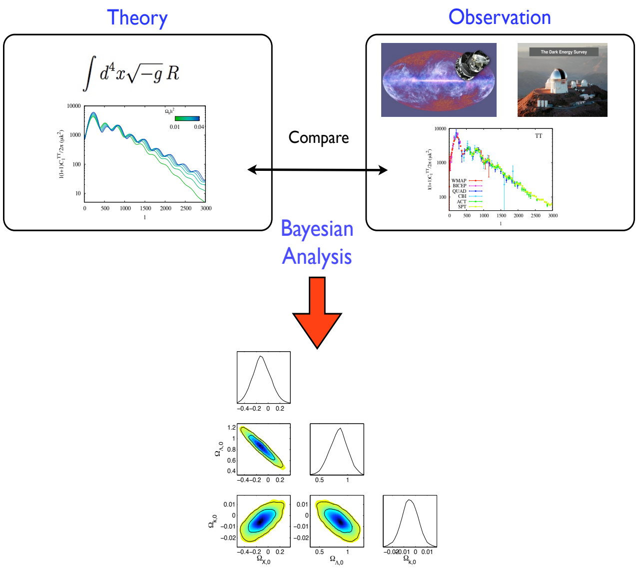
Updated Cosmology

with Python



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Thermal history of the Universe

1.1 Introduction

At the beginning of this review, we considered the homogeneous and isotropic universe, and now it is time to take care about its composition, that is, how the formation of light elements was carried out at initial stages.
Some books: The first three minutes, Weinberg's. Baryogenesis, James M. Cline. Baryogenesis, Csaba Balazs (2014). The early universe, Turner.

1.2 History

Highly Speculative.

- $T \sim 10^{19}$GeV, $t \sim 10^{-43}$sec.
  String theory?, quantum gravity?, super gravity?, quantum birth of the Universe?.
  At these, very high, temperatures the energy density is so high that the classical treatment of GR is no longer reliable, and perhaps the necessity of a *Quantum theory of Gravity.*

- $T \sim 10^{16}$GeV, $t \sim 10^{-38}$sec.
  Grand Unified Theories (GUT) phase transition occurs.
  Strong and electroweak interactions are indistinguishable.

- $T \sim 10^{14}$GeV, $t \sim 10^{-34}$sec.
  Inflation, Monopoles.

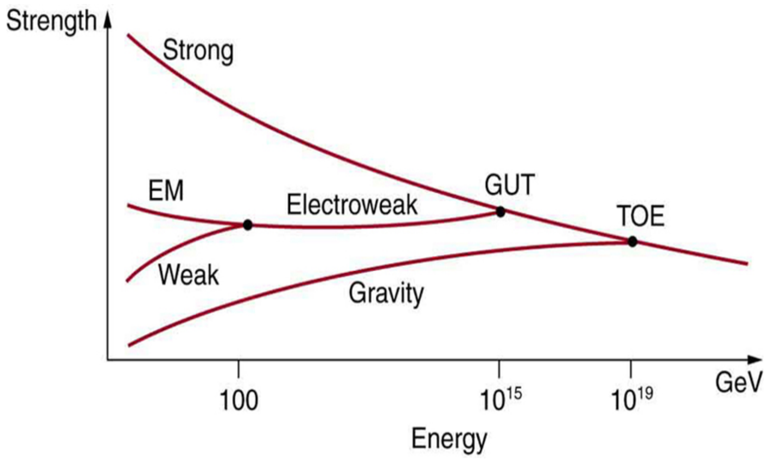

Figure 1.1: Theory of everything

Baryogenesis: Origin of matter-antimatter asymmetry.

Relativistic QFT requires the existence of antiparticles $e^+ + e^- \to \gamma + \gamma$.

- $T \sim 10^{12}$GeV, $t \sim 10^{-30}$sec.

  Peccei-Quinn phase transition. Why QCD does not break CP-symmetry?.

- LHC - 13 TeV (Energies that can be reach).

- $T \sim 100$GeV, $t \sim 10^{-10}$sec.

  Electroweak phase transition.

  Particles receive their masses through the Higgs mechanism (125 GeV).

- $T \sim 10's - 100's$GeV, $t \sim 10^{-8}$sec.

  If dark matter is made up of SUper SYmmetric (SUSY) particles or WIMPs, this is the time when their interactions freeze out and their cosmological abundance is fixed.

- $T \sim 100 - 300$MeV, $t \sim 10^{-5}$sec.

  Quark-hadron phase transition.

  QCD, theory of strong force, which describes the binding of quarks by gluons to first became particles, such as neutrons and protons (baryons - 3 quarks, mesons $\to q + \bar{q}$).

  This is also when axions are produced (if they exist and form the dark matter) – Pretty sure this must have happened.

- $T \sim 0.1 - 10$MeV, $t \sim$ secs-mins.

  Big Bang Nucleosynthesis (BBN) started

  n^o and p^+ first combine to form D, $^3He,^4He,^7Li$.

  The theory agrees very impressively with observations.

