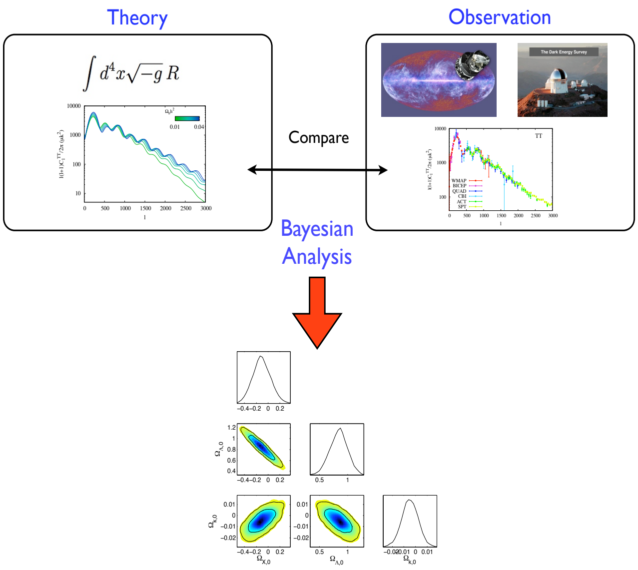
Updated Cosmology

with Python



José-Alberto Vázquez

ICF-UNAM / Kavli-Cambridge

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4

Linearised Evolution Equations

Now it is time for the perturbed Einstein equations $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$.

- We can use two gauge function T y L to set the metric perturbations E and B to zero.

4.1 Conformal Newtonian Gauge

$$ds^2 = a^2[(1+2\psi)d\tau^2 - (1-2\phi)\delta_{ij}dx^i dx^j], \tag{4.1}$$

with $\phi = \Phi$ and $\psi = \Psi$.

Perturbed connections coefficients

We first require to compute the Christoffel symbols for the metric (4.1) and its inverse

$$g^{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} 1-2\psi & 0 \\ 0 & -(1-2\phi)\delta^{ij} \end{pmatrix}. \tag{4.2}$$

$$\text{Remember :} \qquad \Gamma^{\mu}_{\nu\rho} = \frac{1}{2}g^{\mu k}(\partial_\nu g_{k\rho} + \partial_\rho g_{k\nu} - \partial_k g_{\nu\rho}), \tag{4.3}$$

$$\begin{aligned} \Gamma^0_{00} &= \frac{1}{2}g^{00}(2\partial_\tau g_{00} - \partial_\tau g_{00}) = \frac{1}{2}g^{00}\partial_\tau g_{00} \\ &= \frac{1}{2a^2}(1-2\psi)\partial_\tau[a^2(1+2\psi)] = \mathcal{H} + \psi'. \end{aligned} \tag{4.4}$$

Homework

$$\Gamma^0_{0i} = \partial_i\psi \tag{4.5}$$

$$\Gamma^i_{00} = \delta^{ij}\partial_j\psi \tag{4.6}$$

$$\Gamma^0_{ij} = \mathcal{H}\delta_{ij} - [\phi' + 2\mathcal{H}(\phi+\psi)]\delta_{ij} \tag{4.7}$$

$$\Gamma^i_{j0} = \mathcal{H}\delta^i_j - \phi'\delta^i_j \tag{4.8}$$

$$\Gamma^i_{jk} = -2\delta^i_{(j}\partial_{k)}\phi + \delta_{jk}\delta^{il}\partial_l\phi \tag{4.9}$$

4.2 Perturbed Stress-Energy Conservation

$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$ implies conservation of energy and momentum via the contracted Bianchi identity

$$\nabla^\mu G_{\mu\nu} = 0 \qquad \rightarrow \qquad \nabla^\mu T_{\mu\nu} = 0,$$

but it's more convenient to work with the mixed components

$$\nabla_\mu T^\mu_{\ \nu} = 0 \qquad \text{or} \qquad \partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\rho}T^\rho_\nu - \Gamma^\rho_{\mu\nu}T^\mu_\rho = 0. \tag{4.10}$$

