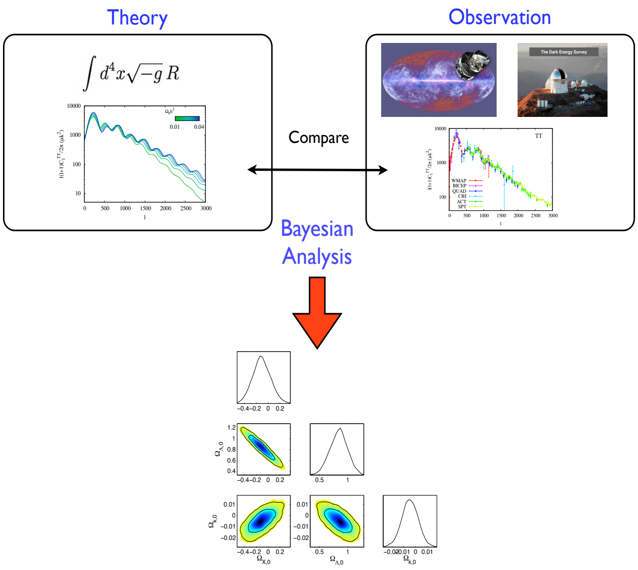
Updated Cosmology

with Python



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1

Inflation

Even though the Hot Big Bang model possesses a strong observational support, there are still certain inconsistencies or unexplained features to deal with: the flatness, horizon and monopole problems, amongst many others. The *inflationary model* offers the most elegant way so far proposed to solve these problems and therefore to understand why the universe is so remarkably in agreement with the standard cosmology. This model was initially introduced by Guth [5], followed by Linde [12]. For an extended review we refer to the textbooks Liddle and Lyth [10], Linde [13], Mukhanov [16]; and papers: Baumann [2], Liddle [8], Lyth and Riotto [15], Olive [17], Riotto [18]. The inflationary universe, Alan Guth, The first three minutes, Steven Weinberg, Endless Universe, P Steinhardt and N Turok. Let us examine some of the problems of the Hot Big Bang model.

1.0.1 Shortcomings of the Hot Big Bang

Flatness problem

HW: Show that a flat universe is an *unstable* fixed point if the strong energy condition is satisfied. hint: Show that the density parameter evolves with the scale factor a as:

$$\frac{d\Omega}{d\ln R} = (1+3w)\Omega(\Omega-1) \tag{1.1}$$

The Friedmann equation (**??**) can be seen in the following form

$$\Omega_T - 1 = \frac{kc^2}{(RH)^2}. \tag{1.2}$$

Written in this way, we notice that $\Omega_T = 1$ is a very special case. If at the beginning the universe was perfectly flat, then it remains so for all time. Nevertheless, a flat geometry is an unstable critical situation, that is, for even a tiny deviation from it, Ω_T would have evolved quite differently and very quickly the universe would become more curved. This can be seen as a consequence of RH being a decreasing function of time during radiation or matter domination epoch. We observe from (1.2) and Table ?? that:

$$\begin{aligned} \mid \Omega_T - 1 \mid &\propto t \qquad \text{radiation domination,} \\ \mid \Omega_T - 1 \mid &\propto t^{2/3} \qquad \text{dust domination.} \end{aligned}$$

Since the present age of the universe is estimated to be $t_0 \simeq 10^{17}\,\text{sec}$ [7], from the above equations we can deduce the required value of $\mid \Omega_T - 1 \mid$ at different early-times in order to obtain the correct value of spatial-geometry at present time $\mid \Omega_{T,0} - 1 \mid$. For instance, let us consider some particular epochs:

- Decoupling ($t \simeq 10^{13}\,\text{sec}$), we would need $\mid \Omega_T - 1 \mid \leq 10^{-3}$.
- Nucleosynthesis ($t \simeq 1\,\text{sec}$), we would need $\mid \Omega_T - 1 \mid \leq 10^{-16}$.
- Planck epoch ($t \simeq 10^{-43}\,\text{sec}$), we would need $\mid \Omega_T - 1 \mid \leq 10^{-64}$.

Consequently, at early times $\mid \Omega_T - 1 \mid$ had to be fine-tuned extremely close to zero in order to reach its actual observed value [6].

