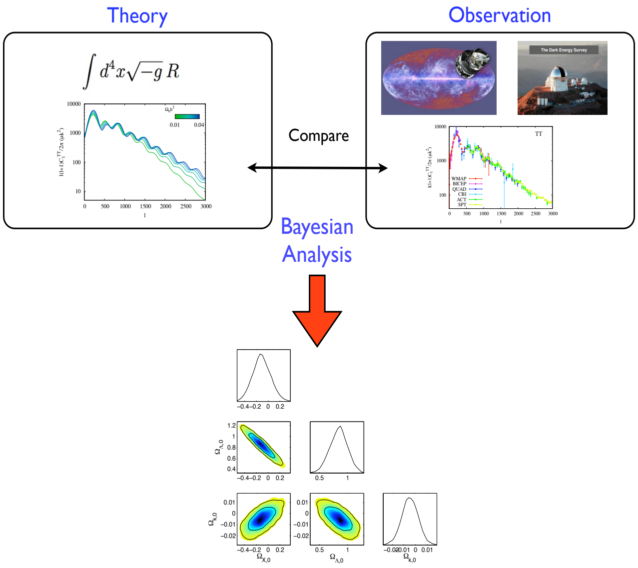
Updated Cosmology

with Python



José-Alberto Vázquez

ICF-UNAM / Kavli-Cambridge

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Big-Bang Nucleosynthesis

Big-Bang Nucleosynthesis refers to how the light elements like H, He and Li were synthesized in the Big Bang. In particular, we are interested on the ratio of the density of helium to hydrogen:

$$\frac{n_{He}}{n_H} \sim \frac{1}{16}. \tag{1.1}$$

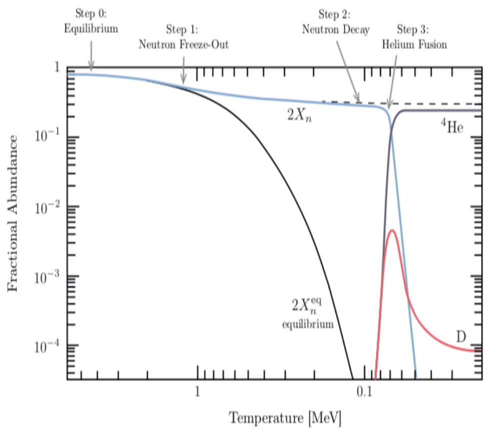

Figure 1.1: Helium production in the Universe.

Step 0: Equilibrium Abundances

No elements heavier than Helium are produced. We need to track hydrogen, helium and isotopes: deuterium, tritium and ^{3}He.

At temperatures of order $T \approx 0.1$ MeV only free protons and neutrons exist $\rightarrow$ we need to solve for the neutron/proton ratio and use it for the synthesis of Deuterium, Helium, etc.

The Relative abundances of n^o and p^+

In the early universe, n^0 and p^+ are coupled by weak interactions

$$n + \nu_{e^-} \quad \longleftrightarrow \quad p^+ + e^-,$$
$$n + e^+ \quad \longleftrightarrow \quad p^+ + \bar{\nu}_{e^-}.$$

taking into account that μ_{e^-} and μ_ν are negligibly small, we have that $\mu_n = \mu_p$ and therefore

$$\left(\frac{n_n}{n_\rho}\right)_{\rm eq} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-(m_n - m_p)/T}. \tag{1.2}$$

A fair assumption is that the mass of neutron is comparable with the mass of the proton $m_n \sim m_p$ to get

$$\left(\frac{n_n}{n_p}\right)_{\rm eq} = e^{-Q/T}, \tag{1.3}$$

with $Q \equiv m_n - m_p$=1.3MeV, the bounding energy of hydrogen. Therefore, for temperatures $T \gg$ 1MeV there are as many neutrons as protons, and for T <1 MeV the neutron fractions gets reduced.

