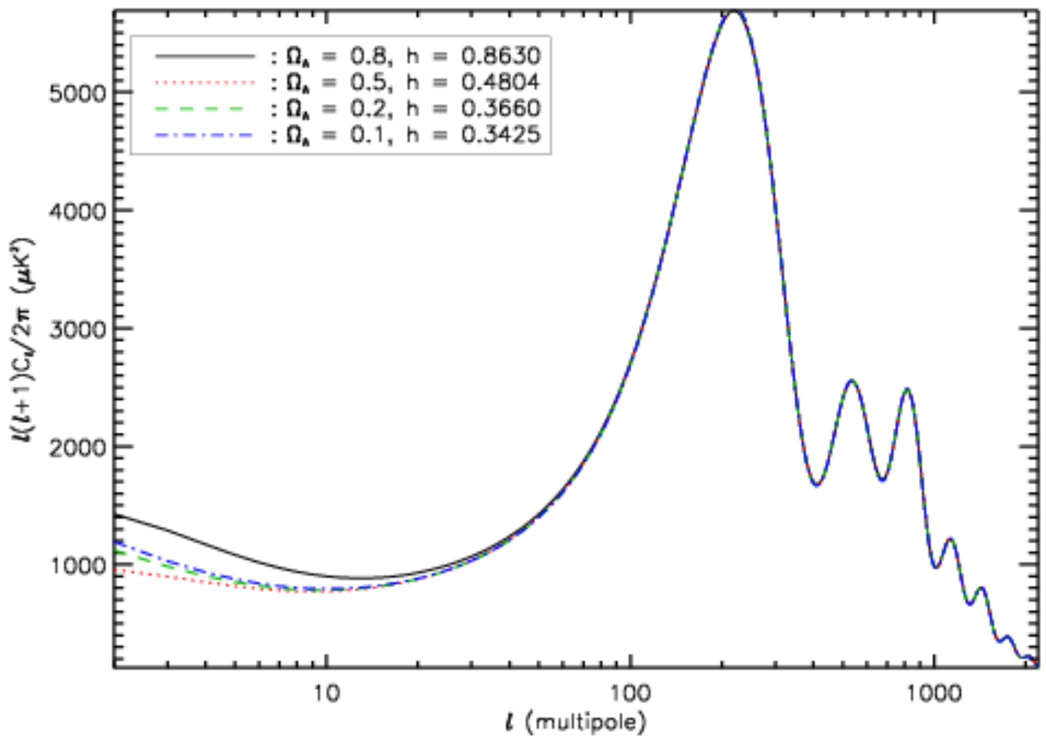


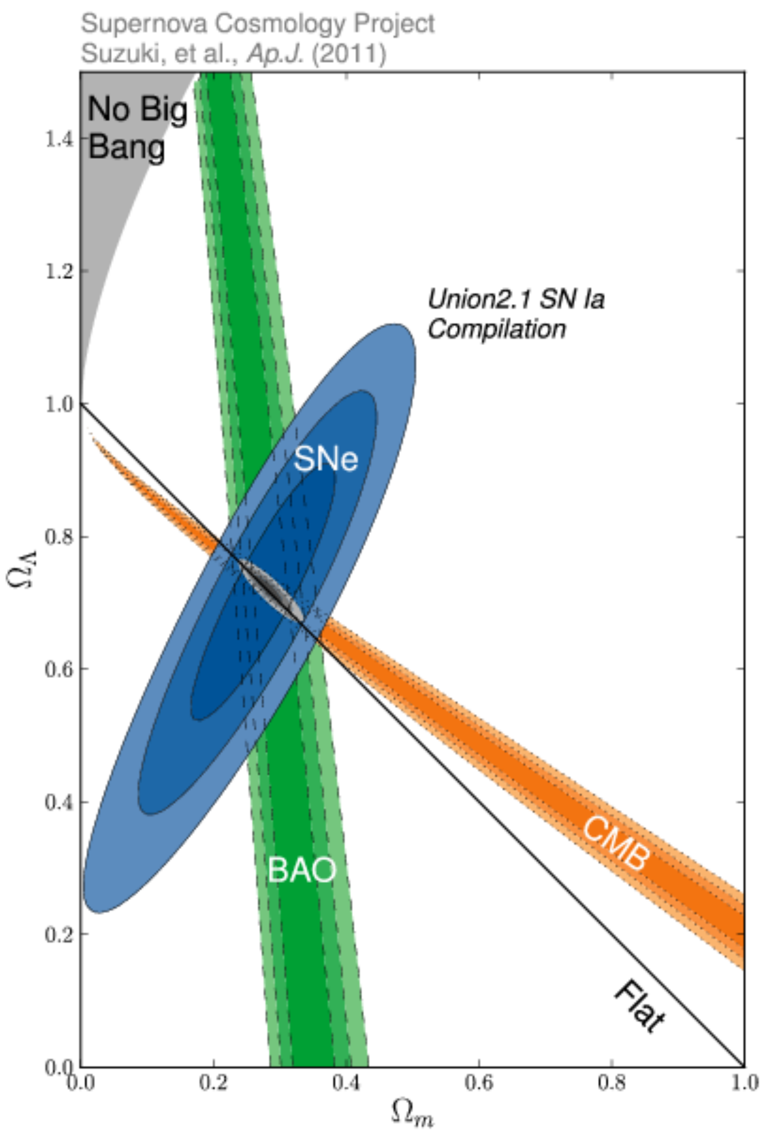
B-mode polarisation only produced by **tensor** perturbations.

measurements of B-modes are important tests for the existence of **primordial gravitational waves**.

Degeneracies

The existence of **strong degeneracies** amongst different combinations of parameters is also noticeable. In particular the well-known **geometrical degeneracy** involving Ω_m, Ω_Λ and the curvature parameter $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$.





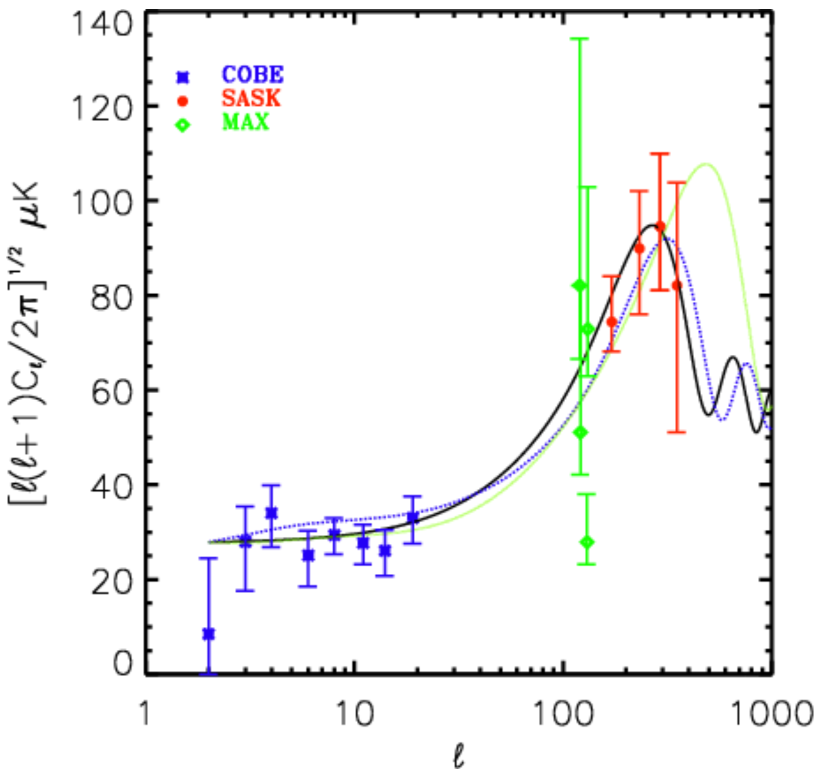
Observations

Model	Abbreviation	Parameters
Cosmological constant	ΛCDM	Ω_k, Ω_m
Constant w	wCDM	Ω_k, Ω_m, w
Varying w (CPL)	CPL	Ω_k, Ω_m, w_0, w_a
Generalized Chaplygin Gas	GCG	Ω_k, A_s, α
Dvali-Gabadadze-Porrati	DGP	Ω_k, Ω_m
Modified Polytropic Cardassian	MPC	Ω_k, Ω_m, q, n
Interacting Dark Energy	IDE	Ω_k, Ω_m, w_x, δ
Early Dark Energy	EDE	Ω_k, Ω_m, Ω_e, w_0

let the observations decide …

Observations

Rapid advance in the development of powerful observational-instruments has led to the establishment of **precision cosmology**.

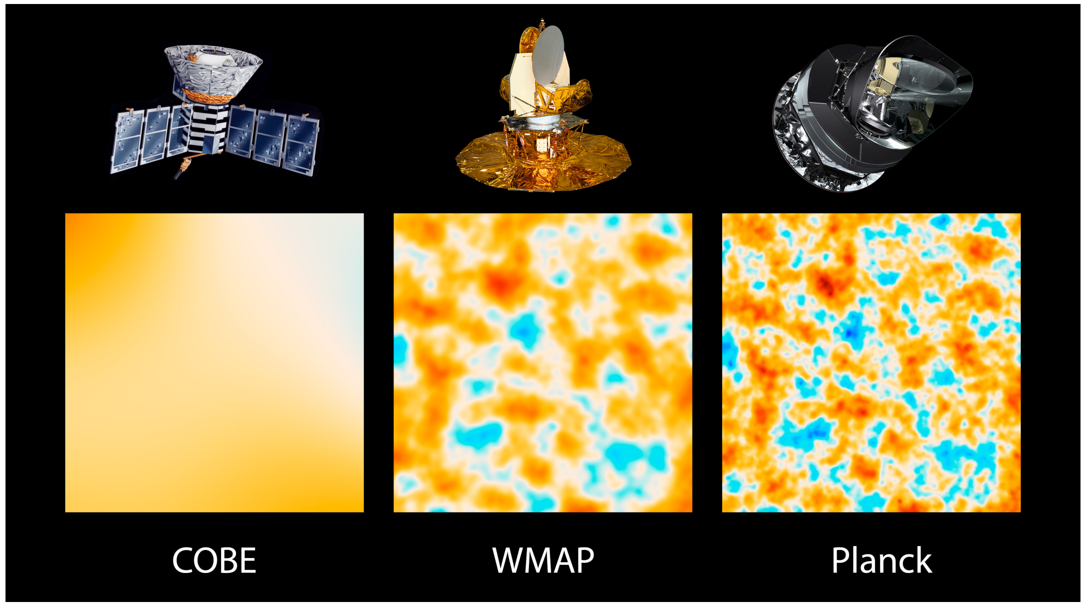


Satellite experiments:

COBE

Wilkinson Microwave Anisotropy Probe

Planck



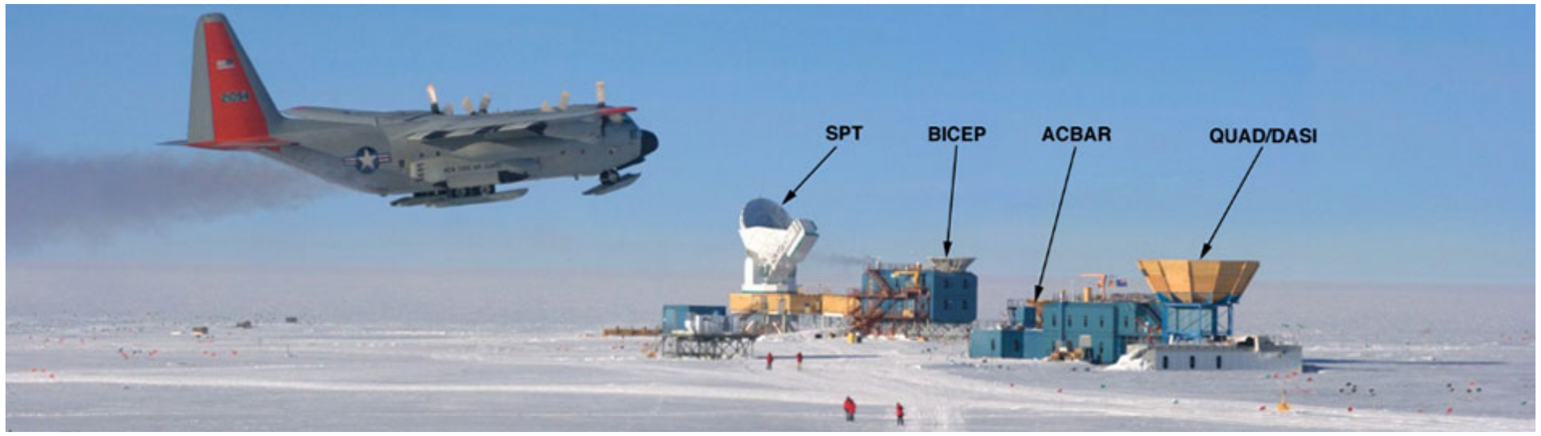
Observations

Ground-based telescopes:

- The Background Imaging of Cosmic Extragalactic Polarization
- The Quest (Q and U Extra-Galactic Sub-mm Telescope) at DASI (Degree Angular Scale Interferometer)
- The Atacama Cosmology Telescope [ACT
- The South Pole Telescope [SPT

Ballon-borne experiments:

- Balloon Observations Of Millimetric Extragalactic Radiation AND Geophysics[BOOMERanG



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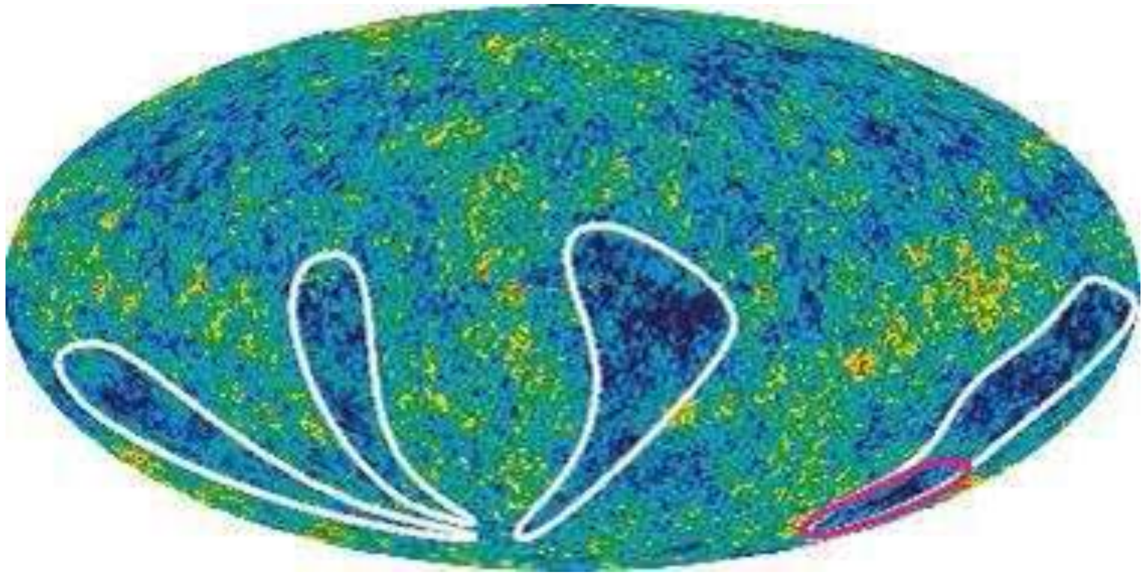
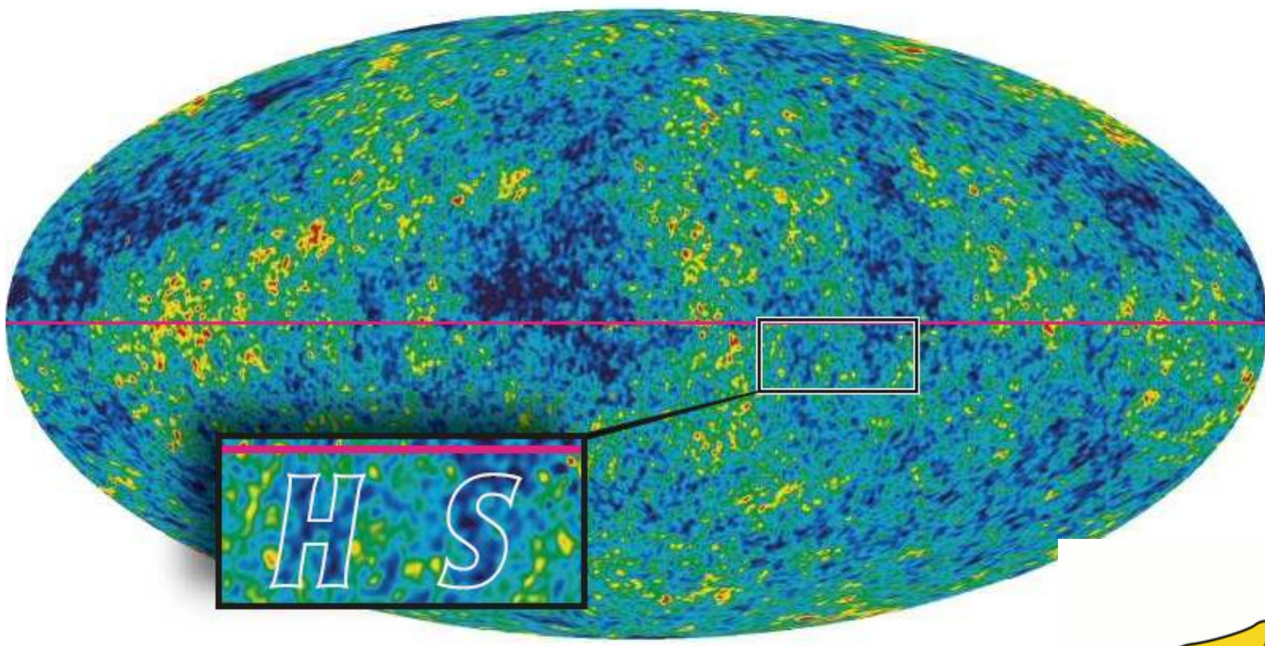
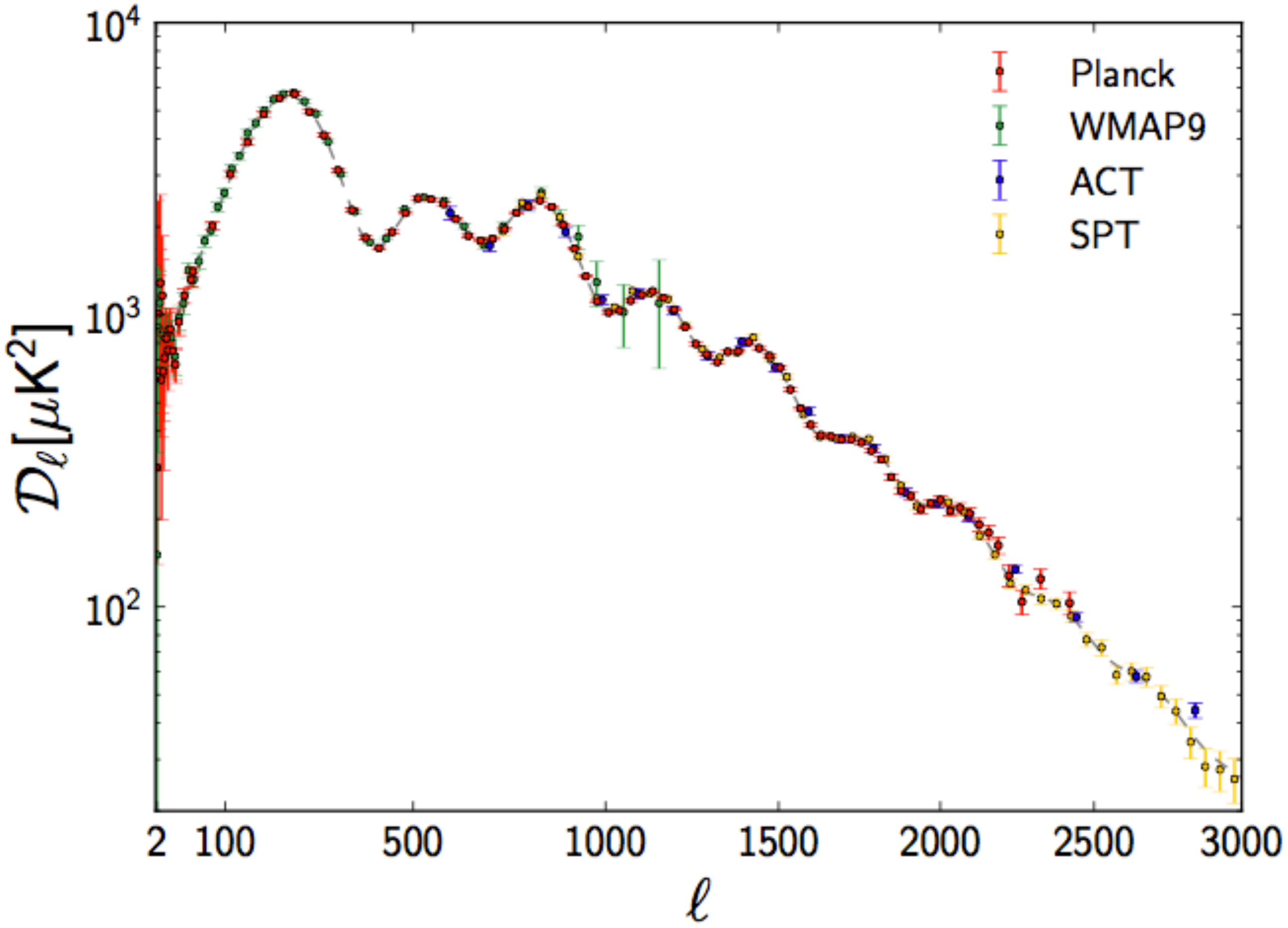
more Observations