Figure 1.1: (jav: Figure that Ω_k drives away from a=1)

Horizon problem

The *horizon problem* is one of the most important problems within the Big Bang model, as it refers to the communication between different regions of the universe. The age of the universe

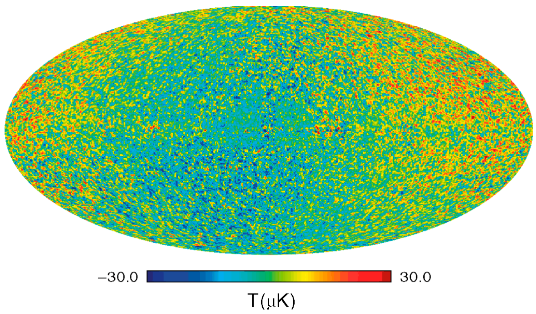

Figure 1.2: Temperature fluctuations observed in the CMB measured by the WMAP-7 experiment. The colour scale represents temperature fluctuations: from -30μK to 30μK. Figure reprinted from [7]. (jav: use an updated figure)

is finite and hence even light should have only travelled a finite distance by any given time. According to the standard cosmology, photons decoupled from the rest of the components at temperatures about $T_{\rm dec} \approx 0.3\,eV$ ($z_{\rm dec} \approx 1100$), from this time on photons free-streamed and travelled basically uninterrupted until reach us, giving rise to the region known as the *observable universe*. This spherical surface at which decoupling process occurred is called the *surface of last scattering*. The primordial photons are responsible for the CMB radiation we observe today. Looking at their fluctuations is thus analogous to taking a snapshot of the universe at that time (about $t_{\rm dec} \approx 380,000$ years after the Big Bang), as seen in Figure 1.2.

Figure 1.2 shows light seen in all directions of sky. These primordial photons have nearly the same temperature $T_{\rm cmb} = 2.725$ K plus small fluctuations (about one part in one hundred thousand). Being at the same temperature is a property of thermal equilibrium, hence observations are easily explained if different regions of the sky have been able to interact and moved towards thermal equilibrium before decoupling. Oddly, the comoving horizon over which causal interactions occurred before photons decoupled was significantly smaller than the comoving distance that radiation travelled after decoupling. This means that photons coming from sky regions separated by more than the horizon scale at last scattering, typically about 1°, would not have been able to interact and establish thermal equilibrium before decoupling. Therefore, the Big Bang model by itself does not offer an explanation of why temperatures seen in opposite directions of the sky are so nearly the same; the homogeneity must have been part of the initial conditions.

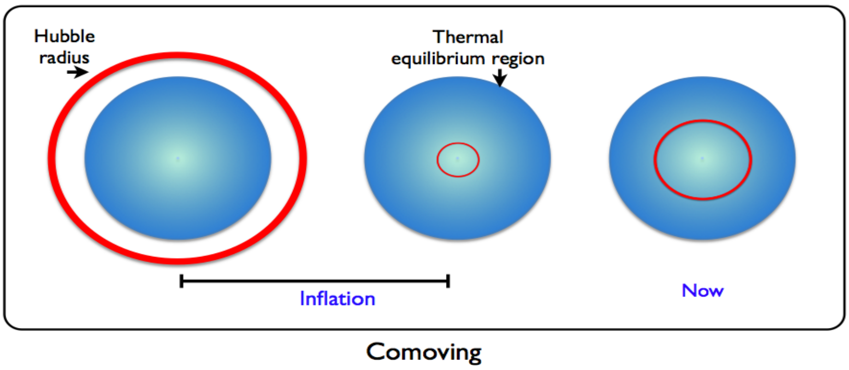

Figure 1.3: Schematic behaviour of the comoving Hubble radius during the Inflationary period (sketched by the red circle). (jav: Now should be CMB, check notes)

Monopole problem

The *monopole problem* was initially the motivation to develop the Inflationary cosmology [4]. The monopole, and other relics, are components of the universe that are expected to emerge as a consequence of unified models. From particle physics models, monopoles would have a mass of 10^6 orders the proton mass. Hence, based on their non-relativistic character, a crude calculation predicts an extremely high abundance at present time, $\Omega_{\rm M,0} \simeq 10^{16}$ [3]. According to this prediction, the universe would be dominated by magnetic monopoles, in contrast with current observations: no one has found any monopole yet [1]. (jav: add textures, string, etc...)

