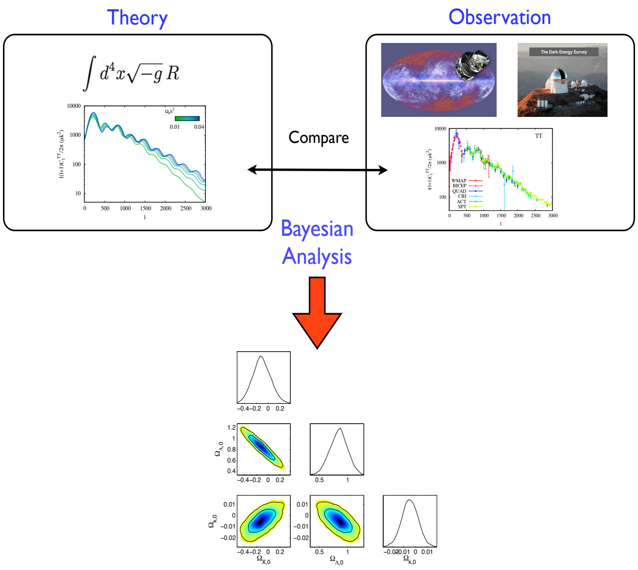
Updated Cosmology

with Python



José-Alberto Vázquez

ICF-UNAM / Kavli-Cambridge

In progress

August 12, 2017

Acronyms

1

Notation and conventions

Some of the notation adopted throughout this book is as follows:

- Whenever indicated we use the **natural unit system**, where measurements are based on universal physical constants. That is, the speed of light c, the reduced Planck's constant $\hbar = h/2\pi$ and the Boltzmann's constant $k_{\rm B}$, are set equal to one (length and time have the same units):
$$c = \hbar = k_{\rm B} = 1. \tag{1.1}$$

- An **overdot** in any quantity denotes time (t) derivative, prime represents conformal-time (η) derivatives and a comma-subscript derivatives with respect to space coordinates or fields (ϕ):
$$\dot{f} \equiv \frac{\partial f}{\partial t}, \quad f' \equiv \frac{\partial f}{\partial \eta}, \quad \text{and} \quad f_{,\phi} \equiv \frac{\partial f}{\partial \phi}. \tag{1.2}$$

- Quantities evaluated at present time ($t = t_0$) are also expressed by a subscript '0', whereas vectors are represented by any of these forms: $\mathbf{x}, x^i, \vec{x}$.

- Greek indices run over time coordinate (labelled by '0') and three spatial-Latin coordinates:
$$\alpha, \beta, \cdots \in \{0, 1, 2, 3\}, \qquad \text{and} \qquad i, j, \cdots \in \{1, 2, 3\}. \tag{1.3}$$

- We adopt the sign convention commonly used in relativity and cosmology. :
$$[\eta_{\mu\nu}] = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

where $\eta_{\mu\nu}$ are the components of the tensor metric, and in a shorthand notation $[\eta_{\mu\nu}] = \mathrm{diag}(+1, -1, -1, -1)$, such that the square line element is $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = c^2dt^2 - d\mathbf{x}^2$ for the Minkowski spacetime. This is the same **signature** used in Particle physics but often opposite in General Relativity.

- We adopt the Einstein's *summation convention*: whenever occur repeated indices (one as a covariant-subscript and one as a contravariant-superscript) in an expression they are summed over their range.

Example 1.0.1: Summation convention

In three dimensions:

$$a^i a_i \equiv \sum_{i=1}^{3} a_i a_i = (a_1)^2 + (a_2)^2 + (a_3)^2,$$

and

$$a^i{}_j a_{ik} = \sum_{i=1}^{3} a_{ij} a_{ik} = a_{1j}a_{1k} + a_{2j}a_{2k} + a_{3j}a_{3k}.$$

Here, the repeated index i is summed over all the components, and it is called a *dummy index*, as it can be replaced by any other index. Whereas the indices j and k, appearing on both sides of this equation, are said to be *free indices* as they can take any value from one to the dimension of the vector.

1.1 Fundamental constants

As we mentioned above, throughout this book we will mostly use natural units, however to make direct comparisons to physical quantities it is necessary to introduce the units back. Scipy (constants) has a library to perform this task (see Part II of the course).

