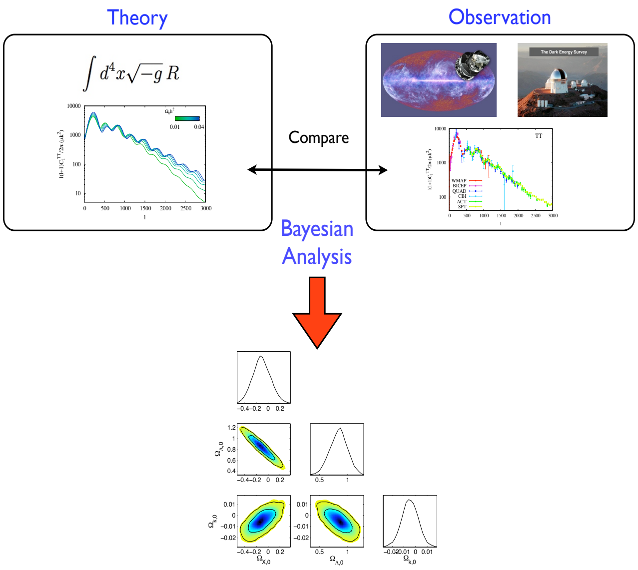
Updated Cosmology

with Python



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In progress

August 12, 2017

0.4 World Models

Video: 9 types of Universes that will surprise you: link.

In the previous chapter, we have considered different types of Universes modelled by only single components, however in order to do our description of the Universe more realistic we need to incorporate some mixtures of these ingredients.

Let us take the equation for the total density $\Omega + \Omega_k = 1$ (39) and the equation for acceleration $q = \frac{1}{2}\sum_i \Omega_i(1+3w_i)$ (40). There are some lines useful to draw in order to identify the type of Universe we live on:

- **Open-closed line** $(k = 0)$

$$\Omega_{\Lambda,0} = 1 - \Omega_{\mathrm{m},0}. \tag{50}$$

- **Accelerating-decelerating line** $(q = 0)$

$$\Omega_{\Lambda,0} = \frac{1}{2}\Omega_{\mathrm{m},0}. \tag{51}$$

- **Expand-forever-recollapse** & **big bang - no big bang**. It requires a little more work.

In general

$$\dot{a}^2 = a^2 H_0^2(\Omega_{\mathrm{r},0}a^{-4} + \Omega_{\mathrm{m},0}a^{-3} + \Omega_{k,0}a^{-2} + \Omega_{\Lambda,0}) \tag{52}$$

with the condition (39) $\Omega_{\mathrm{r}} + \Omega_{\mathrm{m}} + \Omega_k + \Omega_\Lambda = 1$ for all time, in particular for $a = 1$. The FRW Universes dominated by matter and vacuum energy are named as **Lemaitre models**. From Figure 5, taking the joint constraints, we have that (considering present data): we live in a nearly flat accelerating universe that presents a big bang in the past and will expand forever in the future.

Cosmological models with zero cosmological constant $(\Omega_{\Lambda,0} = 0)$, and strictly non-zero matter or radiation density, are known as the **Friedmann models**.

Dust only Friedmann model $(\Omega_{\mathrm{r},0} = 0,\ \Omega_{k,0} = 1 - \Omega_{\mathrm{m},0})$

From the equation (52), we have

$$\dot{a}^2 = H_0^2(\Omega_{\mathrm{m},0}a^{-1} + 1 - \Omega_{\mathrm{m},0}) \qquad \rightarrow \qquad t = \frac{1}{H_0}\int_0^a \left[\frac{x}{\Omega_{\mathrm{m},0} + (1-\Omega_{\mathrm{m},0})x}\right]^{1/2} dx. \tag{53}$$