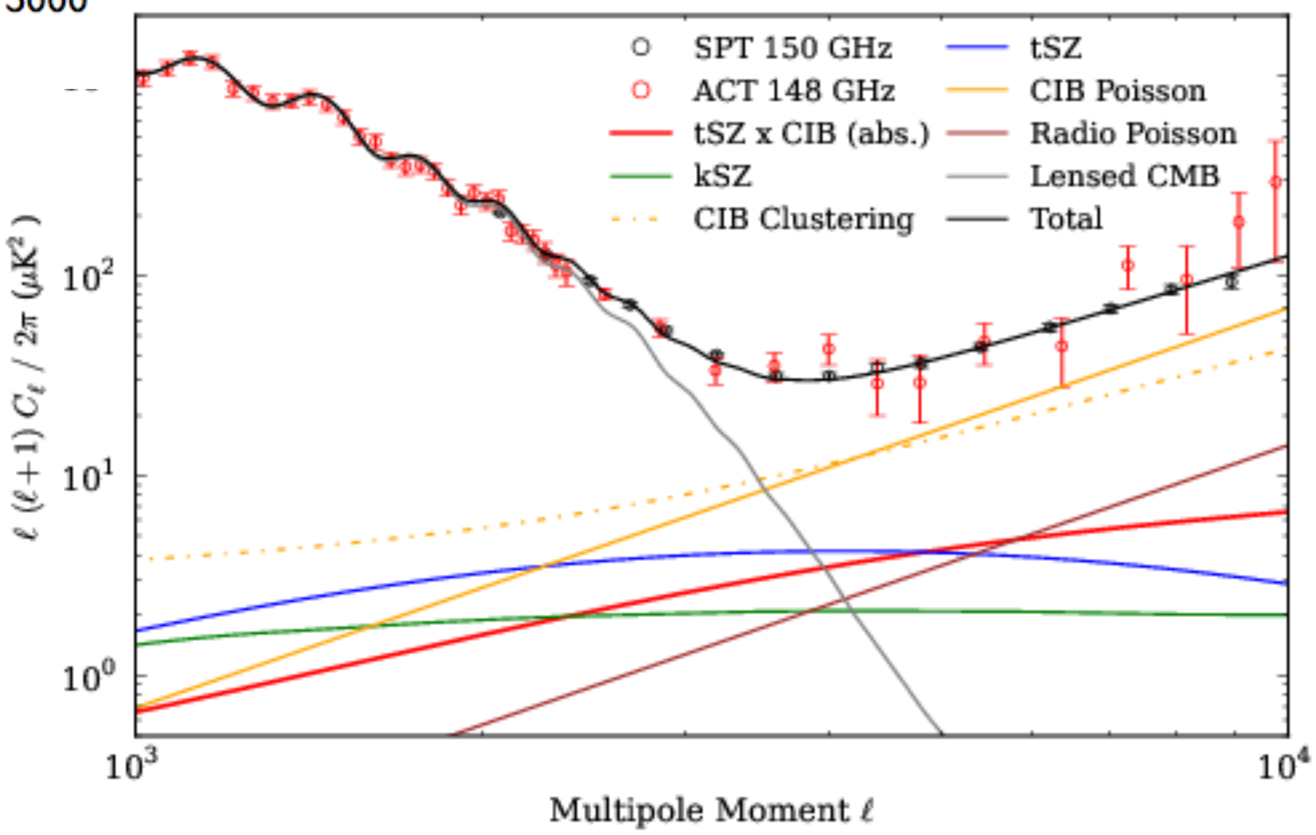



H S

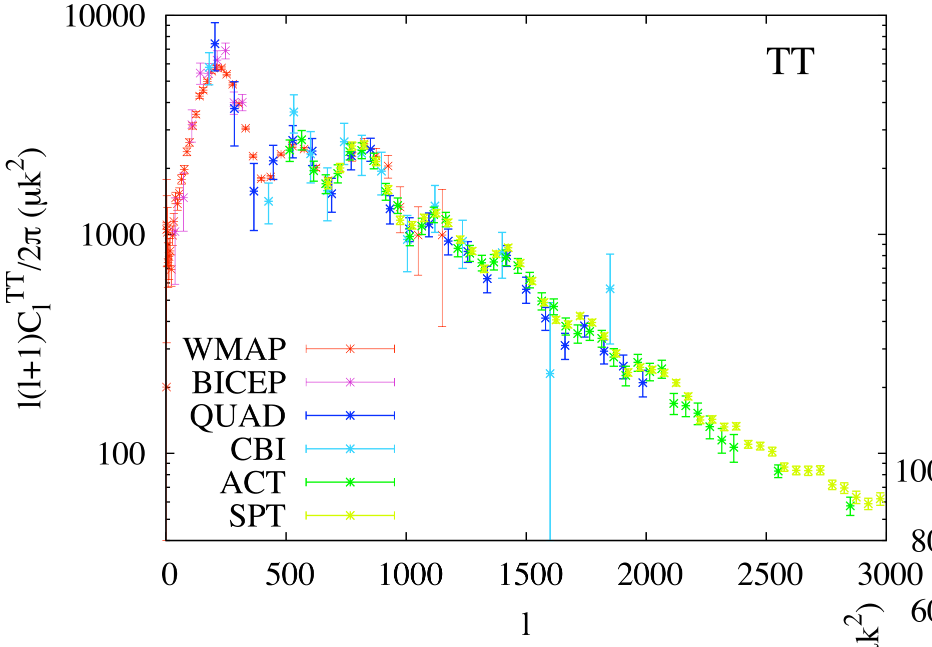


Planck
WMAP9
ACT
SPT
10^4
10^3
10^2
$\mathcal{D}_\ell[\mu K^2]$
2 100 500 1000 1500 2000 2500 3000
ℓ

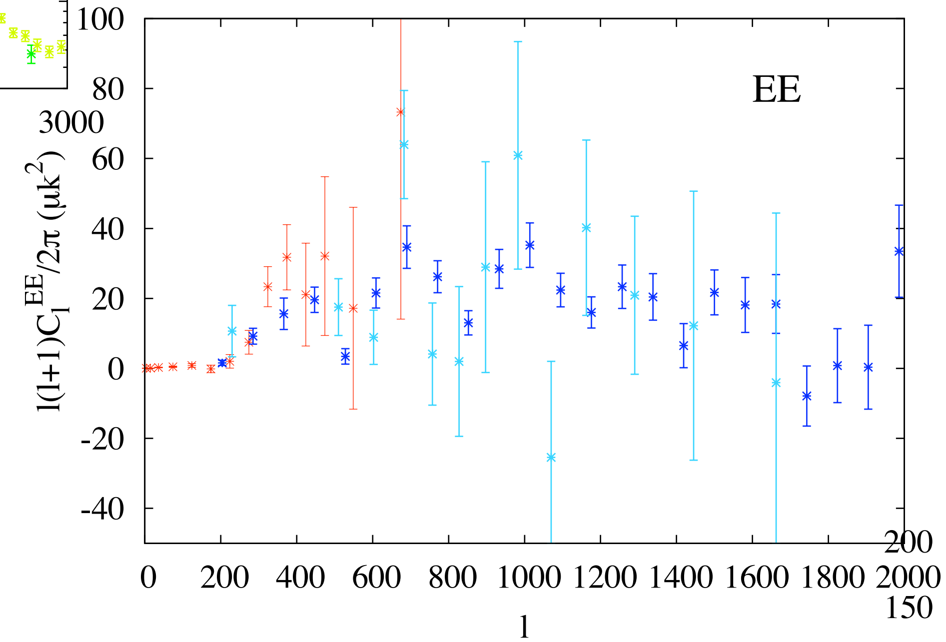
TT

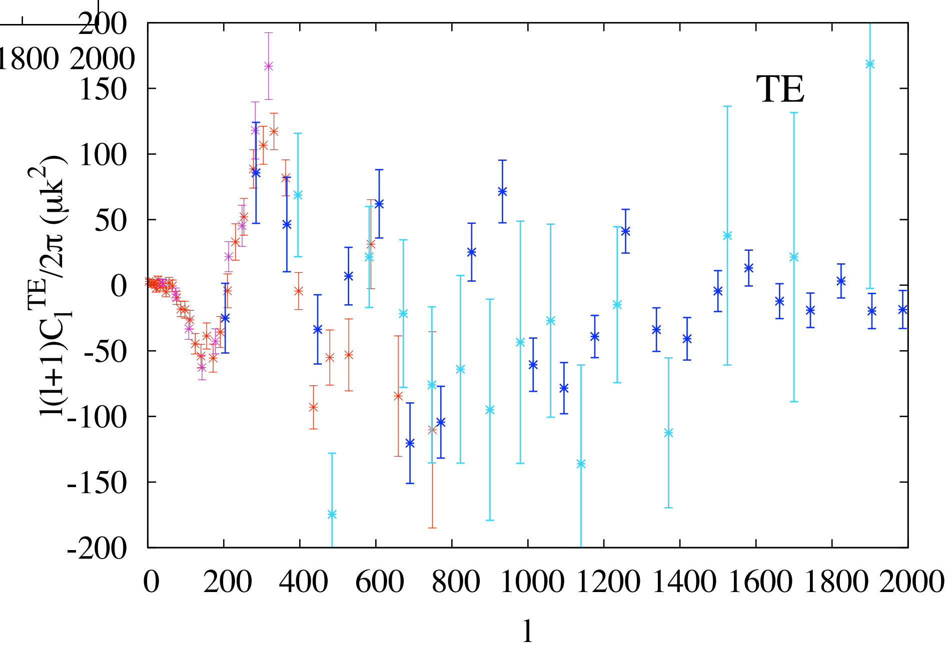


SPT 150 GHz
ACT 148 GHz
tSZ x CIB (abs.)
kSZ
CIB Clustering
tSZ
CIB Poisson
Radio Poisson
Lensed CMB
Total
$\ell(\ell+1)C_\ell/2\pi\ (\mu K^2)$
10^0
10^1
10^2
10^3
10^4
Multipole Moment ℓ