1.0.2 Cosmological Inflation

Inflation is defined as the epoch in the evolution of the universe in which decreases the comoving horizon (comoving Hubble radius) or equivalently the scale factor is quickly accelerated in just a fraction of a second:

$$\begin{aligned}
\text{INFLATION} \quad &\Longleftrightarrow \quad \frac{d}{dt}\left(\frac{1}{RH}\right) < 0, && (1.3)\\
&\Longleftrightarrow \quad \ddot{R} > 0, && (1.4)\\
&\Longleftrightarrow \quad \rho + 3p < 0, && (1.5)\\
&\Longleftrightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} < 1. && (1.6)
\end{aligned}$$

HW: Probe the equivalence amongst the above relations.

The first term corresponds to the comoving Hubble radius (**??**), which is interpreted as the observable universe becoming smaller during the inflationary period (sketched by the red circle in Figure 1.3). This process allowed our present observable universe to lie within a region located well inside the Hubble radius early on during inflation [10].

Accelerated expansion:

Shrinking the comoving Hubble radius implies an accelerated expansion.

$$\frac{d}{dt}(RH)^{-1} = \frac{d}{dt}(\dot{R})^{-1} = -\frac{\ddot{R}}{\dot{R}^2} < 0 \quad \rightarrow \quad \ddot{R} > 0. \tag{1.7}$$

Slowly-varying Hubble parameter:

We introduce the fractional change of the Hubble parameter per e-fold ϵ, as

$$\frac{d}{dt}(RH)^{-1} = -\frac{\dot{R}H + R\dot{H}}{(RH)^2} = -\frac{1}{R}(1-\epsilon). \tag{1.8}$$

where the last term is defined as follows

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{d\ln R} = -\frac{d\ln H}{dN} < 1. \tag{1.9}$$

where $dN \equiv d\ln R = Hdt$ defines de number of e-folds of the inflationary expansion. This represents that the fractional change of the Hubble parameter per e-fold is small, so the last term tell us that if ϵ is small, then inflation happens. The case $\epsilon = 0$ describes a de-Sitter space (H=constant). We want inflation to last for a sufficiently long time, then we introduce

$$\eta \equiv \frac{d\ln\epsilon}{dN} = \frac{\dot{\epsilon}}{H\epsilon} \tag{1.10}$$

therefore if ϵ needs to remain small for a sufficiently large number of Hubble times, then η should be a small quantity (we'll show that later) (jav: η here or better use conformal time as τ?)

$$|\eta| \ll 1. \tag{1.11}$$

Accelerated expansion:

From the acceleration equation, we can write the condition for inflation in terms of the required material to drive the expansion:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}\left(1 + \frac{p}{\rho}\right) < 1 \quad \rightarrow \quad w = \frac{p}{\rho} < -\frac{1}{3}. \tag{1.12}$$

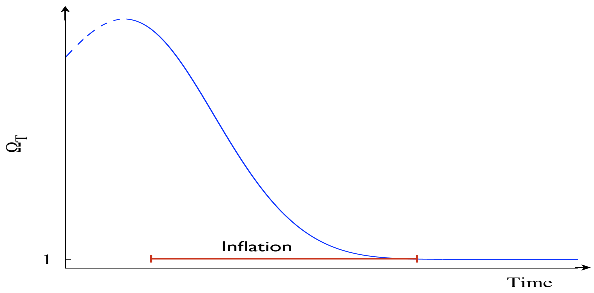

Figure 1.4: Evolution of the density parameter Ω_T, during the inflationary period. Ω_T is driven towards unity.

If this brief period of accelerated expansion occurred, then it is possible that the aforementioned problems of the Big Bang can be solved. However, because in standard physics it is commonly assumed ρ as positive, then to satisfy the acceleration condition it is necessary for the overall pressure to have $p < -\rho/3$.

Nonetheless, neither a radiation nor a matter dominated epoch satisfies such condition. A typical solution would be a universe dominated by a cosmological constant Λ at the earliest stages. As we have shown in Table **??**, a cosmological constant leads to an exponential expansion, a *de Sitter stage*, and hence the condition (1.4) would be naturally fulfilled. Let us postpone for a bit the problem of finding a component which may satisfy this inflationary condition, and look what happens when a general solution is considered.