Continuity equation

The $\nu = 0$ component of (4.10)

$$\partial_0 T^0_{\ 0} + \partial_i T^i_{\ 0} + \cancel{\Gamma^0_{00}T^0_{\ 0}} + \Gamma^i_{i0}T^0_{\ 0} + \underbrace{\Gamma^0_{0i}T^i_{\ 0}}_{\mathcal{O}(2)} + \underbrace{\Gamma^j_{ji}T^i_{\ 0}}_{\mathcal{O}(2)} - \cancel{\Gamma^0_{00}T^0_{\ 0}} - \underbrace{\Gamma^0_{i0}T^i_{\ 0}}_{\mathcal{O}(2)} - \underbrace{\Gamma^i_{00}T^0_{\ i}}_{\mathcal{O}(2)} - \Gamma^i_{j0}T^j_{\ i} = 0.$$

Substitute the perturbed stress-energy tensor and connection coefficients

$$\partial_\tau(\bar\rho + \delta\rho) + \partial_i q^i + (3\mathcal{H} - 3\phi')(\bar\rho+\delta\rho) - (\mathcal{H}-\phi')\delta^i_j[-(\bar P + \delta P)\delta^j_i + \Pi^j_i] = 0,$$

$$\Rightarrow \bar\rho' + \partial_\tau\delta\rho + \partial_i q^i + 3\mathcal{H}(\bar\rho+\delta\rho) - 3\bar\rho\phi' + 3\mathcal{H}(\bar P + \delta P) - 3\bar P\phi' = 0.$$

The contribution of the zero part is

$$\bar\rho' + 3\mathcal{H}(\bar\rho + \bar P) = 0.$$

and the first order part becomes

$$\partial_\tau\delta\rho + \partial_i q^i + 3\mathcal{H}(\delta\rho + \delta P) - 3(\bar\rho+\bar P)\phi' = 0. \tag{4.11}$$

- $\partial_i q^i$ - describes changes due to the local fluid flow because peculiar velocity.
- $3\mathcal{H}$ - dilution due to the background expansion.
- ϕ' - density changes caused by perturbations to the local expansion rate $(1-\phi)a$ is the local scale factor.

Remember $\delta \equiv \frac{\delta\rho}{\bar{\rho}}$ - fractional overdensity and $q^i = (\bar{\rho}+\bar{P})v^i$ in terms of the peculiar velocity

$$\bar{\rho}\delta' - 3\mathcal{H}(\cancel{\bar{\rho}}+\bar{P})\delta + (\bar{\rho}+\bar{P})\partial_i v^i + 3\mathcal{H}(\cancel{\delta\rho}+\delta P) - 3(\bar{\rho}+\bar{P})\phi' = 0.$$

$$\delta' - 3\mathcal{H}\left(\frac{\bar{P}}{\bar{\rho}} - \frac{\delta P}{\delta\rho}\right)\delta + \left(1+\frac{\bar{P}}{\bar{\rho}}\right)(\partial_i v^i - 3\phi') = 0. \tag{4.12}$$

This is the relativistic version of the continuity equation, in the limit $P \ll \rho \to$ recover the Newtonian continuity equation in conformal time

$$\delta' + \partial_i v^i - 3\phi' = 0, \tag{4.13}$$

but with a general-relastivistic correction [small on sub-horizon scales $k \gg \mathcal{H}$].

4.3 Euler equation

The $\nu = i$ component of (4.10)

$$\partial_\mu T^\mu_{\ i} + \Gamma^\mu_{\mu\rho}T^\rho_{\ i} - \Gamma^\rho_{\mu i}T^\mu_{\ \rho} = 0, \tag{4.14}$$

$$\partial_0 T^0_{\ i} + \partial_j T^j_{\ i} + \Gamma^0_{00}T^0_{\ i} + \Gamma^j_{j0}T^0_{\ i} + \Gamma^0_{0j}T^j_{\ i} + \Gamma^k_{kj}T^j_{\ i} - \Gamma^0_{0i}T^0_{\ 0} - \Gamma^0_{ji}T^j_{\ 0} - \Gamma^j_{0i}T^0_{\ j} - \Gamma^j_{ki}T^k_{\ j} = 0,$$

we have not written down $T^0_{\ i}$ explicitly before

$$\begin{aligned} T^0_{\ i} &= g_{i\mu}T^{0\mu} = g_{i0}T^{00} + g_{ij}T^{0j} \\ &= 0 - a^2(1-2\phi)\delta_{ij}a^{-2}q^i = -q^i. \end{aligned} \tag{4.15}$$

