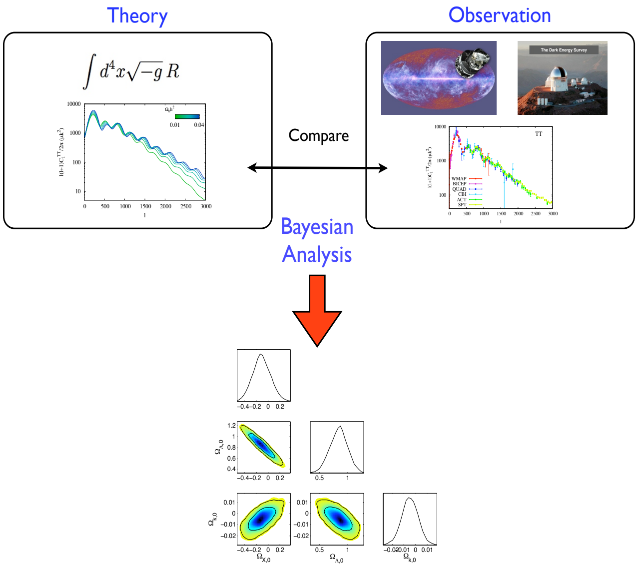
Updated Cosmology

with Python



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Beyond the Equilibrium

2.1 Recombination

An important event in the history of the early universe is the formation of the first atoms. At temperatures above about 1 eV, the universe still consisted of a plasma of free electrons and nuclei. Photons were tightly coupled to the electrons via Compton scattering, which in turn strongly interacted with protons via Coulomb scattering. There was very little neutral hydrogen. When the temperature became low enough, the electrons and nuclei combined to form neutral atoms (recombination), and the density of free electrons fell sharply. The photon mean free path grew rapidly and became longer than the horizon distance. The photons decoupled from the matter and the universe became transparent. Today, these photons form the cosmic microwave background.

2.1.1 Saha Equilibrium

Let us start at $T >$1eV, when baryons and photons were still in equilibrium through electromagnetic reactions such as

$$e^- + p^+ \rightleftharpoons H + \gamma. \tag{2.1}$$

Since $T < m_i$, $i = \{e, p, H\}$, from Eqn. (1.22) we have the following equilibrium abundances

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right), \tag{2.2}$$

where $\mu_p + \mu_e = \mu_H$.

Consider the ratio

$$\left(\frac{n_H}{n_e n_p}\right)_{\rm eq} = \frac{g_H}{g_e g_p}\left(\frac{m_H}{m_e m_p}\frac{2\pi}{T}\right)^{3/2} e^{(m_p+m_e-m_H)/T}, \tag{2.3}$$

we can use $m_H \approx m_p$ (but the difference $B_H \equiv m_p + m_e - m_H = -13.6$eV, the binding energy of Hydrogen). Also, the degrees of freedom $g_p = g_e = 2, g_H = 4$, and considering the universe is electrically neutral, hence $n_e = n_p$, therefore

$$\left(\frac{n_H}{n_e^2}\right)_{eq} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}. \tag{2.4}$$

We wish to follow the *free electron fraction* defined as the ratio

$$X_e \equiv \frac{n_e}{n_b}, \tag{2.5}$$

where n_b is the baryon density

$$n_b = \eta n_\gamma = \eta \times \frac{2\zeta(3)}{\pi^2}T^3, \tag{2.6}$$

and η is the *baryon-to-photon ratio*, $\eta = 5.5 \times 10^{-10}(\Omega_b h^2/0.020)$; we will see more about this number below.