HW 1.1.a: Covert the following quantities:

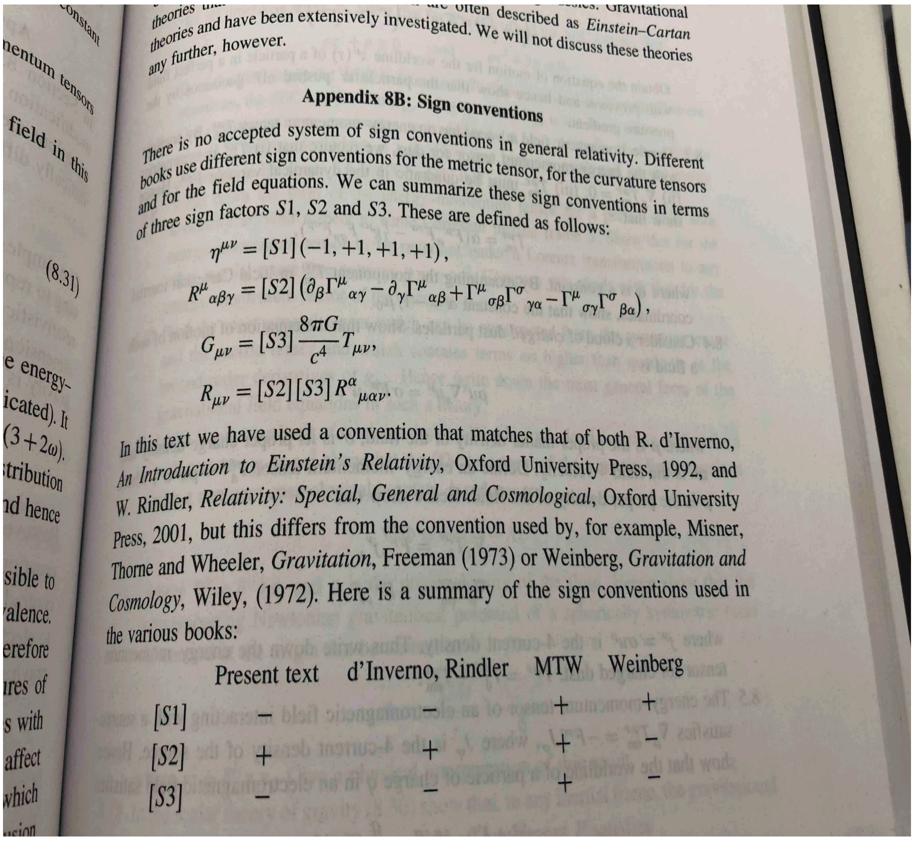
- $T_0 = 2.725K \to 0.2348$ meV, CMB temperature today.
- $\rho_{\gamma,0} = 4\sigma T_0^4/c \to 0.260$ eV cm^{-3} (411γ's cm^{-3}), CMB energy density.
- $c/H_0 \to$ Mpc, with $H_0 = 70$km sec^{-1}Mpc^{-1}, Radius of the observable universe.

HW 1.1.b: Show the equivalence:

- $4.48162009 \times 10^{-7}$= see box below.
- $0.2776566337 = 45 * \zeta(3)/(2 * \pi^4)$.
- $\rho_{c,0} = 1.87840\, h^2 \ \times 10^{-26}$ kg m^{-3} $= 2.775\, h^{-1} \ \times 10^{11} M_\odot/(h^{-1}\mathrm{Mpc})^3$.

type it into google:

```
8*pi^5*(boltzmann constant)^4/(15*(h*c)^3))*(1 Kelvin)**4/(3*(100 km/s/Mpc)^2/(8*Pi*G)*(speed of light)^2)
```


theories and have been extensively investigated. We will not discuss these theories any further, however.