Then, the Euler equation becomes

$$\begin{aligned} -q_i' \quad &+ \quad \partial_j[-(\bar{P}+\delta P)\delta^j_i + \Pi^j_i] - \mathcal{H}q_i - 3\mathcal{H}q_i - \partial_j\psi\bar{P}\delta^j_i + \underbrace{[-2\delta^k_{(k}\partial_{j)}\phi + \delta_{kj}\delta^{kl}\partial_l\phi]}_{[-(\delta^k_k\partial_j+\delta^k_j\partial_k)+\delta^l_j\partial_l]\phi=-3\partial_i\phi}[-\bar{P}\delta^j_i] \\ &- \quad \partial_i\psi\bar{\rho} - \cancel{\mathcal{H}\delta_{ij}q^j} + \cancel{\mathcal{H}\delta^j_i}q_j - \underbrace{[-2\delta^j_{(k}\partial_{i)}\phi + \delta_{ki}\delta^{jl}\partial_l\phi][-\bar{P}\delta^k_j]}_{3\partial_i\phi\bar{P}} = 0. \end{aligned}$$

$$\Rightarrow \qquad -q_i' - \partial_i \delta P + \partial_j \Pi_i^j - 4\mathcal{H} q_i - (\bar{\rho} + \bar{P})\partial_i \psi = 0,$$

using $q_i = (\bar{\rho} + \bar{P})v_i$, and the continuity equation, the relativistic version of the Euler equation becomes

$$v_i' + \frac{\bar{P}'}{\bar{\rho} + \bar{P}} v_i + \frac{1}{\bar{\rho} + \bar{P}} \partial_i \delta P - \frac{1}{\bar{\rho} + \bar{P}} \partial_i \Pi_i^j + \mathcal{H} v_i + \partial_i \psi = 0. \tag{4.16}$$

- $\mathcal{H} v_i$ - redshifting of peculiar velocities.
- $\bar{P}'/\bar{\rho}'$ - small corrections for relativistic fluids.
- $\bar{P}'/\bar{\rho}' = c_s^2$ for adiabatic fluctuations.

We just need the gravitational potentials ψ and ϕ to the close the system of equations.

4.4 Perturbed Einstein tensor

Because we requiere the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ so we first need to compute the Ricci $R_{\mu\nu}$ tensor and scalar R:

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\nu \Gamma^\rho_{\mu\rho} + \Gamma^\alpha_{\mu\nu}\Gamma^\rho_{\alpha\rho} - \Gamma^\alpha_{\mu\rho}\Gamma^\rho_{\nu\alpha}. \tag{4.17}$$

For 00:

$$R_{00} = \partial_\rho \Gamma^\rho_{00} - \partial_0 \Gamma^\rho_{0\rho} + \Gamma^\alpha_{00}\Gamma^\rho_{\alpha\rho} - \Gamma^\alpha_{0\rho}\Gamma^\rho_{0\alpha}.$$

The terms with $\rho = 0$ cancel out, by pairs, then we need to include only $\rho = i$

$$\begin{aligned}
R_{00} &= \partial_i \Gamma^i_{00} - \partial_\eta \Gamma^i_{0i} + \Gamma^\alpha_{00}\Gamma^i_{\alpha i} - \Gamma^\alpha_{0i}\Gamma^i_{0\alpha} \\
&= \partial_i \Gamma^i_{00} - \partial_\eta \Gamma^i_{0i} + \Gamma^0_{00}\Gamma^i_{0i} + \underbrace{\Gamma^j_{00}\Gamma^i_{ji}}_{\mathcal{O}(2)} - \underbrace{\Gamma^0_{0i}\Gamma^i_{00}}_{\mathcal{O}(2)} - \Gamma^j_{0i}\Gamma^i_{0j} \\
&= \nabla^2 \psi - 3\partial_\eta(\mathcal{H} - \phi') + 3(\mathcal{H} + \psi')(\mathcal{H} - \phi') - \underbrace{(\mathcal{H} - \phi')^2 \delta^j_i \delta^i_j}_{3\mathcal{H}^2 - 6\mathcal{H}\phi'}, \\
R_{00} &= -3\mathcal{H}' + \nabla^2 \psi + 3\phi'' + 3\mathcal{H}(\phi' + \psi'). \qquad (4.18)
\end{aligned}$$