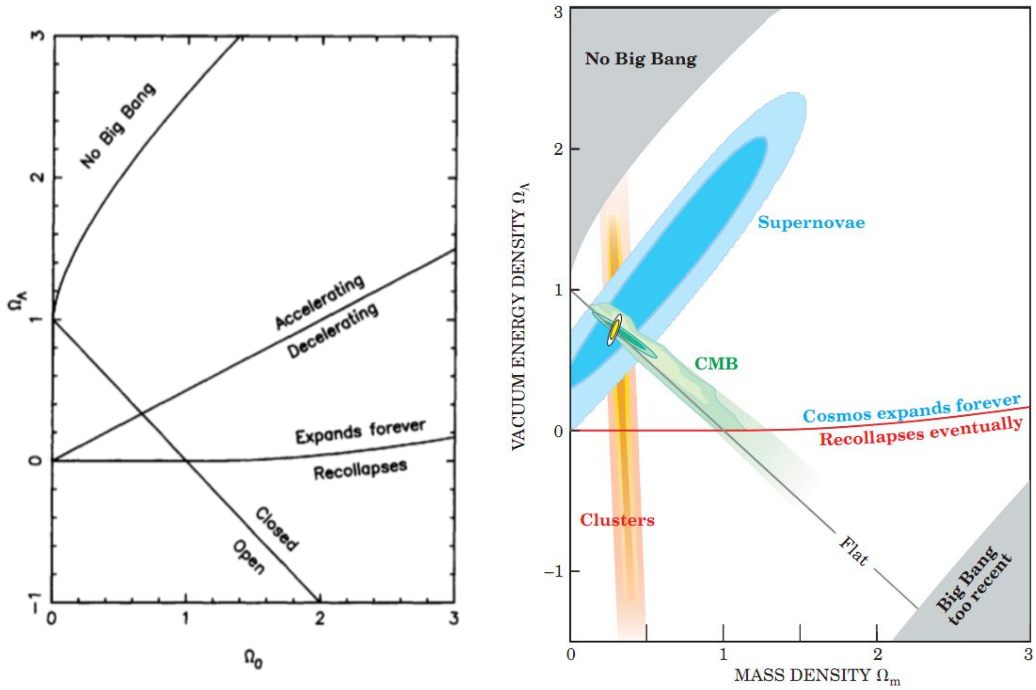

Figure 5: Cosmological constraints using different datasets [I'll do it later].

- Flat Universe ($k = 0$) $\Omega_{m,0} = 1$: This type of Universe is called the **Einstein de-Sitter model**, and we have seen the behaviour before:

$$a(t) = \left(\frac{3}{2} H_0 t\right)^{2/3}. \tag{54}$$

- For spherical Universe $\Omega_{m,0} > 1$ ($k = 1$), we write

$$x = \left[\frac{\Omega_{m,0}}{\Omega_{m,0} - 1} \sin^2 \psi/2\right], \qquad \psi = [0, \pi], \tag{55}$$

and have

$$a(t) = \frac{\Omega_{m,0}}{2(\Omega_{m,0} - 1)}(1 - \cos\psi), \qquad t = \frac{\Omega_{m,0}}{2H_0(\Omega_{m,0} - 1)^{3/2}}(\psi - \sin\psi), \tag{56}$$

where the first term represents the expression for a cycloid, see Figure 6.

- For hyperbolic Universe $\Omega_{m,0} < 1$ ($k = -1$), we write

$$x = \left[\frac{\Omega_{m,0}}{1 - \Omega_{m,0}} \sinh^2 \psi/2\right], \qquad \psi = [0, \pi]. \tag{57}$$

and have

$$a(t) = \frac{\Omega_{m,0}}{2(1 - \Omega_{m,0})}(\cosh\psi - 1), \qquad t = \frac{\Omega_{m,0}}{2H_0(1 - \Omega_{m,0})^{3/2}}(\sinh\psi - \psi). \tag{58}$$

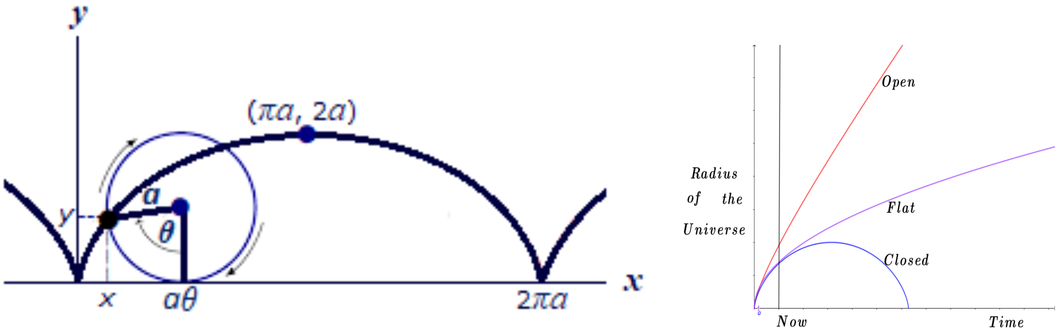

Figure 6: [Redo this universe]

Radiation domination ($\Omega_{\mathrm{m},0} = 0$, $\Omega_{k,0} = 1 - \Omega_{\mathrm{r},0}$)

$$\dot{a}^2 = H_0^2(\Omega_{\mathrm{r},0}a^{-2} + 1 - \Omega_{\mathrm{r},0}) \qquad \rightarrow \qquad t = \frac{1}{H_0}\int_0^a \left[\frac{x}{\sqrt{\Omega_{\mathrm{r},0} + (1-\Omega_{\mathrm{r},0})x^2}}\right] dx. \tag{59}$$

- Flat Universe $\Omega_{\mathrm{r},0} = 1$ ($k = 0$):

$$a(t) = (2H_0 t)^{1/2}. \tag{60}$$

- Spherical $\Omega_{\mathrm{r},0} < 1$ ($k = -1$) or Hyperbolic $\Omega_{\mathrm{r},0} > 1$ ($k = 1$):

$$a(t) = (2H_0\Omega_{\mathrm{r},0}^{1/2}t)^{1/2}\left(1 + \frac{1-\Omega_{\mathrm{r},0}}{2\Omega_{\mathrm{r},0}^{1/2}}H_0 t\right)^{1/2}. \tag{61}$$