Appendix 8B: Sign conventions

There is no accepted system of sign conventions in general relativity. Different books use different sign conventions for the metric tensor, for the curvature tensors and for the field equations. We can summarize these sign conventions in terms of three sign factors $S1$, $S2$ and $S3$. These are defined as follows:

$$\eta^{\mu\nu} = [S1](-1,+1,+1,+1),$$

$$R^{\mu}{}_{\alpha\beta\gamma} = [S2]\left(\partial_\beta\Gamma^{\mu}{}_{\alpha\gamma} - \partial_\gamma\Gamma^{\mu}{}_{\alpha\beta} + \Gamma^{\mu}{}_{\sigma\beta}\Gamma^{\sigma}{}_{\gamma\alpha} - \Gamma^{\mu}{}_{\sigma\gamma}\Gamma^{\sigma}{}_{\beta\alpha}\right),$$

$$G_{\mu\nu} = [S3]\frac{8\pi G}{c^4}T_{\mu\nu},$$

$$R_{\mu\nu} = [S2][S3]R^{\alpha}{}_{\mu\alpha\nu}.$$

In this text we have used a convention that matches that of both R. d'Inverno, *An Introduction to Einstein's Relativity*, Oxford University Press, 1992, and W. Rindler, *Relativity: Special, General and Cosmological*, Oxford University Press, 2001, but this differs from the convention used by, for example, Misner, Thorne and Wheeler, *Gravitation*, Freeman (1973) or Weinberg, *Gravitation and Cosmology*, Wiley, (1972). Here is a summary of the sign conventions used in the various books:

	Present text	d'Inverno, Rindler	MTW	Weinberg
[S1]	−	−	+	+
[S2]	+	+	+	−
[S3]	−	−	+	−

Figure 1.1: Comparison with several books/notes, i.e. Doran, Lasenby and Challinor, Baumman.

1.2 Distances

Measuring distances is not an easy task, specially when such humongous lengths are considered in cosmology. Let us mention some of the most useful ones.

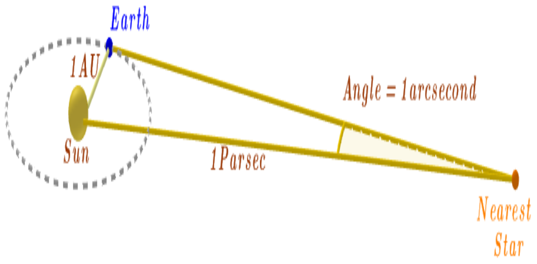


Figure 1.2: One Parsec (pc) is defined as the distance at which one astronomical unit subtends an angle of one second of arc.

- The radius of the Earth's orbit around the Sun, that is, the mean distance from the centre of the earth to the centre of the sun is defined as 1 Astronomical Unit (AU) or equivalently 1 AU= 1.496×10^{13} cm. Note: astronomers mostly used units in cm, gr, etc.

- 1 Parsec (pc) is defined as the distance at which one astronomical unit subtends an angle of one second of arc (1 pc = 1 AU/tan(1")), as shown in Figure 1.2.

$$1 \text{ parsec} \equiv 1.496 \times 10^{13}\text{cm} \times \frac{360}{2\pi} \times 60 \times 60 = 3.086 \times 10^{18}\text{cm} \simeq 3.261 \text{ light years (ly)},$$

  where a light year is the distance that light travels over the lapse of one year.

- Parsec, Kiloparsec (kpc) and Megaparsec (Mpc) are the most commonly used distance units in astronomy.

  For instance:

- Proxima Centauri, meaning the *nearest [star] of Centaurus*, is 1.301 pc (4.244 lys) away to the Sun. Proxima Centauri, being the third member of the Alpha Centauri system, is a red dwarf star with a mass of about an eighth the mass of the Sun $M_\odot$. The other two members, Alpha Centauri A (Rigil Kentaurus) and Alpha Centauri B (Toliman), together form the binary star -Alpha Centauri AB- with an average distance of 4.3 lys from the sun.

- 8.5 kpc is the distance from the Sun to the Galactic Centre of the Milky Way[1], a barred spiral galaxy with the following features
  - Radius of the disk = 12.5 kpc.
  - Thickness of the disk = 0.3 kpc.
  - At our location, the galaxy rotates with a period of 200 million years.

Figure 1.3: The Origin of the Milky Way (source: Tintoretto).

- The *Canis Major Dwarf Galaxy* (a disputed dwarf irregular galaxy) is the closest neighbour galaxy, being located at about 7.7 kpc away from the Solar System and 13 kpc from the Galactic Centre. The previous contender for the closest galaxy corresponds to the *Sagittarius Dwarf Elliptical Galaxy* with a distance of ∼16kpc.
- Another satellite galaxy of the Milky Way is the Large Magellanic Cloud (LMC) [2], located at about 50 kpc. The Milky Way and the LMC are expected to collide in approximately 2.4 billion years [1].
- Andromeda, also known as Messier 31 or just M31, is the closest galaxy with similar size to our own with a distance of 780 kpc. This spiral galaxy, with a diameter of ∼67 kpc, is the largest within the Local Group. Andromeda is approaching the Milky Way at about

[1] Was formed after the trickster god Hermes suckled the infant Heracles at the breast of Hera, the queen of the gods, while she was asleep. When Hera awoke, she tore Heracles away from her breast and splattered her breast milk across the heavens, see Figure 1.3.