- $T \sim 0.8$MeV, $t \sim 1000$sec.

  Neutrino decoupling.

  Neutrino only interacts with the primordial plasma through weak interaction.

- $T \sim 3$eV, $t \sim 10^{4-5}$yrs.

  Before the matter radiation equality $\rightarrow$ energy density is dominated by radiation.

  Perturbations in the dark-matter density can begin to grow.

- $T \sim$eV, $t \sim 400'000$yrs.

  Recombination: $e^- + p^+ \rightarrow H + \gamma$.

  e^- and p^+ combine to form hydrogen atoms.

  γ decouple and CMB happens $\rightarrow$ photons are tightly coupled to the baryon fluid through Thompson scattering from free e^-. CMB observations.

- $T \sim 10^{1-2}$eV, $t \sim$ millions of years.

  Baryon drag ends $\rightarrow$ baryons are still coupled to the CMB photons so perturbations in the baryon cannot grow.

- $T \sim 10^{-3}$eV, $t \sim 10^9$yrs.

  The first stars and (small) galaxies begin to form.

- $T \sim 0.33$meV, $t \sim 9$Gyr.

  Dark energy-matter equality.

- $T \sim 10^{-4}$eV, $t \sim 10^{10}$yrs.

  Baryons and CMB are entirely decoupled.

  Stars and galaxies have been around for a long time.

  Clusters of galaxies (~ 1000s) are becoming common.

- Dark Energy domination.

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Figure 1.2: Thermal History of the Universe [redo this figure].

1.2.1 The Hot Big Bang

The key quantity to understand the thermal history of the universe is the comparison between the rate of interactions Γ and the rate of expansion H. When $\Gamma \gg H$, that is, when the time-scale of particle interactions t_c is much smaller than the characteristic expansion time-scale t_H, such that

$$t_c \equiv \frac{1}{\Gamma} \quad \ll \quad t_H \equiv \frac{1}{H}, \tag{1.1}$$

then, local thermal equilibrium is reached before the effect of expansion becomes relevant.

As the universe cools down, the rate of interactions may decrease faster than the expansion rate. When $t_c \sim t_H$ is reach, particles decouple from the thermal bath. Different particles may decouple at different times depending of its features, as we shall see below.

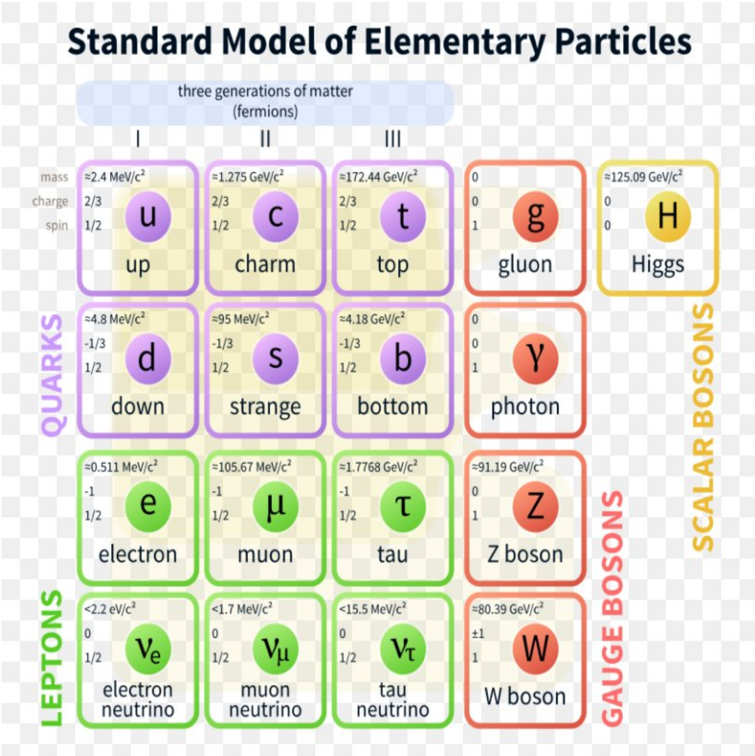

Figure 1.3: Thermal History of the Universe [redo this figure].

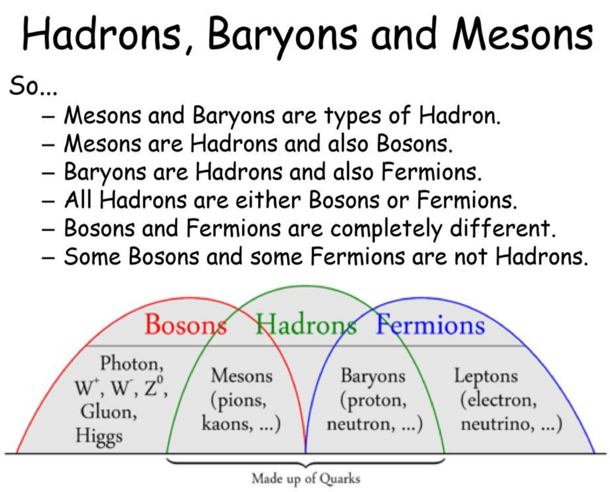

Figure 1.4: Thermal History of the Universe [redo this figure].