Next, Deuterium (isotope of hydrogen, with one p^+ and one n^0)

$$n + p^+ \quad \longleftrightarrow \quad D + \gamma,$$
$$\mu_n + \mu_p = \mu_D,$$
$$\left(\frac{n_D}{n_n n_p}\right)_{\rm eq} = \frac{3}{4}\left(\frac{m_D}{m_n m_p}\frac{2\pi}{T}\right)^{3/2} e^{-(m_D - m_n - m_p)/T}, \tag{1.4}$$

using that the internal degrees of freedom ($g_D = 3$, $g_p = g_n = 2$), $2m_n \approx 2m_p \approx 1.9$ GeV and in the exponential the binding energy of deuterium is given by$B_D \equiv m_n + m_p - m_D = 2.22$ MeV.

$$\left(\frac{n_D}{n_p}\right)_{\rm eq} = \frac{3}{4} n_n^{eq}\left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_D/T}. \tag{1.5}$$

To get an order of magnitude

$$n_n \sim n_b = \eta\, n_\gamma = \eta \frac{2\zeta(3)}{\pi^2} T^3, \tag{1.6}$$

$$\Rightarrow \quad \left(\frac{n_D}{n_p}\right)_{\rm eq} \approx \eta \left(\frac{T}{m_p}\right)^{3/2} e^{B_D/T}. \tag{1.7}$$

$\eta \rightarrow$ inhibits the production of deuterium until T drops well beneath the binding energy B_D (approximately up to $T \sim 0.1$MeV).

Step 1: Neutron Freeze-out

All neutrons incorporate into ^{4}He.

Thermal equilibrium happened until $T_{\rm dec} \sim 0.8$ MeV, where neutrinos decouple and then on we need $\quad\rightarrow\quad$ Boltzmann eqn.

Define the neutron fraction as $X_n \equiv \frac{n_n}{n_n+n_p}$, then

$$X_n^{\rm eq}(T) = \frac{e^{-Q/T}}{1+e^{-Q/T}}. \tag{1.8}$$

Therefore,

$$\begin{aligned} &X_n^{\rm eq}(0.8MeV) = 0.17, \\ &X_n^{\infty} \sim X_n^{\rm eq}(0.8MeV) \sim \frac{1}{6}. \end{aligned} \tag{1.9}$$

Step 2 :Neutron decay

At temperatures $T > 0.2$ MeV ($t \sim 100$ sec) the lifetime of the neutron becomes important. Take the freeze-out abundance by an exponential decay factor

$$X_n(T) = X_n^{\infty} e^{-t/\tau_n} = \frac{1}{6} e^{-t/\tau_n} \qquad (\tau_n = 886.7 \pm 0.8 \text{ sec}). \tag{1.10}$$

Step 3: Helium Fusion

The Universe is mostly made of p^+ and n^0 and heavier nuclei have to be built sequentially from lighter nuclei in two-particle reactions.

First Deuterium

$$n + p^+ \quad \longleftrightarrow \quad D + \gamma.$$

Only when D is available can helium be formed (as long as enough free neutrons are available)

$$\begin{aligned} D + p^+ \quad &\longleftrightarrow \quad {}^3He + \gamma, \\ D +^3 He \quad &\longleftrightarrow \quad {}^4He + p^+. \end{aligned}$$

• The abundance of ^{4}He cannot be formed until sufficient deuterium has become available $\rightarrow$ this effect is known as the *Deuterium bottle neck*. Only when there is enough Deuterium $\rightarrow$ the Helium is produced.

When Nucleosynthesis happened

When the Deuterium fraction in equation would be of order one i.e $(n_D/p)_{eq} \sim 1$, that is (using Eqn. (1.7))

$$T_{\rm nuc} \sim 0.06 \text{ MeV}, \qquad \left(\frac{T}{1 \text{ MeV}}\right) \simeq 1.5 g_*^{-1/4} \left(\frac{1 \text{ sec}}{t}\right)^{1/2}, \tag{1.11}$$

with $g_* = 3.38$

$$\Rightarrow \quad t_{\rm nuc} = 120 sec \left(\frac{0.1 \text{ MeV}}{T_{\rm nuc}}\right)^2 \sim 330 \text{ sec}, \tag{1.12}$$

and from Eqn. (1.10)

$$X_n(t_{nuc}) \sim \frac{1}{8}. \tag{1.13}$$

Helium is produced almost immediately after deuterium, since two n^0 go into one nucleus of the ^4He

$$n_{He} = \frac{1}{2} n_n(t_{\rm nuc}) \qquad \text{or} \tag{1.14}$$

$$\frac{n_{He}}{n_H} = \frac{n_{He}}{n_p} \simeq \frac{\frac{1}{2} X_n(t_{\rm nuc})}{1 - X_n(t_{\rm nuc})} \sim \frac{1}{2} X_n(t_{\rm nuc}) \sim \frac{1}{16}, \tag{1.15}$$

sometime as the mass fraction of Helium

$$\frac{4n_{He}}{n_H} \sim \frac{1}{4}. \tag{1.16}$$

1.1 BBN as a Probe of BSM physics

The Helium mass fraction depends on several inputs parameters.