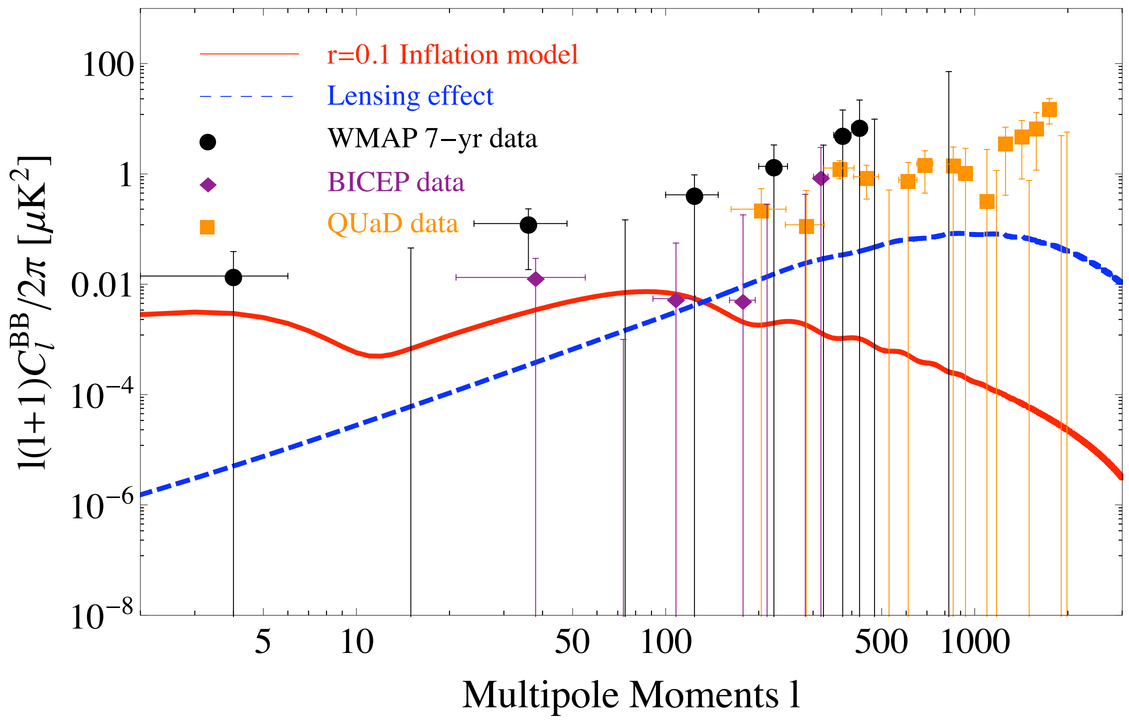
EE
TE

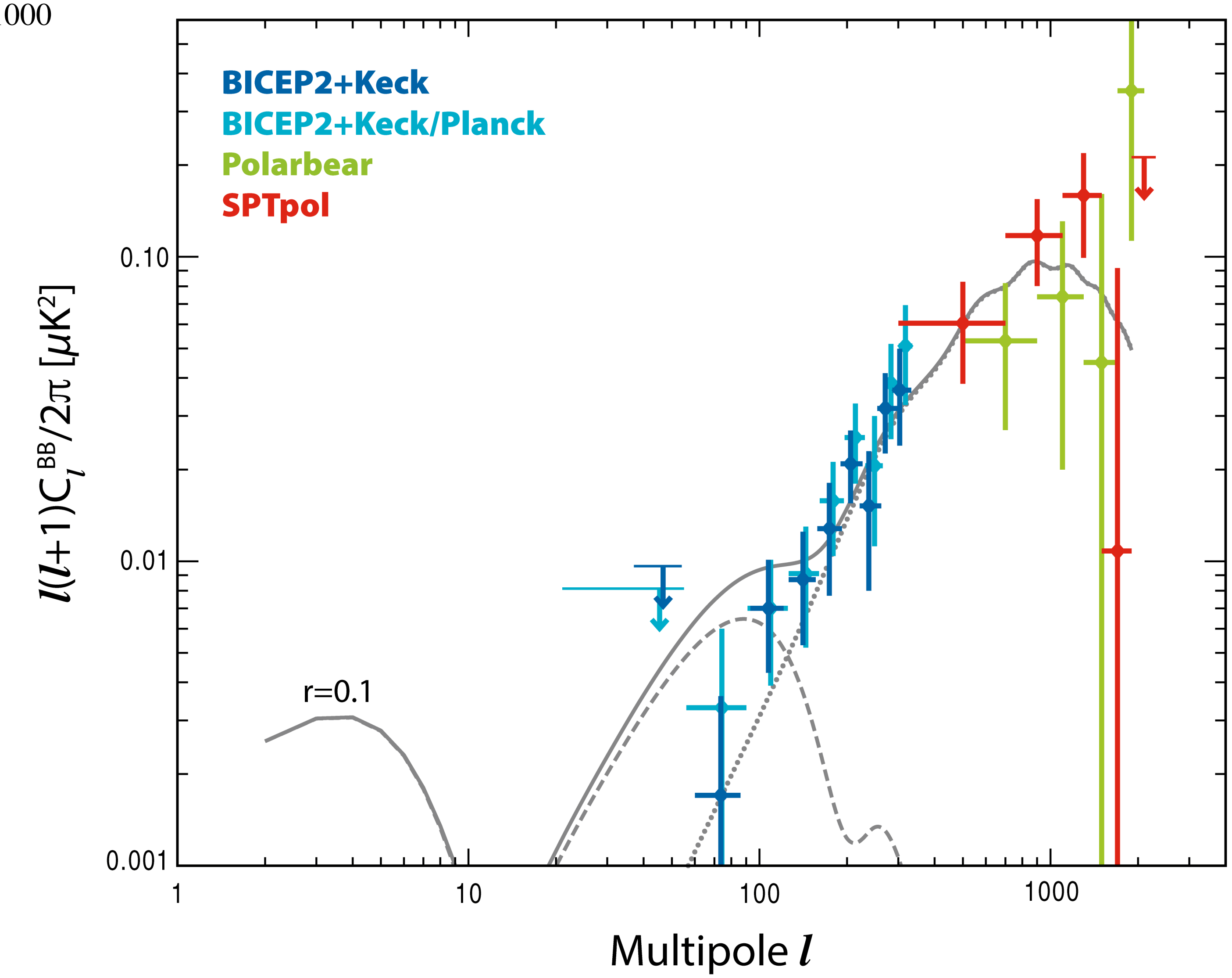




type of noise ?

BB





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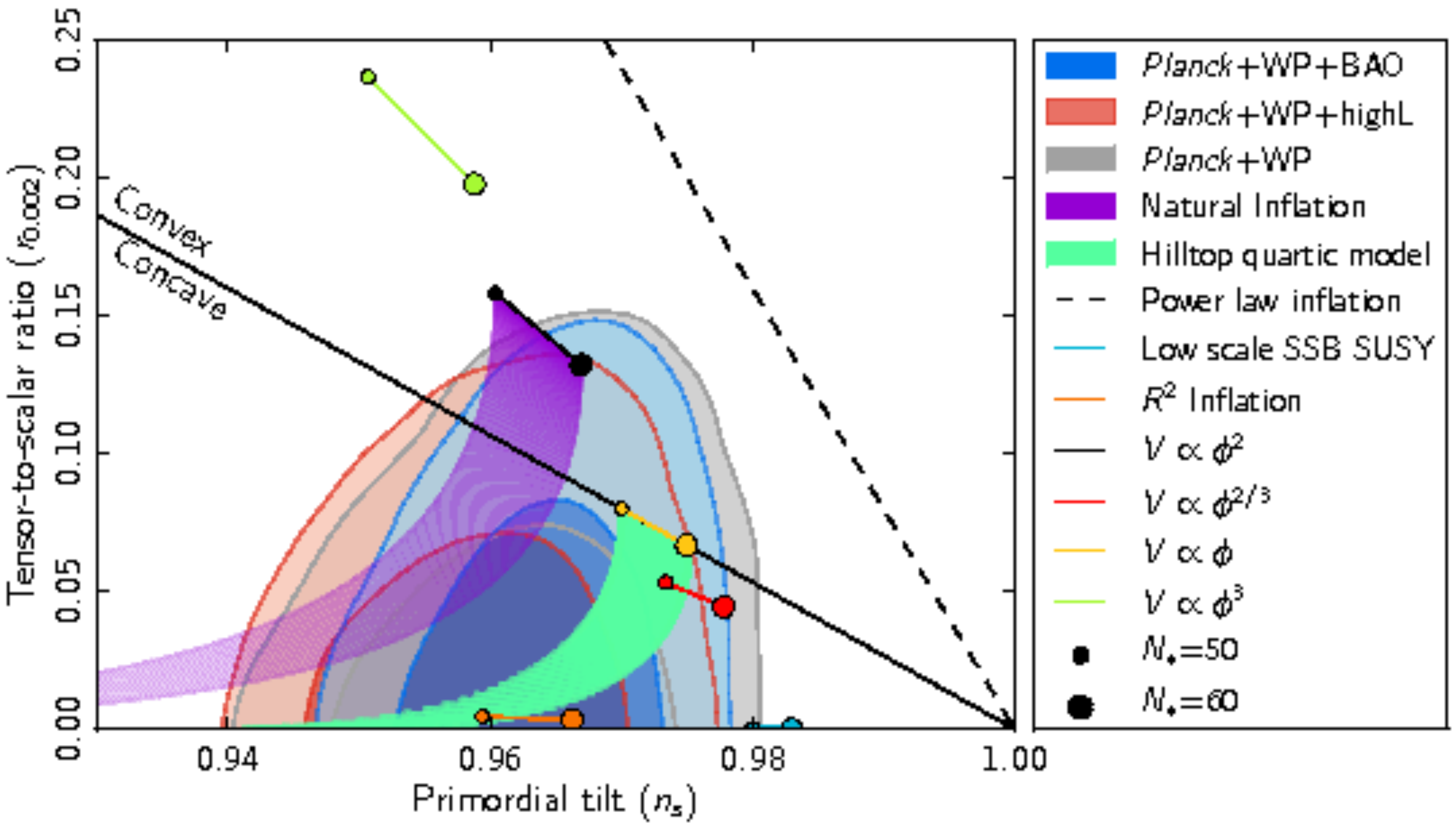
Constraints on inflationary models