1.2.2 Local Thermal equilibrium

Standard model $\gtrapprox$ 100GeV.

The rate of particle interactions can be defined as

$$\Gamma \equiv n\sigma v. \tag{1.2}$$

where

- n: the number density of particles.
- σ: the interaction cross section.
- v: the average velocity of the particles.

For such temperatures ($T \geq 100\text{GeV}$), particles are ultra-relativistic $\rightarrow v \sim 1$. The particle mass can be ignored and therefore $n \sim T^3$. Interactions are mediated by *gauge bosons*, which are massless $\rightarrow$ the cross sections for strong and electroweak interactions is

$$\sigma \sim \frac{\alpha^2}{T^2}, \tag{1.3}$$

where $\alpha \equiv \frac{g_A^2}{4\pi}$ is the generalised structure constant associated to the gauge boson A.

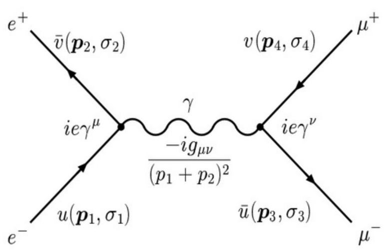

Figure 1.5: A Feynman diagram.

Then

$$\Gamma = n\sigma v \sim T^3 \times \frac{\alpha^2}{T^2} = \alpha^2 T, \tag{1.4}$$

compare to the Hubble rate $H \sim \sqrt{\rho}/M_{Pl}$ with $\rho \sim T^4$.

$$H \sim \frac{T^2}{M_{Pl}^2}. \tag{1.5}$$

Then

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_{Pl}}{T} \sim \frac{10^{16}\text{GeV}}{T}, \tag{1.6}$$

with $\alpha \sim 0.01$. Below $T \sim 10^{16}$GeV but above 100GeV the condition is therefore satisfied, and hence the standard model is indeed in thermal equilibrium.

1.2.3 Equilibrium Thermodynamics

From CMB observations we observe that the early universe was in local thermal equilibrium.

Statistical mechanics: Turning the microscopic laws to understand the macroscopic world, and its description is based on the *distribution function* $f(t, \vec{x}, \vec{p})$:

- Homogeneity $\rightarrow f$ is independent of position $\vec{x}$.
- Isotropy $\rightarrow$ momentum dependence is only in terms of its magnitude $p = |\vec{p}|$.

Therefore the particle density in phase space is then the density of states times the distribution function

$$\frac{g}{(2\pi)^3} \times f(p), \tag{1.7}$$

where g represents the internal degrees of freedom (i.e. spin). The number density of particles (in real space) is then

$$n = \frac{g}{(2\pi)^3} \int d^3p f(p). \tag{1.8}$$

The energy density of a gas of particles ρ, assuming that the particles in the early universe were weakly interactions $\rightarrow$, that is, ignore interactions densities. Using $E(p) = \sqrt{p^2 + m^2}$, we get

$$\rho = \frac{g}{(2\pi)^3} \int d^3p f(p) E(p), \tag{1.9}$$

$$P = \frac{g}{(2\pi)^3} \int d^3p f(p) \frac{p^2}{3E}. \tag{1.10}$$

A system is said to be in **kinetic equilibrium** if the particles exchange energy and momentum efficiently. In this case, the type of particles can be differentiated by the Fermi-Dirac $(+)$ and Bose-Einstein $(-)$ distributions at temperature T

$$f(p) = \frac{1}{e^{(E-\mu)/T} \pm 1}, \tag{1.11}$$

where μ is the chemical potential. At low temperatures $(E - \mu) > T$ both reduce to Maxwell-Boltzmann distribution

$$f(p) \approx e^{-(E-\mu)/T}. \tag{1.12}$$

If a specie i is in chemical equilibrium, then its chemical potential μ_i is related to the chemical potential μ_j of the other species it interacts with. **Chemical equilibrium** implies that

$$\mu_1 + \mu_2 = \mu_3 + \mu_4. \tag{1.13}$$

Since the number of photons is not conserved (i.e. double compton $e^- + \gamma \rightleftharpoons e^- + \gamma + \gamma$) [1] hence $\mu_\gamma = 0$. If the chemical potential of a particle X is μ_X and its corresponding anti-particle is $\bar{X}$, then $\mu_{\bar{X}} = -\mu_X$:

$$X + \bar{X} \rightleftharpoons \gamma + \gamma. \tag{1.14}$$

Therefore **thermal equilibrium** is achieved for species that are both in kinetic and chemical equilibrium, that is, they share a common temperature $T_i = T$ ('Temperature of the Universe').