Flatness solution

If somehow there was an accelerated expansion, $1/(RH)$ tends to decrease with time, and hence from the expression (1.2), Ω_T is driven towards the unity rather than away from it. In this sense, inflation magnifies the curvature radius of the universe, so locally the universe seems to be flat with great precision, as shown in Figure 1.4. Then, we may ask ourselves by how much should $1/(RH)$ decrease. If the inflationary period started at time $t = t_i$ and ended approximately at the beginning of the radiation dominated era ($t = t_f$), then

$$\mid \Omega_T(10^{-34}\text{sec}) - 1 \mid_{t=t_f} \sim 10^{-54},$$

and

$$\frac{\mid \Omega_T - 1 \mid_{t=t_f}}{\mid \Omega_T - 1 \mid_{t=t_i}} = \left(\frac{R_i}{R_f}\right)^2 \equiv e^{-2N}. \tag{1.13}$$

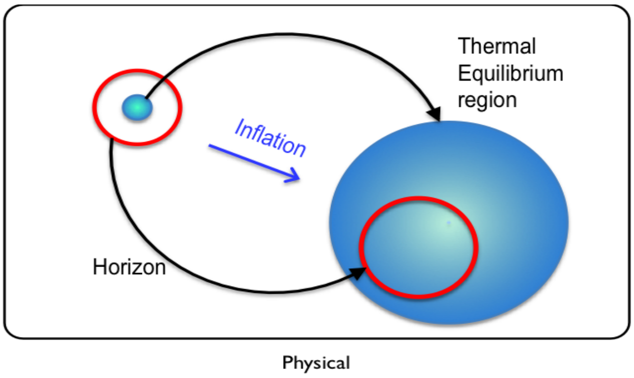

Figure 1.5: Physical evolution of the observable universe during the inflationary period.

So, the required condition to reproduce the value of $\Omega_{T,0}$ today is that inflation lasted for at least $N \equiv \ln R \gtrsim 50$, then Ω_T will be extraordinarily close to one that we still observe it today.

Horizon problem

During inflation the universe expanded drastically and there was a reduction in the comoving Hubble length. That is, a tiny region located inside the Hubble radius evolved and constituted our present observable universe, as seen in Figure 1.5, which represents the physical process of Figure 1.3. Scales that were outside the horizon at CMB decoupling were in fact inside the horizon before inflation. The region of space corresponding to the observable universe therefore was in thermal equilibrium before inflation and the uniformity of the CMB is essentially explained.

Monopole problem

The monopole problem is partially solved by noticing that during the inflationary epoch the universe led to a dramatic expansion over which the density of the unwanted particles were diluted away. Generating enough expansion, the dilution made sure that particles stayed completely out of our observable universe, making pretty difficult to localise any single monopole.

1.0.3 Single-field Inflation

As we have pointed out, a period of accelerated expansion can be created by a cosmological constant Λ, and hence solve the aforementioned problems. After a brief period of time, however, inflation must end and its energy be converted into conventional matter/radiation; this process is called *reheating*. In a universe dominated by a cosmological constant, the reheating process is seen as Λ decaying into conventional particles. Nevertheless, claiming that Λ is able to decay is still a naive way to face the problem. On the other hand, scalar fields (spin-0 particles) can behave like a *dynamical cosmological constant*. There currently exists a broad diversity of models suggested to give rise the Inflationary period, see for instance [14, 15, 17]. Here, we limit ourselves to single scalar-field models based on general gravity, i.e. derived from the Einstein-Hilbert action. (jav: later present BD models, Higgs and so on) (jav: scalar field models, inflaton, curvaton, quintessence).

Let us consider a scalar field minimally coupled to gravity, with an arbitrary potential $V(\phi)$, specified by the action (jav: check Riotto for signature) (jav: Include the E-L equations for Scalar fields)

$$S = \int d^4x\, \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \right]. \tag{1.14}$$

From the action, the Euler-Lagrange equations with a FLRW universe ($\sqrt{-g} = R^3$) lead to the Klein-Gordon equation.