HW:

$$R_{0i} = 2\partial_i \phi' + 2\mathcal{H}\partial_i \psi. \tag{4.19}$$

$$\begin{aligned}
R_{ij} &= [\mathcal{H}' + 2\mathcal{H}^2 + \nabla^2 \phi - \phi'' - \mathcal{H}\psi' - 5\mathcal{H}\phi' - 2(\mathcal{H}' + 2\mathcal{H}^2)(\phi + \psi)]\delta_{ij} \\
&+ \partial_i \partial_j (\phi - \psi). \qquad (4.20)
\end{aligned}$$

Ricci scalar

$$R = g^{00}R_{00} + \underbrace{2g^{0i}R_{0i}}_{=0} + g^{ij}R_{ij},$$

$$\begin{aligned}
a^2R &= (1-2\psi)R_{00} - (1+2\phi)\delta^{ij}R_{ij} \\
&= (1-2\psi)[-3\mathcal{H}' + \nabla^2\psi + 3\phi'' + 3\mathcal{H}(\phi' + \psi')] \\
&\quad - \underbrace{\delta^{ij}\delta_{ij}}_{3}(1+2\phi)[\mathcal{H}' + 2\mathcal{H}^2 + \nabla^2\phi - \phi'' - \mathcal{H}\psi' - 5\mathcal{H}\phi' - 2(\mathcal{H}' + 2\mathcal{H}^2)(\phi + \psi)] \\
&\quad -(1+2\phi)\nabla^2(\phi - \psi),
\end{aligned}$$

to linear order

$$\begin{aligned}
a^2R &= [-3\mathcal{H}' + \nabla^2\psi + 3\phi'' + 3\mathcal{H}(\phi' + \psi')] + 6\psi\mathcal{H}' \\
&\quad -3[\mathcal{H}' + 2\mathcal{H}^2 + \nabla^2\phi - \phi'' - \mathcal{H}\psi' - 5\mathcal{H}\phi' - 2(\mathcal{H}' + 2\mathcal{H}^2)(\phi + \psi)] - 6\phi(\mathcal{H}' + 2\mathcal{H}^2) \\
&\quad -\nabla^2(\phi - \psi),
\end{aligned}$$

$$a^2R = -6(\mathcal{H}' + \mathcal{H}^2) + 2\nabla^2\psi - 4\nabla^2\phi + 6\phi'' + 6\mathcal{H}(3\phi' + \psi') + 12(\mathcal{H}' + \mathcal{H}^2)\psi. \quad (4.21)$$

Then, the Einstein tensor 00 is

$$\begin{aligned}
G_{00} &= R_{00} - \frac{1}{2}g_{00}R \\
&= \cancel{-3\mathcal{H}'} + \cancel{\nabla^2\psi} + \cancel{3\phi''} + 3\mathcal{H}(\phi' + \cancel{\psi'}) + 3(1+2\psi)(\cancel{\mathcal{H}'} + \mathcal{H}^2) \\
&\quad -\frac{1}{2}[\cancel{2\nabla^2\psi} - 4\nabla^2\phi + \cancel{6\phi''} + 6\mathcal{H}(3\phi' + \cancel{\psi'}) + 12(\cancel{\mathcal{H}'} + \mathcal{H}^2)\psi] \\
G_{00} &= 3\mathcal{H}^2 + 2\nabla^2\phi - 6\mathcal{H}\phi'. \quad (4.22)
\end{aligned}$$

On the other hand, since $g_{0i} = 0$ (newtonian)

$$G_{0i} = 2\partial_i\phi' + 2\mathcal{H}\partial_i\psi. \quad (4.23)$$

And, finally

$$\begin{aligned}
G_{ij} &= R_{ij} - \frac{1}{2}g_{ij}R \\
&= [\mathcal{H}' + 2\mathcal{H}^2 + \nabla^2\phi - \phi'' - \mathcal{H}\psi' - 5\mathcal{H}\phi' - 2(\mathcal{H}' + 2\mathcal{H}^2)(\phi + \psi)]\delta_{ij} \\
&\quad +\partial_i\partial_j(\phi - \psi) - 3(1-2\phi)(\mathcal{H}' + \mathcal{H}^2)\delta_{ij} \\
&\quad +\frac{1}{2}[2\nabla^2\psi - 4\nabla^2\phi + 6\phi'' + 6\mathcal{H}(\psi' + 3\phi') + 12(\mathcal{H}' + \mathcal{H}^2)\psi]\delta_{ij},
\end{aligned}$$