1.2.4 Densities and Pressure

Combining the previous equations, using that μ can be neglected (for now), we have

$$n = \frac{g}{2\pi^2}\int_0^\infty dp \frac{p^2}{\exp[\sqrt{p^2+m^2/T}] \pm 1}, \tag{1.15}$$

$$\rho = \frac{g}{2\pi^2}\int_0^\infty dp \frac{p^2\sqrt{p^2+m^2}}{\exp[\sqrt{p^2+m^2/T}] \pm 1}, \tag{1.16}$$

$$P = \frac{g}{6\pi^2}\int_0^\infty dp \frac{p^4(p^2+m^2)^{-1/2}}{\exp[\sqrt{p^2+m^2/T}] \pm 1}. \tag{1.17}$$

These integrals have to be evaluated numerically, however in the (ultra) relativistic and non-relativistic limits, we can get analytical results.

Some useful integrals:

$$\int_0^\infty d\xi \frac{\xi^n}{e^\xi - 1} = \zeta(n+1)\Gamma(n+1),$$

$$\int_0^\infty d\xi \xi^n e^{-\xi^2} = \frac{1}{2}\Gamma\left(\frac{1}{2}(n+1)\right),$$

$$\int_0^\infty d\xi \frac{\xi}{e^\xi + 1} = \frac{\pi}{12},$$

with $\zeta(z)$ is the Riemann zeta-function, and

$$\frac{1}{e^\xi + 1} = \frac{1}{e^\xi - 1} - \frac{2}{e^{2\xi - 1}}$$

[1] Compton is the Thompson low energy limit.

HW: In the relativistic limit ($T \gg m$) show that:

$$
\begin{aligned}
n &= \frac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 & \text{bosons} \\ \frac{3}{4} & \text{fermions,} \end{cases} && (1.18) \\
\rho &= \frac{\pi^2}{30} g T^4 \begin{cases} 1 & \text{bosons} \\ \frac{7}{8} & \text{fermions,} \end{cases} && (1.19) \\
P &= \frac{\rho}{3}. && (1.20)
\end{aligned}
$$

Using the current temperature of the Universe $T_0 = 2.73\text{K}$, we have

$$
\begin{aligned}
n_{\gamma,0} &= \frac{2\zeta(3)}{\pi^2} T_0^3 \approx 410 \text{ photons cm}^{-3}. \\
\rho_{\gamma,0} &= \frac{\pi^2}{15} T_0^4 \approx 4.6 \times 10^{-34} \text{g cm}^{-4} \quad \rightarrow \quad \Omega_{\gamma,0} h^2 \approx 2.5 \times 10^{-5}.
\end{aligned}
$$

If we add the chemical potential, the excess of fermion species over its antiparticle, assuming $\mu_+ = -\mu_-$ ($X + \bar{X} \rightleftharpoons \gamma + \gamma$). The net particle number for $T \gg m$ (an exact result) is

HW: Show that the net particle number for $T \gg m$ and $T \gg |\mu|$ is

$$
\begin{aligned}
n - \bar{n} &= \frac{g}{2\pi^2} \int_0^\infty dp p^2 \left(\frac{1}{e^{(p-\mu)/T} + 1} - \frac{1}{e^{(p+\mu)/T} + 1} \right) \\
&= \frac{1}{6\pi^2} g T^3 \left[\pi^2 \left(\frac{\mu}{T} \right) + \left(\frac{\mu}{T^3} \right)^3 \right]. && (1.21)
\end{aligned}
$$

hint: expand the expression over $|\mu| \sim 0$.

HW: Show that, in the non-relativistic limit ($m \gg T$):

$$
n = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}. \tag{1.22}
$$

Therefore, massive particles are exponentially rare at low temperature ($m \gg T$). In this limit $E(p) = \sqrt{p^2 + m^2} \approx m$ and from the expressions (1.15) and (1.16) we have $\rho \approx mn$

$$
P = nT \ll nm = \rho, \tag{1.23}
$$

therefore $P \ll \rho$, non-relativistic gas of particles behaves as a pressurless matter or *dust*.

$P = nT$ - ideal gas law (or $PV = Nk_BT$), and adding the chemical potential

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}, \tag{1.24}$$

$$n - \bar{n} = 2g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}\sinh\left(\frac{\mu}{T}\right). \tag{1.25}$$

The excess of fermion species in the non-relativistic limit, n^0, e^-, p^+, fall exponentially (are *Boltzmann supressed*) as the temperature drops below the mass of the particle.

Interpretation of the annihilation of $X + \bar{X}$: At high energies the annihilation occurred but they are balanced by $X, \bar{X}$ production. At low temperatures the thermal particle energies are not sufficient for pair production.