The total baryon number density $n_b \approx n_p + n_H = n_e + n_H$, hence

$$\left(\frac{1-X_e}{X_e^2}\right)_{\rm eq} = \frac{n_H}{n_e^2}n_b. \tag{2.7}$$

We arrive at the so-called Saha equation

$$\left(\frac{1-X_e}{X_e^2}\right)_{\rm eq} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}. \tag{2.8}$$

Fig. (2.1) shows the redshift evolution of the free electron fraction as predicted both by the Saha approximation and by a more exact numerical treatment (see below). The Saha approximation correctly identifies the onset of recombination, but it is clearly insufficient if the aim is to determine the relic density of electrons after freeze-out.

2.1.2 Hydrogen Recombination

Let us define the recombination temperature $T_{\rm rec}$ as the temperature where $X_e = 10^{-1}$, i.e. when 90% of the electrons have combined with protons to form hydrogen. We find

$$T_{\rm rec} \approx 0.3 \text{ eV} \simeq 3600 \text{ K}. \tag{2.9}$$

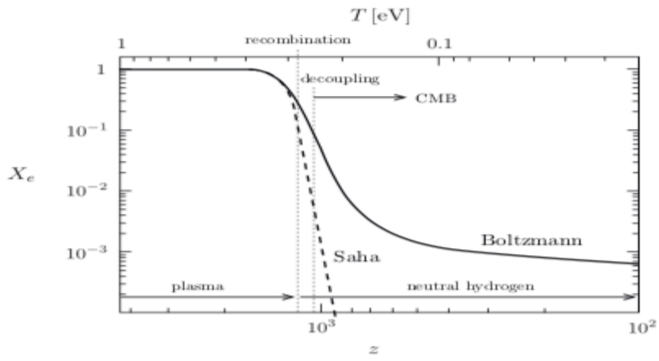

Figure 2.1: Free electron fraction as a function of redshift.

Using $T_{\rm rec} = T_0(1 + z_{\rm rec})$, with $T_0 = 2.7$K, gives the redshift of recombination,

$$z_{\rm rec} \approx 1320. \tag{2.10}$$

Since matter-radiation equality is at $z_{\rm eq} \simeq 3500$, we conclude that recombination occurred in the matter-dominated era. Using $a(t) = (t/t_0)^{2/3}$, we obtain an estimate for the time of recombination

$$t_{\rm rec} = \frac{t_0}{(1 + z_{\rm rec})^{3/2}} \sim 290,000 \text{ yrs}. \tag{2.11}$$

Recombination was not an instantaneous process but proceeded relatively quickly, nevertheless, with the fractional ionisation decreasing from $X = 0.9$ to $X = 0.1$ over a time interval $\Delta t \sim$ 70 000 yrs. With the number density of free electrons dropping rapidly, the time when photons and baryons decoupled follows soon.

2.2 Photon Decoupling

Photons are most strongly coupled to the primordial plasma through their interactions with electrons, through Thomson scattering

$$e^- + \gamma \rightleftharpoons e^- + \gamma, \tag{2.12}$$

i.e. the elastic scattering of electromagnetic radiation by a free charged particle. Thomson scattering is the low-energy limit of Compton scattering and it is a valid description in the regime where the photon energy is much less than the rest-mass energy of the electron. An important feature of Thomson scattering is that it introduces polarisation along the direction of motion of the electron

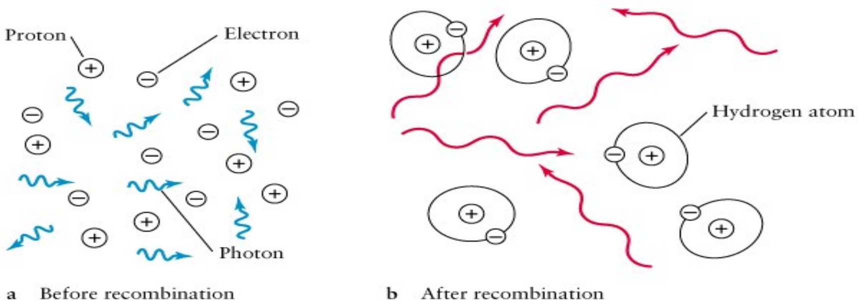

Figure 2.2: Recombination process.

