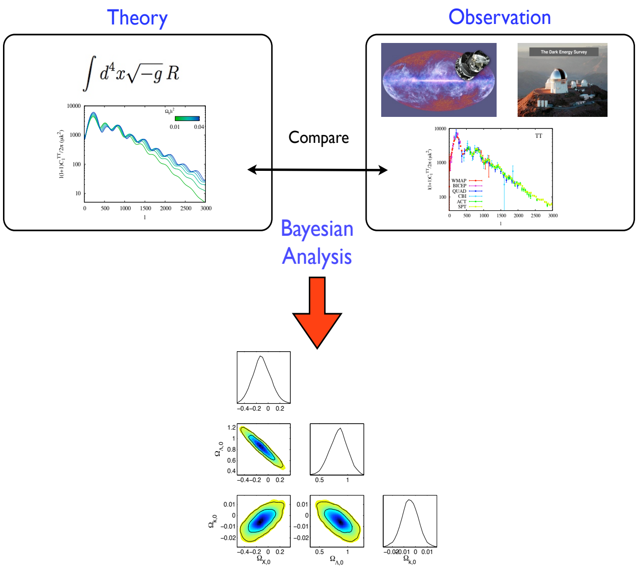
Updated Cosmology

with Python



José-Alberto Vázquez

ICF-UNAM / Kavli-Cambridge

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Applications to cold-Dark matter

2.1 Solutions in an Einstein -de Sitter universe ($\bar{P} \approx 0,\ k = \Lambda = 0$)

Large scales, after matter-radiation equality, but before dark energy dominates.

$\Rightarrow$ Einstein -de Sitter model $\bar{P} \approx 0$ and zero curvature and $\Lambda = 0$.

The fractional overdensity in the baryons δ_b approaches the CDM δ_c, and the matter behaves like a single pressure-free fluid whit total density contrast

$$\delta_m = \frac{\bar{\rho}_b \delta_b + \bar{\rho}_c \delta_c}{\bar{\rho}_b + \bar{\rho}_c} \approx \delta_c,$$

$c_s = 0$ for linearized CDM fluctuations, $k^2 \ll a^2$ and, at this stage, $H^2 \propto \bar{\rho} \propto a^{-3}$, $a \propto t^{2/3} \Rightarrow H = \frac{2}{3t}$ and $4\pi G\bar{\rho} = \frac{2}{3t^2}$.

• Scales of cosmological interest are much larger than Jean's scale ($k^2 \ll a^2$), for baryons and CDM fluctuations the Jean's equation (1.19) becomes

$$\Rightarrow \quad \partial_t^2 \delta_m + \frac{4}{3t} \partial_t \delta_m - \frac{2}{3t^2} \delta_m = 0,$$

assuming $\delta_m \sim t^p$ we get that $\delta_m \propto t^{-1} \propto a^{-3/2}$, and $\delta_m \propto t^{2/3} \propto a$. The growing-mode $\rightarrow$ grows like scale factor.

In an expanding universe $\rightarrow$ power law-growth of δ. Exponential growth predicted in a non-expanding universe though (middle term cancels out).

• **The Poisson equation**

From (1.17), we observe the gravitational potential is constant

$$-k^2\phi = 4\pi G \underbrace{\bar{\rho}}_{a^{-3}} \underbrace{\delta}_{a} a^2 = \text{constant}.$$

For δ_r we need relativistic perturbation theory.

2.2 The Meszaros effect

CDM grows only logarithmically on scales inside the sound horizon during radiation domination

$$\partial_t^2\delta_i + 2H\partial_t\delta_i - 4\pi G\sum_j \bar{\rho}_j\delta_j - \frac{1}{a^2\bar{\rho}_i}\nabla^2\delta P_i = 0,$$

for radiation perturbations, $i = r$ we shall show it properly when we develop relativistic perturbation theory. For CDM:

$$\partial_t^2\delta_c + 2H\partial_t\delta_c - 4\pi G\sum_j \bar{\rho}_j\delta_j = 0,$$

radiation fluctuations oscillates as sound waves, vanishing the density contrast, then, in the radiation domination epoch $a \propto t^{1/2}$, $H = \frac{1}{2t}$, thus

$$\partial_t^2\delta_c + \frac{1}{t}\partial_t\delta_c - 4\pi G\bar{\rho}_c\delta_c = 0,$$

δ_c evolves on cosmological timescales, then

$$\partial_t^2\delta_c \sim H\partial_t\delta_c \sim H^2\delta_c \sim \frac{8\pi G}{3}\bar{\rho}_r\delta_c \gg 4\pi G\bar{\rho}_c\delta_c,$$

since $\bar{\rho}_r \gg \bar{\rho}_c$ during radiation domination. Therefore

$$\partial_t^2\delta_c + \frac{1}{t}\partial_t\delta_c = 0,$$

that has solutions: δ_c =constant, and the growing mode $\delta_c \propto \ln t$.