1.2.5 Effective number of Relativistic Species

Let us consider the temperature of the photon gas as T. Then, the total radiation density (1.19) is the sum over the energy densities of all relativistic species

$$\rho_r = \sum \rho_i = \frac{\pi^2}{30} g_*(T) T^4, \tag{1.26}$$

where $g_*(T)$ is the effective number of relativistic degrees of freedom at temperature T. There are two contributions:

- Relativistic species in thermal equilibrium with photons, $T_i = T \gg m_i$

$$g_*^{\text{ther}}(T) = \sum_{i=b} g_i + \frac{7}{8}\sum_{i=f} g_i. \tag{1.27}$$

- Relativistic species not in thermal equilibrium with photons $T_i \neq T \gg m_i$

$$g_*^{\text{dec}}(T) = \sum_{i=b} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=f} g_i \left(\frac{T_i}{T}\right)^4. \tag{1.28}$$

- At $T \gtrapprox 100$ GeV, all particles of the Standard model are relativistic

$$g_b = 28, g_f = 90 \rightarrow g_* = g_b + \frac{7}{8} g_f = 106.75. \tag{1.29}$$

- As the temperature T drops, various species became non-relativistic and annihilate

Quarks	t	174.2 ± 3.3GeV	$\bar{t}$	spin=$\frac{1}{2}$	$g = 2 \cdot 2 \cdot 3 = 12$	
	b	4.20 ± 0.07GeV	$\bar{b}$	3 colors		
	c	1.25 ± 0.09GeV	$\bar{c}$			
	s	95 ± 25MeV	$\bar{s}$			
	d	3–7MeV	$\bar{d}$			
	u	1.5–3.0MeV	$\bar{u}$			
						72
Gluons	8 massless bosons			spin=1	$g = 2$	16
Leptons	τ^-	1777.0 ± 0.3MeV	τ^+	spin=$\frac{1}{2}$	$g = 2 \cdot 2 = 4$	
	μ^-	105.658MeV	μ^+			
	e^-	510.999keV	e^+			
						12
	ν_τ	< 18.2MeV	$\bar{\nu}_\tau$	spin=$\frac{1}{2}$	$g = 2$	
	ν_μ	< 190keV	$\bar{\nu}_\mu$			
	ν_e	< 2 eV	$\bar{\nu}_e$			
						6
Electroweak gauge bosons	W^+	80.403 ± 0.029GeV		spin=1	$g = 3$	
	W^-	80.403 ± 0.029GeV				
	Z^0	91.1876 ± 0.0021GeV				
	γ	0 $(< 6 \times 10^{-17}$eV)			$g = 2$	
						11
Higgs boson (SM)	H^0	> 114.4GeV		spin=0	$g = 1$	1
						$g_f = 72 + 12 + 6 = 90$
						$g_b = 16 + 11 + 1 = 28$

Figure 1.6: Thermal History of the Universe [redo this table].

- The heaviest particles (top q)[1] annihilate first at $T \sim 30$GeV ($\frac{1}{6}m_t$), and the effective number of relativistic species is reduced

$$g_* = 106.75 - \frac{7}{8} \times 12 = 96.25. \tag{1.30}$$

- $W^\pm, Z^0$, Higgs boson (Gauge bosons) annihilate next, $T \sim 10$GeV

$$g_* = 96.25 - (1 + 3 \times 3) = 86.25. \tag{1.31}$$

- b quarks follow

$$g_* = 86.25 - \frac{7}{8} \times 12 = 75.75. \tag{1.32}$$

[1] top q decays (99.8%) into W-boson, bottom, and less likely into strange or down. Its mean lifetime is about $\sim 5 \times 10^{-25}$s.

- and finally c, τ quarks

$$g_* = 75.75 - \frac{7}{8} \times (12+4) = 61.75. \tag{1.33}$$

- Before the strange annihilates the matter undergoes to the QCD phase transition $T \sim$ 150MeV. Quarks combine into baryons (p^+, n^0) and mesons (pions). All particles except pions $(\pi^\pm, \pi^0)$ are non-relativistic below the temperature phase transition, T_{QCD}. Thus the only particle species left are pions, e^-, μ, ν, γ, therefore

$$g_* = \underbrace{2}_{\gamma} + 3 + \frac{7}{8} \times (4+4+6) = 17.25. \tag{1.34}$$

- Next, e^- & e^+ annihilate and we need the Entropy.

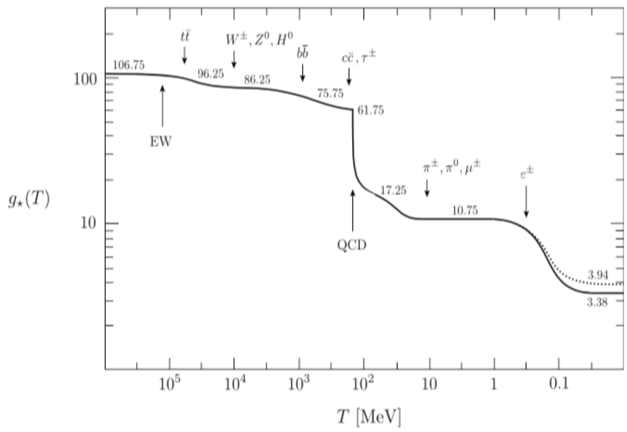

Figure 1.7: Thermal History of the Universe [redo this figure].

1.2.6 Conservation of Entropy

It is useful to track a conserved quantity. To a good approximation we can therefore treat the expansion of the universe as adiabatic, so the total entropy stays constant [even beyond the equilibrium].