Spatially flat ($\Omega_{k,0} = 0$, $\Omega_{\mathrm{m},0} + \Omega_{\mathrm{r},0} = 1$)

$$\dot{a}^2 = H_0^2(\Omega_{\mathrm{m},0}a^{-1} + \Omega_{\mathrm{r},0}a^{-2}) \qquad \rightarrow \qquad t = \frac{1}{H_0}\int_0^a \left[\frac{x}{\sqrt{\Omega_{\mathrm{m},0}x + \Omega_{\mathrm{r},0}}}\right] dx, \tag{62}$$

doing $y = \Omega_{\mathrm{m},0}x + \Omega_{\mathrm{r},0}$

$$H_0 t = \frac{2}{3\Omega_{\mathrm{m},0}^2}\left[(\Omega_{\mathrm{m},0}a + \Omega_{\mathrm{r},0})^{1/2}(\Omega_{\mathrm{m},0}a - 2\Omega_{\mathrm{r},0}) + 2\Omega_{\mathrm{r},0}^{3/2}\right]. \tag{63}$$

Cannot be easily inverted to give $a(t)$. Nevertheless $t = \frac{2}{3}a^{3/2}$ for matter only, and $t = \frac{1}{2}a^2$ for radiation, as expected.

Lemaitre models ($\Omega_{\Lambda,0} \neq 0$) but $\Omega_{\mathrm{r},0} = 0$

- Spatially flat ($\Omega_{\mathrm{m},0} + \Omega_{\Lambda,0} = 1$):

$$\dot{a}^2 = H_0^2[(1-\Omega_{\Lambda,0})a^{-1} + \Omega_{\Lambda,0}a^2] \qquad \rightarrow t = \frac{1}{H_0}\int_0^a \sqrt{\frac{x}{(1-\Omega_{\Lambda,0}) + \Omega_{\Lambda,0}x^3}}dx, \tag{64}$$

writing $y^2 = x^3|\Omega_{\Lambda,0}|/(1-\Omega_{\Lambda,0})$, we have then

$$H_0 t = \frac{2}{3\sqrt{|\Omega_{\Lambda,0}|}} \int_0^{\sqrt{a^3|\Omega_{\Lambda,0}|/(1-\Omega_{\Lambda,0})}} \frac{dy}{\sqrt{1 \pm y^2}}, \tag{65}$$

with solutions

$$H_0 t = \frac{2}{3\sqrt{|\Omega_{\Lambda,0}|}} f(x) = \begin{cases} \sinh^{-1}[\sqrt{a^3|\Omega_{\Lambda,0}|(1-\Omega_{\Lambda,0})}], & \Omega_{\Lambda,0} > 0. \\ . \\ \sin^{-1}[\sqrt{a^3|\Omega_{\Lambda,0}|(1-\Omega_{\Lambda,0})}], & \Omega_{\Lambda,0} < 0. \end{cases} \tag{66}$$

- Arbitrary spatial curvature ($\Omega_{k,0} = 1 - \Omega_{\mathrm{m},0} - \Omega_{\Lambda,0}$)

$$\dot{a}^2 = H_0^2(\Omega_{\mathrm{m},0}a^{-1} + \Omega_{\Lambda,0}a^2 + \Omega_{k,k}). \tag{67}$$

Quite complicated, but it may have solutions by using elliptical functions to get

$$a(t) \quad = \quad \left(\frac{3}{2}H_0\sqrt{\Omega_{\mathrm{m},0}t}\right)^{3/2} \qquad \text{small t, radiation domination.} \tag{68}$$

$$a(t) \quad \propto \quad \exp\left(H_0\sqrt{\Omega_{\Lambda,0}}t\right) \qquad \text{large t, } \Lambda \text{ domination.} \tag{69}$$

De-Sitter model ($\Omega_{\mathrm{m},0} = 0,\ \Omega_{\mathrm{r},0} = 0,\ \Omega_{\Lambda,0} = 1 \quad \rightarrow \quad k = 0$)

Not a physical model but interesting to study, specially during inflation, and as we shall see in the following chapter

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2, \tag{70}$$

wit solutions of the form

$$a(t) = \exp[H_0(t - t_0)] = \exp[\sqrt{\Lambda/3}c(t - t_0)]. \tag{71}$$

Anti de-Sitter space (negative cosmological constant?).

Einstein static Universe

Before the discovery of the expansion, Einstein introduced the cosmological constant Λ to get $\dot{a} = \ddot{a} = 0$ which has the following implications

$$4\pi G\rho_{m,0} = \Lambda c^2 = \frac{c^2 k}{a^2}, \tag{72}$$

from the first equality (using the acceleration equation) we have that $\rho_{\mathrm{m},0} = 2\rho_{\Lambda,0}$ and $\Lambda > 0$, and from the second (using the Friedmann equation) we have that $k = 1$, which is disagreement with current observations and also this type of universe is an unstable one.

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