- g_* - The number of relativistic degrees of freedom determines H during the radiation era, therefore, $H \propto g_F^{1/2}$ and hence affects the freeze-out temperature.

$$G_F^2 T_F^5 \sim \sqrt{G_N g_*} T_F^2 \qquad \Rightarrow \qquad T_F \propto g_*^{1/6}. \tag{1.17}$$

  $\Rightarrow$ increasing g_* increases T_F (weak force?).
  $\Rightarrow$ increases the n/p ratio at freeze-out.
  $\Rightarrow$ increases the final helium abundance.

- τ_n - Large neutron lifetime (15 mins 881.5 sec) would reduce the amount of neutron decay after freeze-out.

  $\Rightarrow$ increases the final helium abundance.

- Q Larger mass difference $(m_n - m_p)$ decreases the n/p ratio at freeze-out

  $\Rightarrow$ decreases the final helium abundance.

- η - The amount fo helium increases with increasing η as nucleosynthesis starts earlier for large baryon density.

- G_N - increasing the strength of gravity increases the freeze-out temperature $T \propto G_N^{1/6}$.

  $\Rightarrow$ increases the final helium abundance.

- G_F - increasing the weak force, decrease the freeze-out temperature $T_f \propto G_F^{-2/3}$.

  $\Rightarrow$ decrease the final helium abundance.

Changing the input, would change the predictions of BBN : BBN is a probe of fundamental physics.

1.2 Light Element Synthesis

The couple Boltzmann equation have to be solved numerically

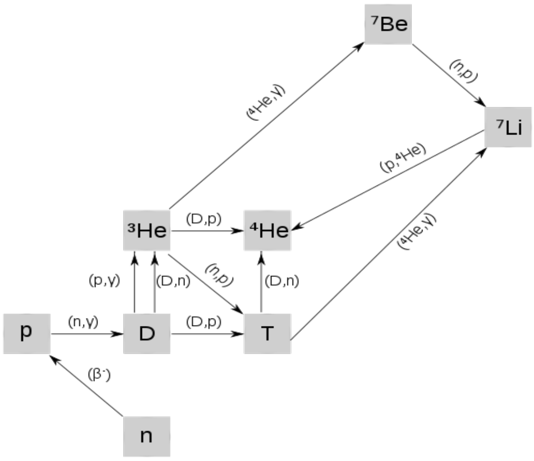

Figure 1.2: Helium production in the Universe

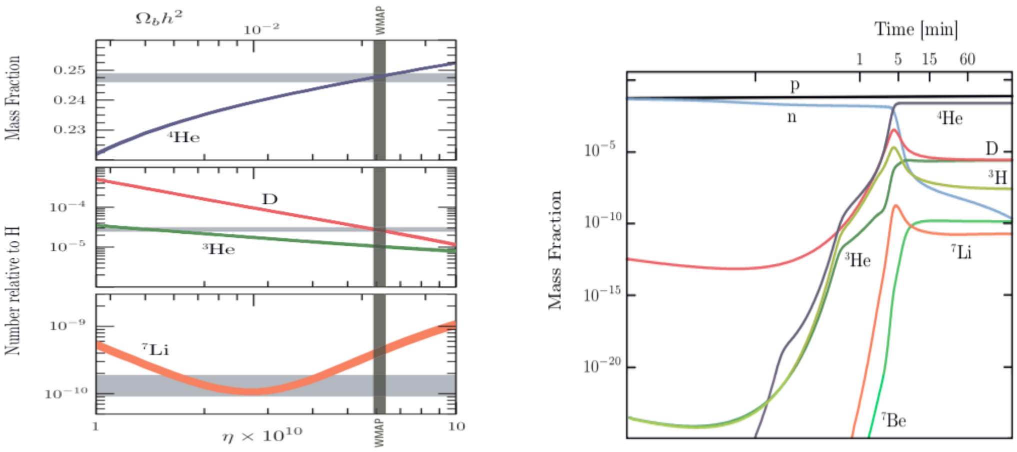

Figure 1.3: Helium production in the Universe

Bibliography