HW: Show that

$$\frac{\partial P}{\partial T} = \frac{\rho + P}{T}. \tag{1.35}$$

Consider the second law of Thermodynamics

$$TdS = dU + PdV,$$

Using $U = \rho V$

$$\begin{aligned} dS &= \frac{1}{T}\left(d[(\rho+P)V] - VdP\right) \\ &= \frac{1}{T} d\left[(\rho+P)V\right] - \frac{V}{T^2}(\rho+P)dT \\ &= d\left(\frac{\rho+P}{T}V\right) \end{aligned}$$

Let us show the conservation of entropy

$$\begin{aligned} \frac{dS}{dt} &= \frac{d}{dt}\left[\frac{\rho+P}{T}V\right] \\ &= \frac{V}{T}\left[\frac{d\rho}{dt} + \frac{1}{V}\frac{dV}{dt}(\rho+P)\right] + \frac{V}{T}\left[\frac{dP}{dt} - \frac{\rho+P}{T}\frac{dT}{dt}\right] = 0. \end{aligned}$$

The first term in equation resembles the continuity equation, while the second term is given by the homework, Eqn. (1.35). We just showed the conservation of entropy whilst in equilibrium. It is convenient to work with the entropy density $s = S/V$. From dS we see that

$$s = \frac{\rho+P}{T}, \tag{1.36}$$

from ρ and P, we have (still in relativistic limit)

$$s = \sum \frac{\rho_i + P_i}{T_i} \equiv \frac{2\pi^2}{45} g_{*S}(T) T^3. \tag{1.37}$$

1.2.7 Effective number of degrees of freedom in entropy

Similarly to pervious calculations, we split the contribution in thermal equilibrium and when the particles decoupled from the thermal bath

$$g_{*S}(T) = g_{*S}^{\text{th}}(T) + g_{*S}^{\text{dec}}(T), \tag{1.38}$$

notice that in thermal equilibrium, the two quantities coincide

$$g_{*S}^{\text{th}}(T) = g_*^{\text{th}}(T). \tag{1.39}$$

On the other hand, for decoupled species

$$g_{*S}^{\text{dec}}(T) \equiv \sum_{i=b} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i=f} g_i \left(\frac{T_i}{T}\right)^3 \neq g_*^{\text{dec}}(T). \tag{1.40}$$

Conservation of entropy has two consequences

1. The number of particles in a comoving volume ($N \equiv Vn$) is defined as the number density n_i divided by the entropy density $N_i \equiv \frac{n_i}{s}$. If no particles are produced or destroy (with $s \propto a^{-3}$) then $n_i \propto a^{-3}$ and N_i is constant. i.e. baryon number after baryogenesis

$$\frac{n_B}{s} \equiv \frac{(n_b - n_{\bar{b}})}{s}$$

2. It implies

$$sV \sim g_{*S}(T)T^3a^3 = const, \qquad \text{or} \qquad T \propto g_{*S}^{-1/3}a^{-1}. \tag{1.41}$$

   Away from the particle mass thresholds, g_{*S} is approximately constant and hence $T \propto a^{-1}$, as expected and used in previous chapters.

   Using the previous eqn, $T \propto g_{*S}^{-1/3}a^{-1}$, into the Friedmann equation

$$H = \frac{1}{a}\frac{da}{dt} \simeq \left(\frac{\rho_r}{3M_P^2}\right)^{1/2} \simeq \frac{\pi}{3}\left(\frac{g_*}{10}\right)^{1/2}\frac{T^2}{M_P},$$

   for a radiation dominated universe $a \propto t^{1/2}$ and for $T \propto t^{-1/2}$.

$$\frac{T}{1\text{MeV}} \simeq 1.5g_*^{-1/4}\left(\frac{1 \text{ sec}}{t}\right)^{1/2} \tag{1.42}$$

   The temperature of the Universe one second after the Big Bang was about 1MeV.

1.2.8 Neutrino decoupling

Neutrinos are coupled to the thermal bath via weak interactions processes like

$$\begin{aligned} \nu_e + \bar{\nu}_e &\rightleftharpoons e^+ + e^-, \\ e^- + \bar{\nu}_e &\rightleftharpoons e^- + \bar{\nu}_e. \end{aligned}$$

The cross section for these interactions is $\sigma \sim G_F^2T^2$ and hence $\Gamma \sim G_F^2T^5$. As the temperature decreased, the interaction rate dropped much more rapidly than the Hubble rate $H \sim T^2/M_P$

$$\frac{\Gamma}{H} \sim \left(\frac{T}{1 \text{ MeV}}\right)^3. \tag{1.43}$$

Therefore, neutrinos decoupled around 1MeV (more accurately $T_{dec} \sim 0.8$MeV).

After decoupling, neutrinos moved freely along geodesics and preserved the relativistic Fermi-Dirac distribution. The neutrino number density (and particle number conservation) requires $n_\nu \propto a^{-3}$ and therefore $T_\nu \propto a^{-1}$.