Forecast

Here, we aim to explore future constraints **coming from experiments**

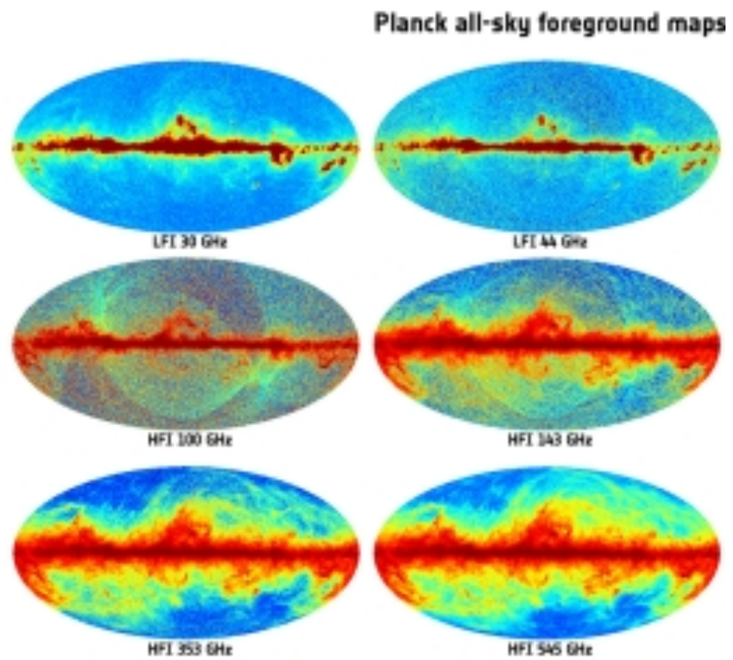
we need to **simulate these experiments** by generating mock data of the $\hat{C}^{XY}$'s from a χ^2_{l+1} distribution with variances

$$(\Delta\hat{C}_l^{XX})^2 = \frac{2}{(2l+1)f_{sky}}\left(C_l^{XX}+N_l^{XX}\right)^2,$$
$$(\Delta\hat{C}_l^{TE})^2 = \frac{2}{(2l+1)f_{sky}}\left[\left(C_l^{TE}\right)^2+\left(C_l^{TT}+N_l^{TT}\right)\left(C_l^{EE}+N_l^{EE}\right)\right],$$

fsky is the fraction of the observed sky. N^{XY} the instrumental noise spectra for each experiment.

In experiments with multiple frequency channels c, the noise spectrum is approximated

$$N_l^X = \left(\sum_c \frac{1}{N_{l,c}^X}\right)^{-1}, \qquad N_{l,c}^X = (\sigma_{\rm pix}\,\theta_{\rm fwhm})^2 \exp\left[l(l+1)\frac{\theta^2_{\rm fwhm}}{8\ln 2}\right]\delta_{XY}.$$



The noise per pixel $\sigma^X_{\rm pix}$ (and $\sigma^P_{\rm pix} = \sqrt{2}\sigma^T_{\rm pix}$) depends on the instrumental parameters;

$\theta_{\rm fwhm}$ full width at half maximum (FHWM) of the Gaussian beam.

Cosmic Variance

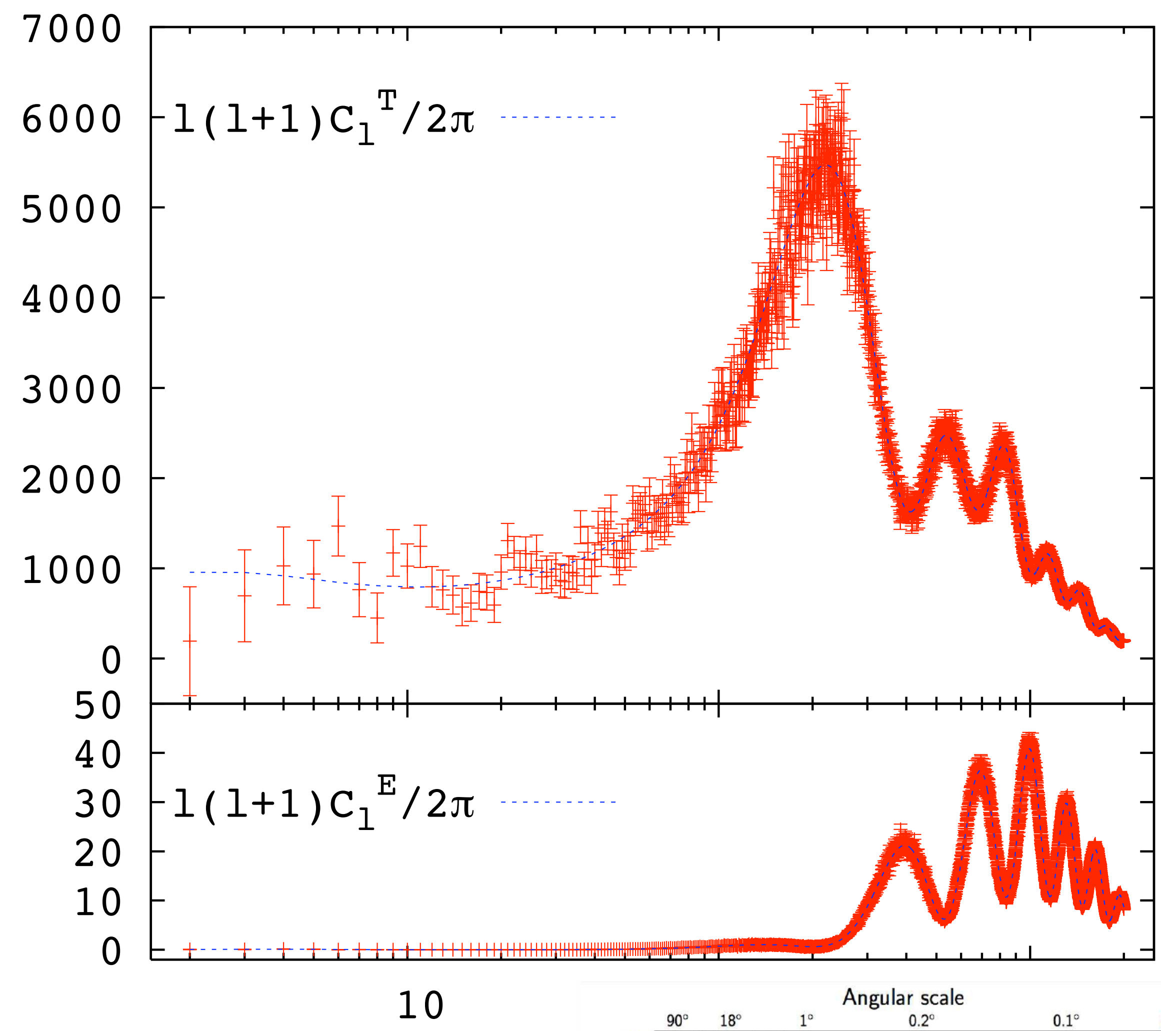
$$(\Delta\hat{C}_l^{XX})^2 = \frac{2}{(2l+1)f_{sky}}\left(C_l^{XX}+N_l^{XX}\right)^2,$$

$$(\Delta\hat{C}_l^{TE})^2 = \frac{2}{(2l+1)f_{sky}}\left[\left(C_l^{TE}\right)^2+\left(C_l^{TT}+N_l^{TT}\right)\left(C_l^{EE}+N_l^{EE}\right)\right],$$

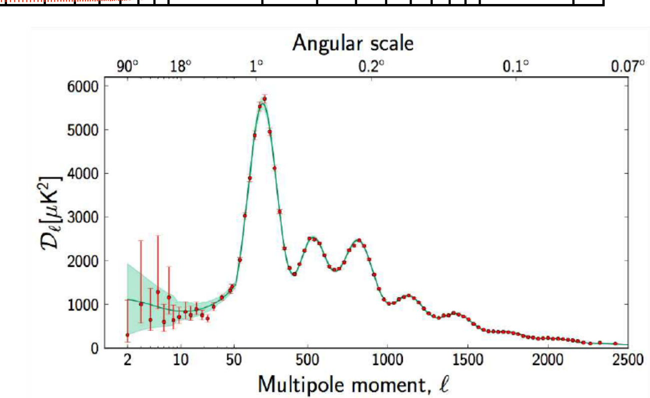
For a given multipole l, we expect to have a variance, **called the cosmic variance**, of the Cl's given by

$$(\Delta C_l)^2 = \frac{2}{2l+1}C_l^2.$$

In real experiments, the error is increased due to the limited sky coverage by f^{-1} .