The mean free path for photons (the mean distance travelled between scattering) is

$$\lambda = \frac{1}{n_e \sigma_T}, \tag{2.13}$$

and therefore the interaction rate at which a photon undergoes scattering, given by

$$\Gamma_\gamma \approx n_e \sigma_T, \tag{2.14}$$

decreases as the density of free electrons drops, where $\sigma_T \approx 2 \times 10^{-3}$ MeV^{-2} is the Thomson cross section. Since $\Gamma_\gamma \propto n_e$, the interaction rate becomes smaller than the expansion rate, and hence photons and electrons decouple roughly when

$$\Gamma_\gamma(T_{\rm dec}) \sim H(T_{\rm dec}), \tag{2.15}$$

$$\Gamma_\gamma(T_{\rm dec}) = n_b X_e(T_{\rm dec}) \sigma_T = \frac{2\zeta(3)}{\pi^2} \eta \sigma_T X_e(T_{\rm dec}) T_{\rm dec}^3, \tag{2.16}$$

and

$$H(T_{\rm dec}) = H_0 \sqrt{\Omega_m} \left(\frac{T_{\rm dec}}{T_0} \right)^{3/2}. \tag{2.17}$$

We get

$$X_e(T_{\rm dec}) T_{\rm dec}^{3/2} \sim \frac{\pi^2}{2\zeta(3)} \frac{H_0 \sqrt{\Omega_m}}{\eta \sigma_T T_0^{3/2}}. \tag{2.18}$$

Using the Saha equation for $X_e(T_{\rm dec}) \sim 0.01$, we find

$$T_{\rm dec} \sim 0.27 \text{ eV}. \tag{2.19}$$

The redshift and time of decoupling are

$$z_{\mathrm{dec}} \sim 1100, \qquad t_{\mathrm{dec}} \sim 380\,000 \text{ yrs.} \tag{2.20}$$

After decoupling the photons stream freely (their mean free path becomes very much longer): the Universe is now transparent to radiation. Observations of the cosmic microwave background today allow us to probe the conditions at last-scattering.

2.3 Last Scattering

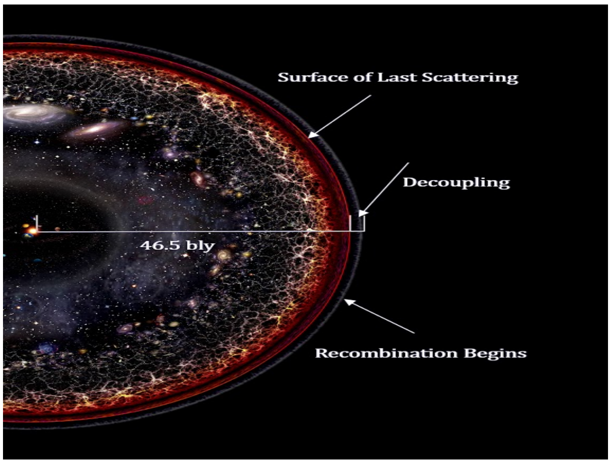

Figure 2.3: A summary of possible geometries. [redo this figure].

After their last scattering of electrons, photons were able to travel unimpeded through the Universe. These are the Cosmic Microwave Background photons we receive today, still with their blackbody distribution, now redshifted by a factor of 1100. They constitute a last scattering surface, or more appropriately a last scattering layer, since (obviously) not all photons underwent their last scattering simultaneously.

Of course, there is nothing special about this particular surface, other than it happens to be at the right distance for the photons to have reached us today. There are photons originating at every point, and observers in different parts of the Universe will see photons originating from different large spheres, of the same radius, centred on their location.

Bibliography

[1] Scientific American. Could silicon be the basis for alien life forms, just as carbon is on earth? https://www.scientificamerican.com/article/could-silicon-be-the-basi/, 1998. 19