1.2.9 Electron-Positron Annihilation

Shortly after neutrinos decouple, T drops below the electron mass and electron positron annihilation occurs:

$$e^+ \ + \ e^- \rightleftharpoons \gamma + \gamma.$$

The energy density and entropy of the electron and positron are transferred to the photons, therefore the photons are thus 'heated'.

Consider the change in the effective number of degrees of freedom in entropy

$$g_{*S}^{\mathrm{th}} = \begin{cases} 2 + \frac{7}{8} \times 4 & T \gtrsim m_e \\ 2 & T < m_e \end{cases} \tag{1.44}$$

equation (1.41), $g_{*S}^{\mathrm{th}}(aT_\gamma)^3$, remains constant, therefore aT_γ increases after $e^+ + e^-$ annihilation by a factor $\left(\frac{4}{11}\right)^{1/3}$, while aT_ν remains the same. Hence, the temperature of neutrinos after the $e^+ \ + \ e^-$ annihilation is slightly lower

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma. \tag{1.45}$$

For $T \ll m_e$, the effective number of relativistic species (in energy density and entropy) becomes

$$\begin{aligned} g_* &= 2 + \frac{7}{8} \times 2N_{\mathrm{eff}} \left(\frac{4}{11}\right)^{4/3} = 3.36, \\ g_{*S} &= 2 + \frac{7}{8} \times 2N_{\mathrm{eff}} \left(\frac{4}{11}\right) = 3.94, \end{aligned}$$

with N_{eff} is the effective number of neutrino species. It explains the previous plot.

If neutrino decoupling was instantaneous, then $N_{\mathrm{eff}} = 3$. Also that neutrino spectrum, after decoupling, deviates slightly from Fermi-Dirac distribution, hence $N_{\mathrm{eff}} = 3.046$. Planck satellite constraints are $N_{\mathrm{eff}} = 3.36 \pm 0.34$.

1.2.10 Cosmic Neutrino Background

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma, \tag{1.46}$$

holds until the present day. $T_{\nu,0} = 1.95K = 0.17\mathrm{meV}$, lower than the CMB.

The number density of neutrinos is

$$n_\nu = \frac{3}{4} N_{\mathrm{eff}} \times \frac{4}{11} n_\gamma, \tag{1.47}$$

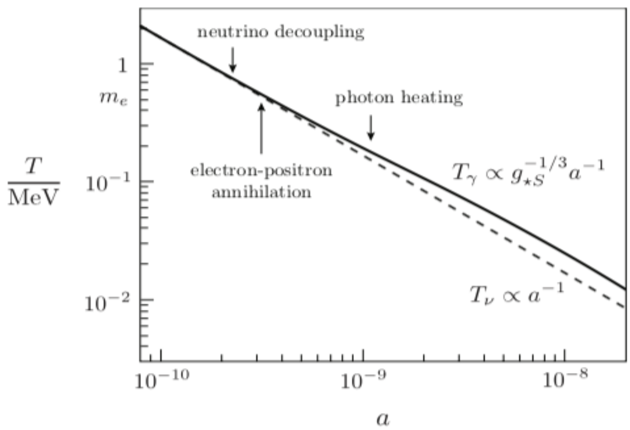

Figure 1.8: Thermal History of the Universe [redo this fig].

using $n_{\gamma,0}$ implies 112 neutrinos cm^{-3} per flavour. If neutrinos are massless

$$\rho_\nu = \frac{7}{8} N_{\rm eff} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma \to \Omega_\nu h^2 \approx 1.7 \times 10^{-5}. \tag{1.48}$$

However, neutrino oscillation experiments show that ν do have mass and the minimum value is $\sum m_{\nu,i} > 60$meV. Massive neutrinos behave as radiation like particles in the early universe, and as matter-like particles in the late universe.

For massive neutrinos

$$\Omega_\nu h^2 \approx \frac{\sum m_{\nu,i}}{94 \text{ eV}}. \tag{1.49}$$

Observation of the CMB and SNIa constrain $\sum m_{\nu,i} < 1eV$, 25 times the energy density of photons but still subdominant $0.001 < \Omega_\nu < 0.02$.

Once Big Bang Nucleosynthesis is over, at time $t \sim 300s$ and temperature $T \sim 8 \times 10^8 K$, the Universe is in a thermal bath of photons, protons, electrons, in addition to neutrinos and the unknown dark matter particle(s). The energy density is dominated by the relativistic component, photons and neutrinos.

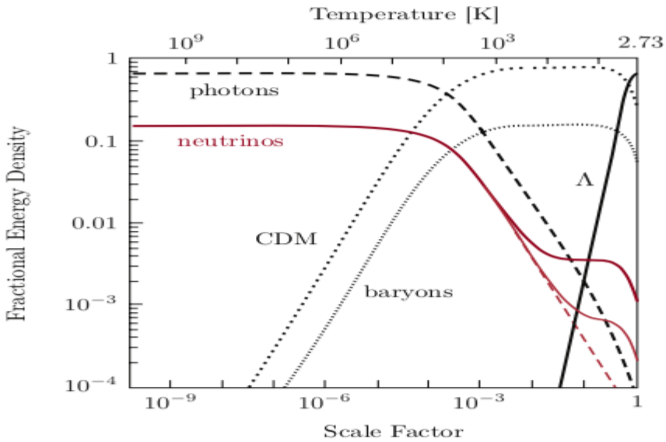

Figure 1.9: Thermal History of the Universe [redo this fig].