[2] An intermediate between dwarf spiral galaxies and irregular galaxies.

110 kilometres per second, and hence they are expected to collide in about 4.5 billion years [2].

- The *Local group* of galaxies, the one we live in, has a total diameter of roughly 3.1 Mpc. The Local Group comprises more than 54 galaxies, most of them are dwarf galaxies, and the two largest members include the Andromeda galaxy and the Milky Way (Triangulum Galaxy being the third largest member). On the other hand, the Virgo cluster, comprises approximately 1300 (and possibly up to 2000) member galaxies and along with the Local Group they lay within the Virgo Supercluster.

- 33 Mpc is the diameter of Virgo Supercluster which contains at least 100 galaxy groups and clusters. The Virgo SC is only one of about 10 million superclusters in the observable universe.

- From the range of 10-100 Mpc are the sizes of Voids – spaces between filaments (Figure 1.4) – which contain very few or no galaxies. The *Local void*, lying adjacent to the Local Group, has a diameter of about 60 Mpc.

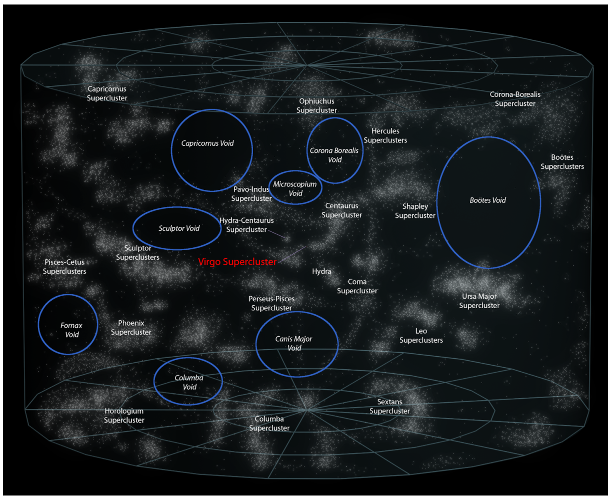


Figure 1.4: List of voids.

- The Laniakea Supercluster ("immense heaven" in Hawaiian), stretched out over 160 Mpc, encompasses approximately 100,000 galaxies. It has the approximate mass of 10^{17} solar masses and the closets supercluster consists of: Hydra-Centaurus (the closest neighbour), Perseus-Pisces and Pavo-Indus.

- The Hubble radius (radius of the observable universe) corresponds to c/H_0 =4450 Mpc (or 14.4 billion light years), as we shall see in the following sections.