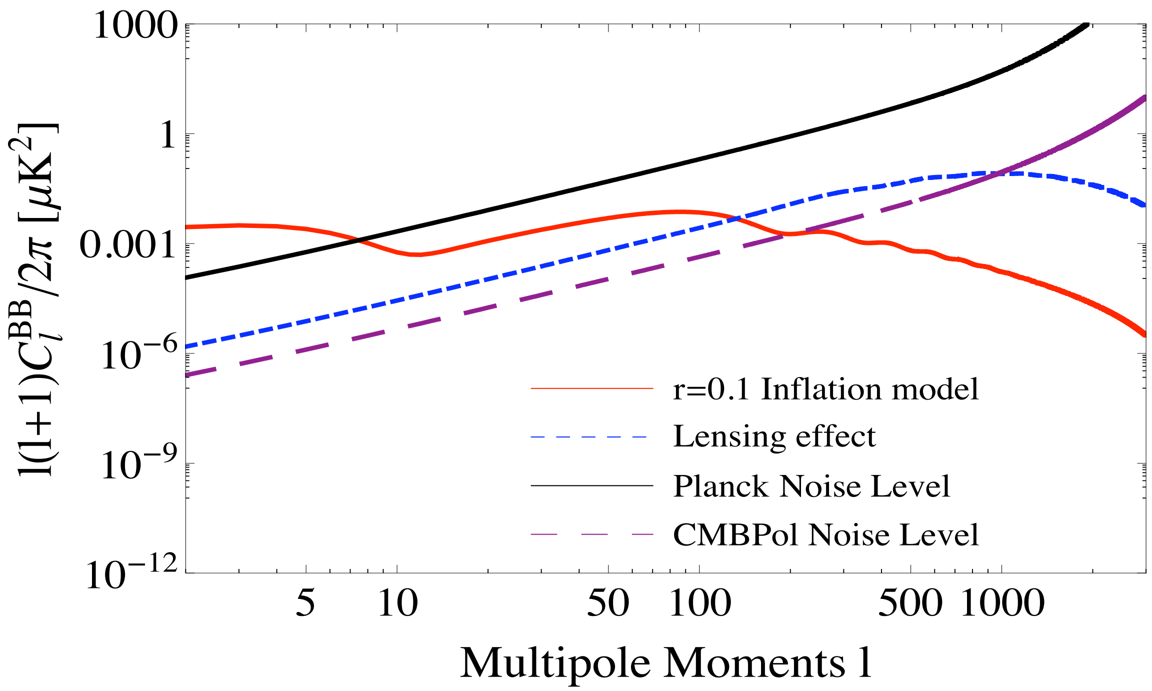


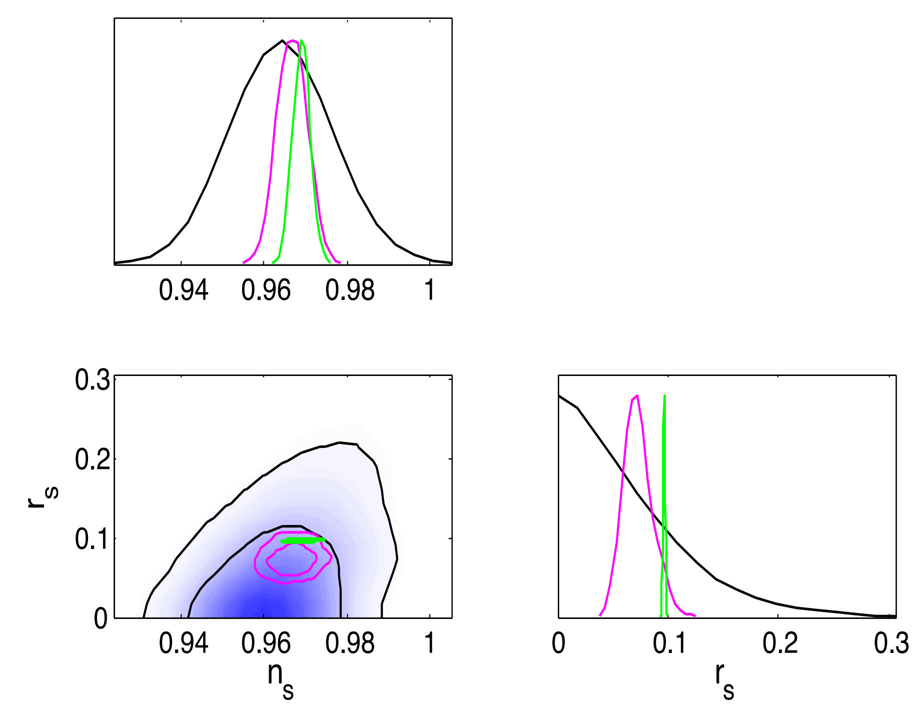
evitarla ?



Planck

For the Planck experiment, we include three channels with frequencies (100 GHz, 143 GHz, 217 GHz) and noise levels per beam $(\sigma_p{}^T{}_{ix})^2$= (46.25 μK^2, 36 μK^2, 171 μK^2). The FHWM of the three channels are θ_{fwhm} =(9.5, 7.1, 5.0) arc-minute.





MCMC Example

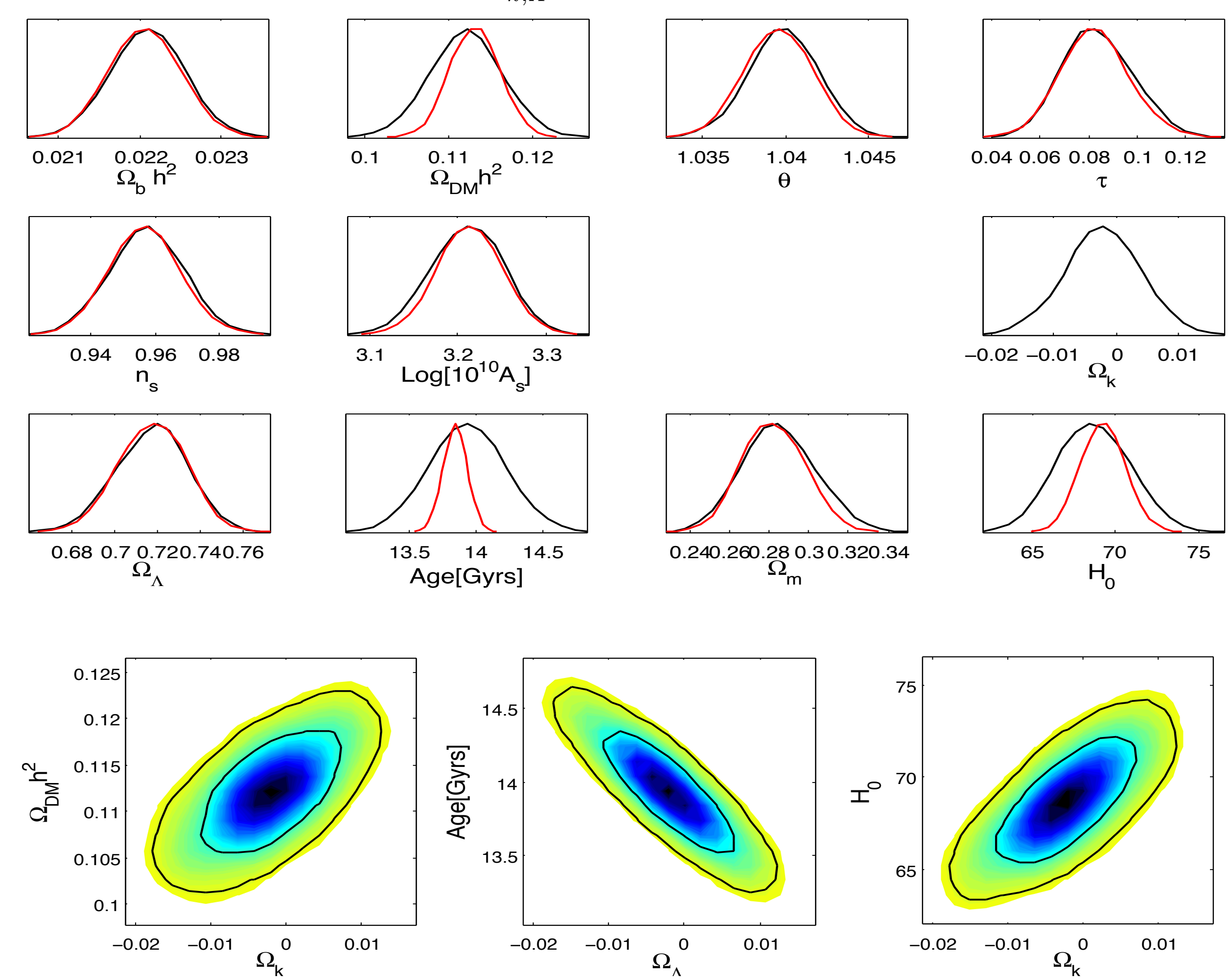
They are not, however, **predicted** by any fundamental theory, rather **we have to fit them by hand** in order to determine which **combination** best describes the current astrophysical observations

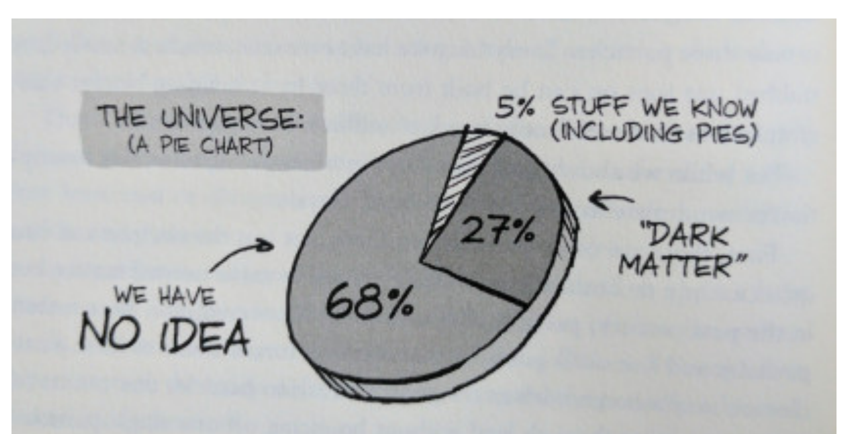
Parameters	Description	Prior range
Background		
$\Omega_{\mathrm{b},0}h^2$	Physical baryon density	$[0.01, 0.03]$
$\Omega_{\mathrm{dm},0}h^2$	Physical cold dark matter density	$[0.01, 0.3]$
θ	Ratio of the sound horizon to the angular diameter distance	$[1, 1.1]$
τ	Reionization optical depth	$[0.01, 0.3]$
Inflationary		
$\log[10^{10}A_{\mathrm{s}}]$	Curvature perturbation amplitude	$[2.5, 4]$
n_{s}	Spectral scalar index	$[0.5, 1.2]$
Secondary		
A_{SZ}	Sunyaev-Zel'dovich amplitude	$[0, 3]$
A_{c}	Total Poisson power	$[0, 20]$
A_{p}	Amplitude of the clustered power	$[0, 30]$