The effectively unclustered radiation reduces the growth of δ_c to only logarithmically. Need to wait until matter-domination so the DM density fluctuations grow significantly.

2.3 Late-time suppression of structure formation by Λ

Dark matter doesn't cluster on the last stages of the universe.

$$\partial_t^2\delta_m + 2H\partial_t\delta_m - 4\pi G\bar{\rho}_m\delta_m = 0,$$

when Λ dominates $a \propto e^{t\sqrt{\Lambda/3}}$, $H \simeq$ constant, and $4\pi G\bar{\rho}_m \ll H^2 \sim \left(\frac{\Lambda}{3}\right) \sim \frac{8\pi G\rho_\Lambda}{3}$, then

$$\Rightarrow \quad \partial_t^2 \delta_m + 2H\partial_t\delta_m \simeq 0,$$

which has solutions δ_m =constant, and $\delta_m \propto e^{-2t\sqrt{\Lambda/3}} \propto a^{-2}$, Λ suppresses the growth of structure [matter fluctuations].

Gravitational potential

$$-k^2\phi = 4\pi G \underbrace{\bar{\rho}}_{a^{-3}} \delta a^2 \quad \propto a^{-1}. \tag{2.1}$$

This affects the integrated Sachs-Wolfe effect as we shall see later.

2.4 Evolution of baryon fluctuations after decoupling

The coupled dynamics of the baryon and CDM fluids after decoupling is described by the following two equations:

$$\begin{aligned} \partial_t^2\delta_b + \frac{4}{3t}\partial_t\delta_b &= 4\pi G(\bar{\rho}_c\delta_b + \bar{\rho}_c\delta_c), \\ \partial_t^2\delta_c + \frac{4}{3t}\partial_t\delta_c &= 4\pi G(\bar{\rho}_c\delta_b + \bar{\rho}_c\delta_c). \end{aligned}$$

Substituting $\Delta \equiv \delta_c - \delta_b$, we have

$$\partial_t^2\Delta + \frac{4}{3t}\partial_t\Delta = 0,$$

which has solutions Δ = constant or $\Delta \propto t^{-1/3}$ while δ_m has solutions t^{-1}, $t^{2/3}$, therefore

$$\frac{\delta_c}{\delta_b} = \frac{\bar{\rho}_m\delta_m + \bar{\rho}_b\Delta}{\bar{\rho}_m\delta_m - \bar{\rho}_c\Delta} \to \frac{\delta_m}{\delta_m} = 1.$$

Bibliography

[1] James M. Bardeen. Gauge-invariant cosmological perturbations. Phys. Rev. D, 22:1882–1905, Oct 1980. doi: 10.1103/PhysRevD.22.1882. URL http://link.aps.org/doi/10.1103/PhysRevD.22.1882. 1

[2] Anthony Challinor. Part III- Large Scale Strucutre Formation. *http://nefertiti.hpc.phys.ucl.ac.uk/cosmology/cosmology.html*, 2008. 1

[3] Scott Dodelson. Modern Cosmology. Academic Press, 2003. 1

[4] Michael Doran. The Theory of the Cosmic Microwave Background. *http://www.thphys.uni-heidelberg.de/ cosmo/view/main/cmbdoran*, 2010. 1

[5] Ruth Durrer. The theory of CMB anisotropies. [arXiv:0109522], 2001. 1

[6] Wayne Hu and Scott Dodelson. COSMIC MICROWAVE BACKGROUND ANISOTROPIES. Annual Review of Astronomy and Astrophysics, 40(1):171–216, 2002. doi: 10.1146/annurev.astro.40.060401.093926. URL http://www.annualreviews.org/doi/abs/10.1146/annurev.astro.40.060401.093926. 1

[7] Christian Knobel. An introduction into the theory of cosmological structure formation. 2012. [arXiv:1208.5931]. 1

[8] Hideo Kodama and Misao Sasaki. Cosmological Perturbation Theory. Progress of Theoretical Physics Supplement, 78:1–166, 1984. doi: 10.1143/PTPS.78.1. URL http://ptp.ipap.jp/link?PTPS/78/1/. 1

[9] Hannu Kurki-Suonio. Cosmological Perturbation Theory. *http://www.helsinki.fi/∼hkurkisu/*, 2008. 1

[10] Andrew R. Liddle and David H. Lyth. Cosmological Inflation and Large-Scale Structure. Cambridge University Press, 2000. 1

[11] Chung-Pei Ma and Edmund Bertschinger. Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges. [arXiv:9506072], 1995. 1

[12] V. Mukhanov. Physical Foundations of Cosmology. Cambridge University Press, 2005. 1

[13] V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger. Theory of cosmological perturbations. Physics Reports, 215:203 – 333, 1992. ISSN 0370-1573. doi: 10.1016/0370-1573(92)90044-Z. URL http://www.sciencedirect.com/science/article/pii/037015739290044Z. 1