this neats up

$$\begin{aligned} G_{ij} &= -(2\mathcal{H}' + \mathcal{H}^2)\delta_{ij} + [\nabla^2(\psi - \phi) + 2\phi'' + 2(2\mathcal{H}' + \mathcal{H}^2)(\phi + \psi) \\ &\quad +2\mathcal{H}\psi' + 4\mathcal{H}\phi']\delta_{ij} + \partial_i\partial_j(\phi - \psi) \end{aligned} \tag{4.24}$$

4.5 Perturbed Einstein equations

Putting altogether, the 00 Einstein equation:

$$G_{00} = 8\pi G T_{00} + \Lambda g_{00}.$$

$$\begin{aligned} 3\mathcal{H}^2 + 2\nabla^2\phi - 6\mathcal{H}\phi' &= 8\pi G(g_{00}T^0{}_0 + \cancel{g_{0i}T^i{}_0}) + \Lambda a^2(1 + 2\psi) \\ &= 8\pi G a^2(1 + 2\psi)\bar{\rho}(1 + \delta) + \Lambda a^2(1 + 2\psi). \end{aligned}$$

Zero-order

$$\mathcal{H}^2 = \frac{8\pi G}{3}a^2\bar{\rho} + \frac{1}{3}\Lambda a^2. \tag{4.25}$$

00 Einstein first order

$$\begin{aligned} \nabla^2\phi &= 3\mathcal{H}\phi' + (8\pi G a^2\bar{\rho} + \Lambda a^2)\psi + 4\pi G a^2\bar{\rho}\delta \\ \nabla^2\phi &= 3\mathcal{H}(\phi' + \mathcal{H}\psi) + 4\pi G a^2\bar{\rho}\delta. \end{aligned} \tag{4.26}$$

Einstein $0i$ component:

$$G_{0i} = 8\pi G T_{0i} + \Lambda g_{0i},$$

but, we need T_{0i}

$$\begin{aligned} T_{0i} &= g_{0\mu}g_{i\nu}T^{\mu\nu} \\ &= g_{00}g_{ij}T^{0j} \qquad (g_{0i} = 0) \\ &= -a^2(1 + 2\psi)a^2(1 - 2\phi)\delta_{ij}a^{-2}q^j \\ &= -a^2 q_i. \end{aligned} \tag{4.27}$$

Hence

$$\begin{aligned} \Rightarrow \partial_i\phi' + \mathcal{H}\partial_i\psi &= -4\pi G a^2 q_i \qquad \text{but} \quad q_i = (\bar{\rho} + \bar{P})\partial_i v \\ \phi' + \mathcal{H}\psi &= -4\pi G a^2(\bar{\rho} + \bar{P})v, \end{aligned}$$

now using G_{00} (4.26)

$$\Rightarrow \nabla^2\phi = 4\pi G a^2[\bar{\rho}\delta - 3\mathcal{H}(\bar{\rho} + \bar{P})v], \tag{4.28}$$

$[\bar{\rho}\delta - 3\mathcal{H}(\bar{\rho}+\bar{P})v]$ poisson with source density
$\bar{\rho}\Delta \equiv [\bar{\rho}\delta - 3\mathcal{H}(\bar{\rho}+\bar{P})(B+v)]$ the gauge-invariant variable ($B=0$), since $B=0$ in the conformal newtonian gauge.

• Introduce comoving hypersurfaces-orthogonal to the wordlines of a set of observers comoving with the total matter. In the comoving gauge ($q^i = 0, B_i = 0$), Δ is the fractional overdensity

ij Einstein equation

$$\begin{aligned}
G_{ij} &= 8\pi G T_{ij} + \Lambda g_{ij} \\
&= 8\pi G g_{ik} g_{jl} T^{kl} - a^2\Lambda(1-2\phi)\delta_{ij} \\
&= 8\pi G a^4(1-2\phi)^2\delta_{ik}\delta_{jl}a^{-2}[\bar{P}\delta^{kl} + (2\bar{P}\phi - \delta P)\delta^{kl} - \Pi^{kl}] - a^2\Lambda(1-2\phi)\delta_{ij} \\
&= 8\pi G a^2(1-4\phi)[\bar{P}\delta_{ij} + (2\bar{P}\phi + \delta P)\delta_{ij} - \Pi_{ij}] + a^2\Lambda(1-2\phi)\delta_{ij} \\
&= a^2(8\pi G\bar{P} - \Lambda)\delta_{ij} + a^2[8\pi G(\delta P - 2\bar{P}\phi) + 2\Lambda\phi]\delta_{ij} - 8\pi G a^2\Pi_{ij}.
\end{aligned}$$