MCMC Example

Description		Flat ΛCDM	Non-flat ΛCDM
	$\Omega_{b,0}h^2$	0.02206 ± 0.00042	0.0221 ± 0.00043
	$\Omega_{dm,0}h^2$	0.1130 ± 0.0028	0.112 ± 0.0041
Base	θ	1.039 ± 0.0019	1.039 ± 0.0020
parameters	τ	0.082 ± 0.013	0.083 ± 0.014
	n_s	0.956 ± 0.010	0.957 ± 0.011
	$\log[10^{10}A_s]$	3.21 ± 0.035	3.21 ± 0.039
	$\Omega_{k,0}$	-	-0.0022 ± 0.0058
	$\Omega_{m,0}$	0.282 ± 0.015	0.285 ± 0.018
Derived	$\Omega_{\Lambda,0}$	0.717 ± 0.015	0.717 ± 0.016
parameters	H_0	69.2 ± 1.27	68.7 ± 2.13
	Age(Gyrs)	13.84 ± 0.086	13.93 ± 0.27
	$-2\ln\mathcal{L}_{max}$	8240.46	8240.80
Bayes factor	$\mathcal{B}_{\Lambda,\Lambda+\Omega_k}$	$+1.6 \pm 0.4$	-
Dataset consistency	$\mathcal{B}_R$	$+5.06 \pm 0.4$	$+5.07 \pm 0.4$

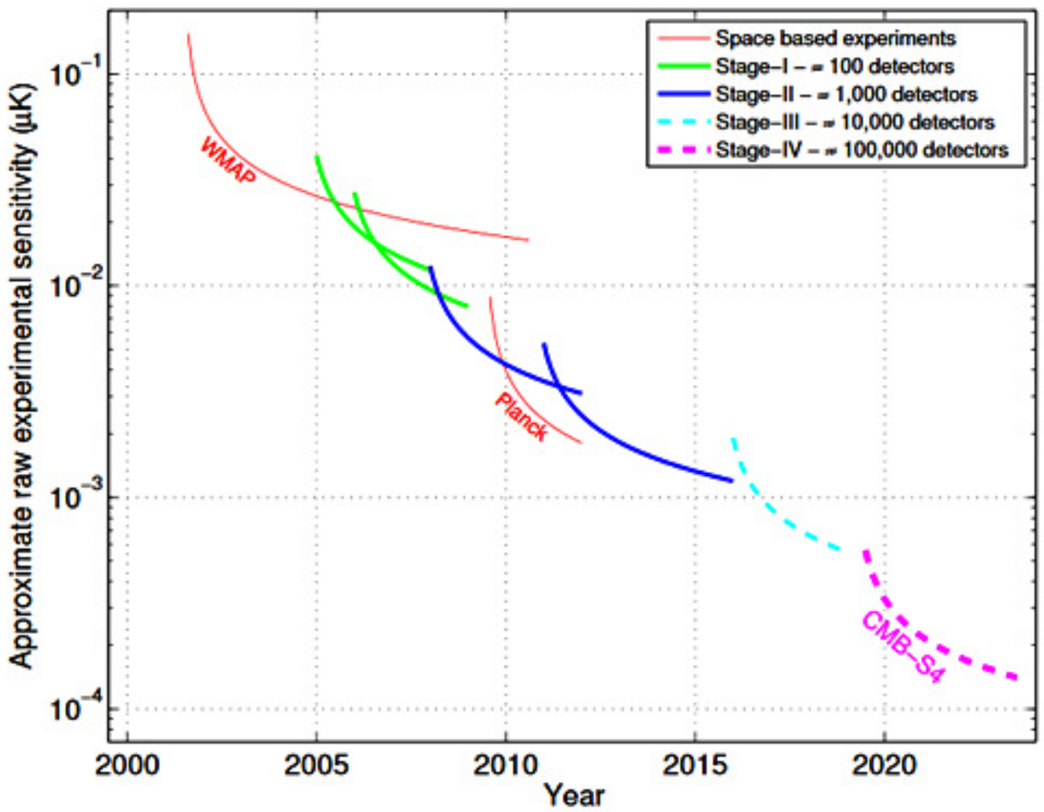
$\mathcal{B}_{k,\Lambda} = -1.90 \pm 0.35$