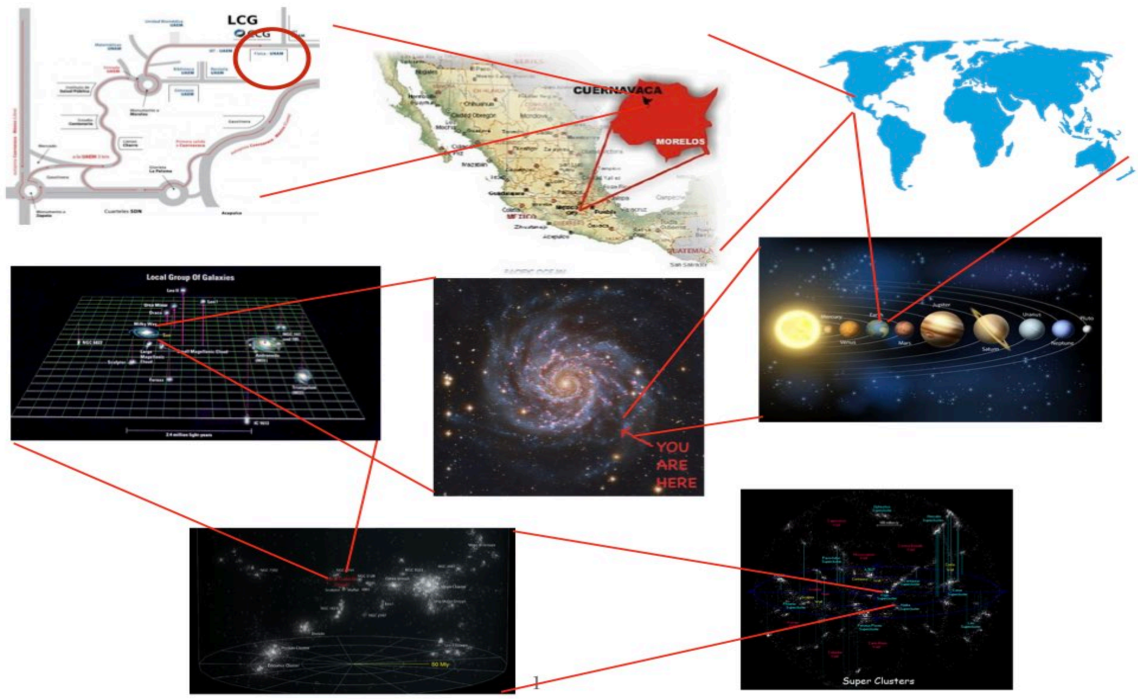


Figure 1.5: Size of the Universe.

1.3 Masses

- Stellar mass of the Milky Way $\simeq 1 \times 10^{11} M_\odot$.
- Dynamical mass of the Milky Way $\simeq 1 \times 10^{12} M_\odot$, measured from the motion of its satellites. Its difference compared to the stellar mass gives rise to the *Dark Matter* idea.
- Mass of giant elliptical galaxy $\simeq 1 \times 10^{13} M_\odot$.
- Mass of the Virgo cluster $\approx 10^{15} M_\odot$ (~ 1300 galaxies).
- Laniakea $\approx 10^{17} M_\odot$ ($\sim 1 \times 10^5$ galaxies).

HW 1.3: Compute the orbital speed of Neptune (5.43 km/s) given the mass of the sun and its distance, and compare with the real measurements (see Figure 1.7). Do the same with the Milky Way and the Sun.

The scale of the Universe: https://scaleofuniverse.com/

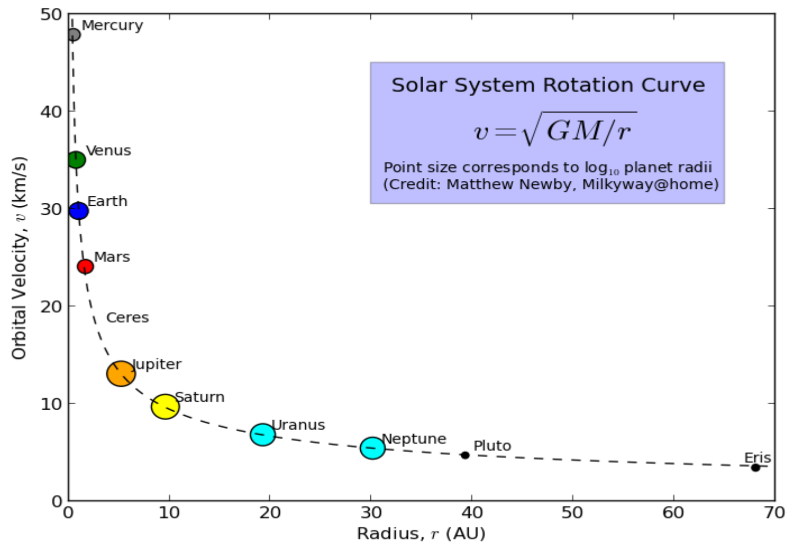


Figure 1.6: Rotation curves.

Bibliography

[1] Marius Cautun, Alis J Deason, Carlos S Frenk, and Stuart McAlpine. The aftermath of the Great Collision between our Galaxy and the Large Magellanic Cloud. Monthly Notices of the Royal Astronomical Society, 483(2):2185–2196, 11 2018. ISSN 0035-8711. doi: 10.1093/mnras/sty3084. URL https://doi.org/10.1093/mnras/sty3084. 5

[2] Ron Cowen. Andromeda on collision course with the milky way. Nature, 2012. doi: 10.1038/nature.2012.10765. URL https://doi.org/10.1038/nature.2012.10765. 6