G_{ij} to zero order (4.24)

$$2\mathcal{H}' + \mathcal{H}^2 = -a^2(8\pi G\bar{P} - \Lambda),$$

the second Friedmann equation.

The first order Einstein ij equation:

$$[\nabla^2(\psi-\phi) + 2\phi'' + 2(2\mathcal{H}'+\mathcal{H}^2)(\phi+\psi) + 2\mathcal{H}\psi' + 4\mathcal{H}\phi']\delta_{ij} + \partial_i\partial_j(\phi-\psi)$$
$$= a^2[8\pi G(\delta P - 2\bar{P}\phi) + 2\Lambda\phi]\delta_{ij} - 8\pi G a^2\Pi_{ij} \tag{4.29}$$

• First, consider the trace-free part

$$\partial_{\langle i}\partial_{j\rangle}(\phi-\psi) = -8\pi G a^2\Pi_{ij}, \tag{4.30}$$

in the absence of anisotropic stress $\rightarrow \phi = \psi$ so there's only one gauge-invariant degree of freedom in the metric (Π_{ij} neutrino decoupling - or not a perfect fluid).
• The trace part $\delta^{ij}/3$

$$\nabla^2(\psi-\phi) + 2\phi'' + 2(2\mathcal{H}'+\mathcal{H}^2)(\cancel{\phi}+\psi) + 2\mathcal{H}\psi' + 4\mathcal{H}\phi' + \frac{1}{3}\nabla^2(\phi-\psi)$$
$$= 8\pi G a^2\delta P - \cancel{2a^2(8\pi G\bar{P}-\Lambda)\phi},$$

using the zero order, and diving by 2

$$\frac{1}{3}\nabla^2(\psi-\phi) + \phi'' + (2\mathcal{H}'+\mathcal{H}^2)\psi + \mathcal{H}\psi' + 2\mathcal{H}\phi' = 4\pi G a^2\delta P,$$

4.6 Summary

Newtonian Gauge, no-anisotropic stress, $\Psi = \Phi$

Metric : $ds^2 = a^2(\tau)\left[(1+2\phi)d\tau^2 - (1-2\phi)\delta_{ij}dx^i dx^j\right]$

Einstein equations:

$$\begin{aligned} \nabla^2\phi - 3\mathcal{H}(\phi' + \mathcal{H}\phi) &= 4\pi G a^2 \delta\rho, && (4.31)\\ \phi' + \mathcal{H}\phi &= -4\pi G a^2(\bar{\rho} + \bar{P})v, && (4.32)\\ \phi'' + 3\mathcal{H}\phi' + (2\mathcal{H}' + \mathcal{H}^2) &= 4\pi G a^2 \delta P, && (4.33) \end{aligned}$$

using the comoving gauge density contrast

$$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\Delta. \qquad (4.34)$$

The conservation equations:

$$\begin{aligned} \delta' + 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - \frac{\bar{P}}{\rho}\right)\delta &= -\left(1 + \frac{\bar{P}}{\bar{\rho}}\right)(\nabla\cdot\bar{v} - 3\phi'), && (4.35)\\ v' + 3\mathcal{H}\left(\frac{1}{3} - \frac{\bar{P}'}{\bar{\rho}'}\right)v &= -\frac{\nabla\delta P}{\bar{\rho} + \bar{P}} - \nabla\phi, && (4.36)\\ \mathcal{R} &= -\phi - \frac{\mathcal{H}(\phi' + \mathcal{H}\phi)}{4\pi G a^2(\bar{\rho} + \bar{P})}, && (4.37) \end{aligned}$$

doesn't evolve on super-Hubble scales $k \ll \mathcal{H}$ with adiabatic perturbations.

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