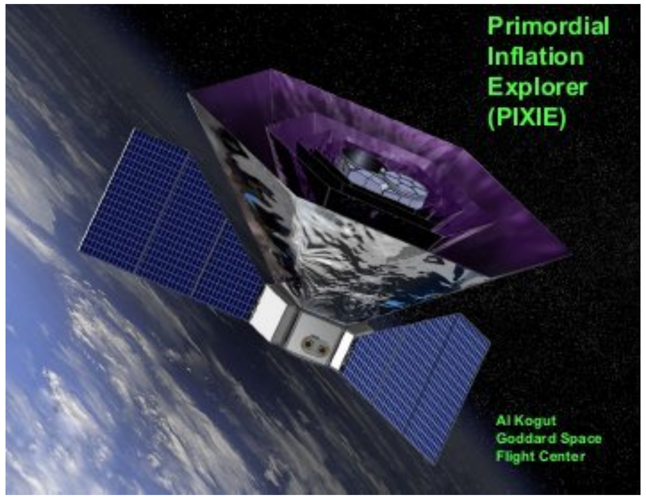




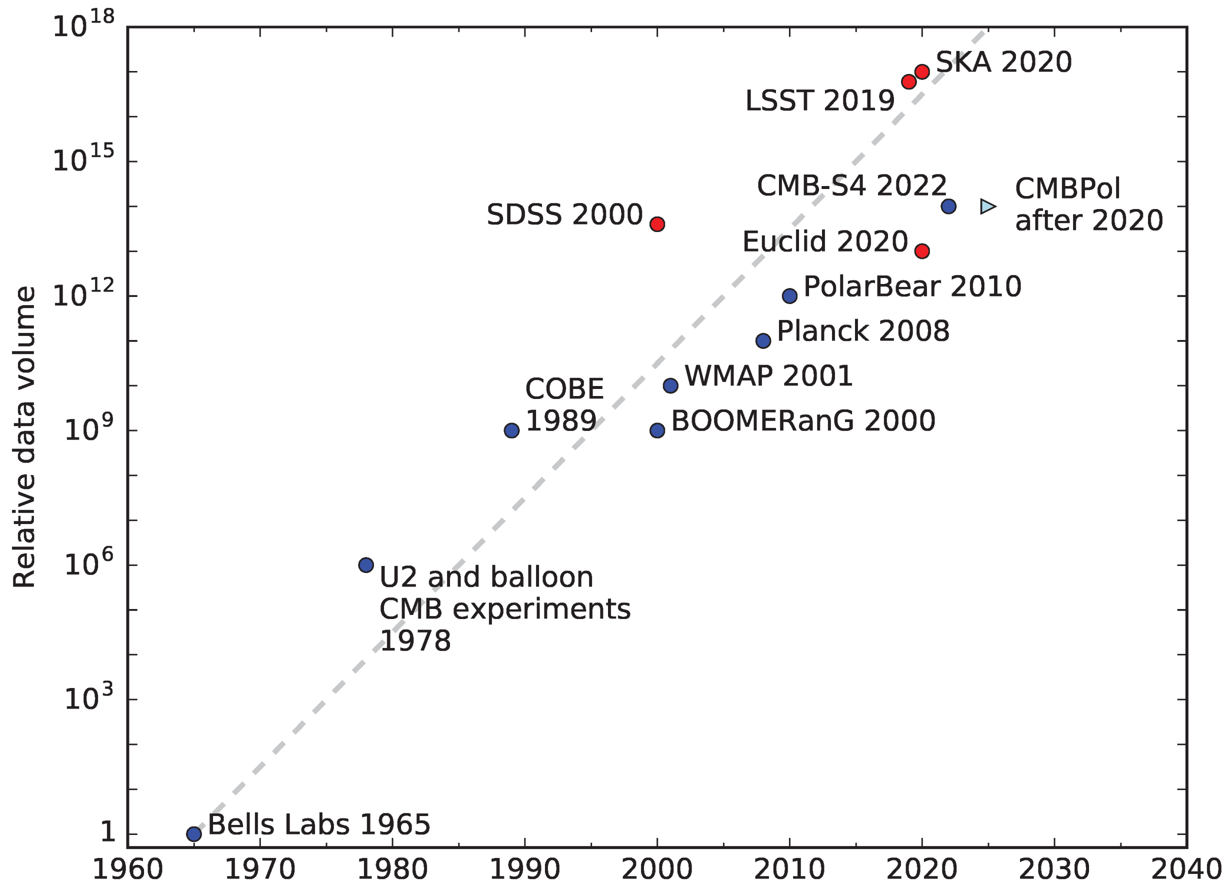
JAVazquez

Future



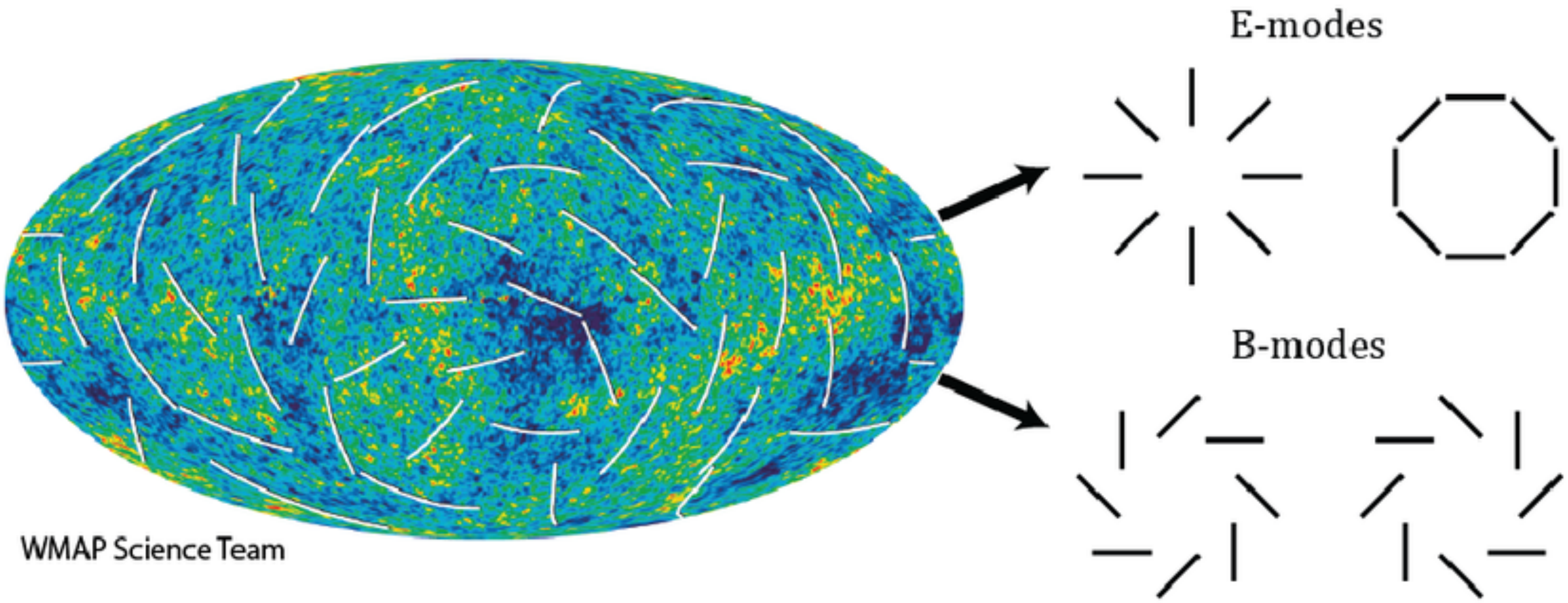


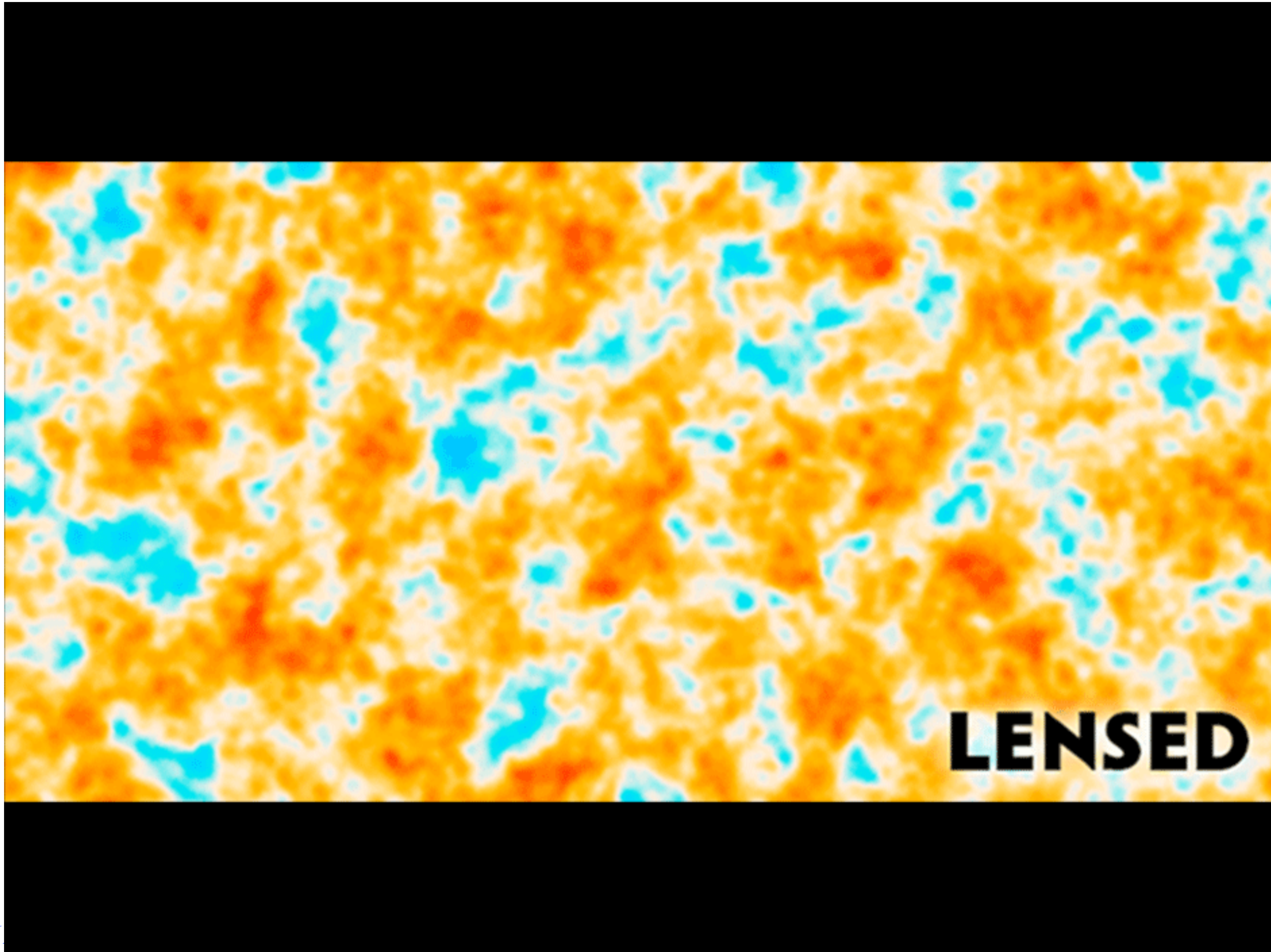
r < 10^{-3} at 5 standard deviations



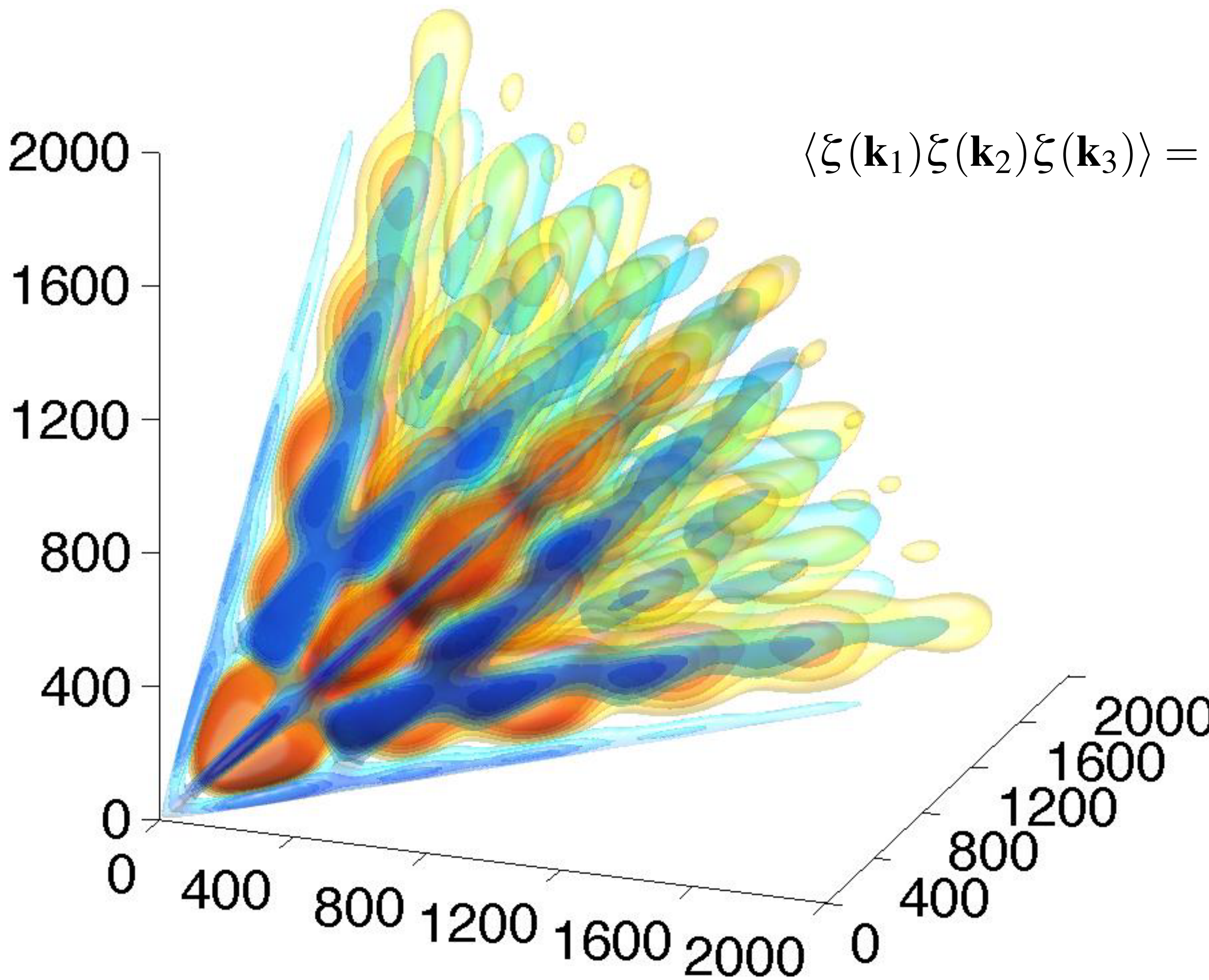
Constraints on inflationary models

Tensor perturbations




LENSED

Non-Gaussianity



$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = B_\zeta(k_1,k_2,k_3)\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,.$$

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL} \left[\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2 \rangle\right]$$

f_{NL}: $-4 < f_{NL} < 80$ (95% C.L.)

Examen sorpresa!