Introduction

Does antimatter feel gravitational interaction in the same way than ordinary matter does? Even though this question might be in principle easy to answer with the obvious response: It does, since if they are massive, they must feel the gravitational interaction in the same way ordinary matter do. Nevertheless, there is still no experimental evidence to determine how antiparticles behave according to the gravitational interaction. In this paper some theoretical ideas and experimental attempts to answer this question are discussed.

Attraction under gravitational interaction

The first idea that someone might conjecture is that the behaviour of antimatter under the gravitational interaction shall be the same than the ordinary matter, i.e., an attractive behaviour among particles. This idea has been supported by arguing that otherwise (repulsion) it would violate CPT invariance (a fundamental symmetry of every physical theory) since C-symmetry does not modify the gravitational behaviour (mass) of particles/antiparticles. Moreover, energy conservation would not be hold leading to vacuum instability because in the case that matter and antimatter responded oppositely to a gravitational field, one could take advantage of the fact that no energy would be needed to move a pair particle-antiparticle. However all these ideas were later turned down in 1991[**?**].

Repulsion under gravitational interaction

In contrast, due to no experimental evidence has been found to confirm that antimatter should act attractively, some ideas have been proposed to support the thought that antimatter with repulsive behaviour should be valid. Therefore, one main idea in this direction is distinguished and was formulated by Kowitt's [**?**]. Inpired by Dirac's ideas about his propose of a particles sea, Kowitt proposed that a positron should act as hole within the sea of electrons of negative energy but possitive mass. Notice that this entails a modification on how C-inversion acts on particles/antiparticles,i.e., this would imply that a positron has positive energy but negative gravitational mass leading to gravitational repulsion.

Experimental tests

Since at high energies (small distances) the gravity is negligible, it is difficult to directly observe gravitational forces at the particle level, for instance, electromagnetic force dominates over the gravitational one for charged particles since this latter is much more weaker at these scales. However, some experiments such as cold neutral anti-hydrogen experiments have been realised [**?**], taking advantage of the fact that anti-hydrogen is electrically neutral, which open the possibility of a direct measurement about the attractive/repulsive nature of antimatter, however no overwhelming results were obtained. Therefore, some recent experiments with a better accuracy have been done recently in this direction [**?**], with the goal in mind of finding a definitive answer about this dilemma.

Conclusion

Since no experimental evidence about the nature of antimatter under gravitational interaction has been found, some ideas arguing a repulsive/attractive behaviour of antiparticles have been discussed. Finally, the ultimate theory describing antiparticles gravitational nature is still unclear and will be determined until overwhelming experimental evidence is found.

Hoyle-Narlikar theory of gravity

In cosmology, some alternatives to the Big Bang Theory of the evolution of the universe have been proposed. Among these alternative one can find the steady state model where the density of matter in the expanding universe remains unchanged due to a continuous creation of matter.

Then, the Hoyle-Narikar theory of gravity is based upon this idea, and will be discussed on this paper.

Once we know what the Mach's principle says, we can define the Hoyle-Narlikar model as [1] a Machian and conformal theory of gravity proposed by Fred Hoyle and Jayant Narlikar that originally fits into the quasi-steady state model of the universe. This theory can be derived from the action

$$S = \sum_a \sum_b \int \int G(a,b)\; da\; db \quad , a \neq b.$$

where $G(a,b)$ is the Green function that holds the equation:

$$\Box G(x,y) + \frac{1}{6} R G(x,y) = \frac{\delta(x-y)}{\sqrt{-g}},$$

where g is the determinant of the spacetime metric.

On the other hand, although the Einstein's theory of relativity has been very successful, one can realise that since it does not provide boundary conditions, a whole family of possible solutions to describe our Universe is in general possible. In contrast, in the Hoyle-Narlikar theory of gravity one can find exactly that, a boundary condition. Interestingly, S. Hawking showed [**?**] that this boundary condition prohibits the existence of a solution type FLRW that is the best model that we have so far to describe our Universe. However, at that time the accelerating expansion of the universe was unknown, and this may allow the existence of such solutions. Furthermore, recently it has been shown[**?**] that in the limit of a smooth fluid model of particle distribution constant in time and space, the model can be reduced to Einstein's general relativity, and thus, FLRW Universe can be recovered.

Conclusion

The main ideas and features of the Hoyle-Narlikar theory of gravity have been discussed. Despite this model can recover Einstein's theory of gravity at a certain limit, perhaps the main problem is still its nature as a quasi-steady state model, since these kind of models do not fit into the observational data of WMAP [**?**], and thus, its approach as a phenomenological model is still no viable.

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