HW: obtener KG

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{R^2}\nabla^2\phi + V_{,\phi} = 0, \tag{1.15}$$

where the second term is referred as the friction due to the expansion.
The energy-momentum tensor corresponding to this scalar field is given by

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} \partial_\sigma \phi \partial^\sigma \phi + V(\phi) \right]. \tag{1.16}$$

By comparing (1.16) to the energy-momentum tensor of perfect fluids (**??**), one can identify an associated energy-density ρ_ϕ and pressure p_ϕ for the scalar-field. In a FRW background, they are found to be

$$T_{00} = \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2R^2}(\nabla\phi)^2, \tag{1.17}$$

$$T_{ii} = p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + \frac{1}{6R^2}(\nabla\phi)^2, \tag{1.18}$$

with its corresponding equation-of-state $p_\phi = w_\phi \rho_\phi$.

HW: Get ρ_ϕ and p_ϕ.

To provide a better understanding of the inflaton field, ϕ can be split up as

$$\phi(t, \mathbf{x}) = \phi(t) + \delta\phi(t, \mathbf{x}), \tag{1.19}$$

where $\phi(t)$ is considered a classical field, that is, the mean value of the inflaton field on the homogeneous and isotropic state; whereas $\delta\phi(t, \mathbf{x})$ describes the quantum fluctuations around $\phi(t)$ (we will see more about perturbations of the field $\delta\phi$ in Section **??**). The evolution equation for the background field ϕ is thus given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{1.20}$$

From the structure of the effective energy-density and pressure, the Friedmann and the acceleration equations for a homogeneous single-scalar field become

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \tag{1.21}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\dot{\phi}^2 - V(\phi)\right]. \tag{1.22}$$

with an analogous equation of state

$$w_\phi = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{1.23}$$

Therefore, the inflationary condition to be satisfied is $\dot{\phi}^2 \ll V(\phi)$, which is easily fulfilled with a suitable flat potential. Inflation is driven by the vacuum energy of the inflaton field $p_\phi \approx -\rho_\phi$.

(jav: If I use Friedmann and acceleration with p_ϕ and ρ_ϕ, combine them we get KG equation.)

1.0.4 Slow-Roll Inflation

HW: show that $\dot{H} = -4\pi G\dot{\phi}^2$.

By Substituting $\dot{H}$ into ϵ (Eqn. 1.9), we have

$$\epsilon = \frac{4\pi G\dot{\phi}^2}{H^2} = 3\left(\frac{\frac{\dot{\phi}^2}{2}}{\frac{\dot{\phi}^2}{2} + V(\phi)}\right) \tag{1.24}$$

Therefore, Inflation ($\epsilon \ll 1$) occurs when the kinetic energy $\frac{1}{2}\dot{\phi}^2$ makes a small contribution to the total energy $\dot{\phi}^2 \ll H^2 \sim \rho_\phi$, or equivalently $\dot{\phi}^2 \ll V(\phi)$.

Also, the acceleration of the scalar field has to be small or analogously the friction term in the KG equation is dominated by the cosmological expansion. By differentiating the above expression, $\dot{\phi}^2 \ll H^2$, we have

$$\begin{aligned} 2\dot{\phi}\ddot{\phi} &\ll 2H\dot{H} \sim -2H\dot{\phi}^2 \\ &\rightarrow \left|\frac{\ddot{\phi}}{H\dot{\phi}}\right| \ll 1. \end{aligned} \tag{1.25}$$

Then, we define the dimensionless acceleration per Hubble time.

$$\delta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}, \tag{1.26}$$

to get (1.10)

$$\begin{aligned} \dot{\epsilon} &= 8\pi G \left(\frac{\dot{\phi}\ddot{\phi}}{H^2} - \frac{\dot{\phi}^2 \dot{H}}{H^3} \right), \\ \rightarrow \eta &= \frac{\dot{\epsilon}}{H\epsilon} = 2\left(\frac{\ddot{\phi}}{H\dot{\phi}} - \frac{\dot{H}}{H^2} \right) = 2(\epsilon - \delta), \end{aligned} \tag{1.27}$$

therefore, the slow-roll parameters

$$\{\epsilon, |\delta|\} \ll 1 \qquad \text{imply} \qquad \{\epsilon, |\eta|\} \ll 1. \tag{1.28}$$

So far, no approximations have been made.

1.0.5 Slow-Roll approximation

Based on the single scalar-field approach, it is useful to suggest a model starting with a nearly flat potential, i.e. initially satisfies the condition $\dot{\phi}^2 \ll V(\phi)$ (and its derivative $\ddot{\phi} \ll V_{,\phi}$). In this case the field is slowly rolling down on its potential; such an approximation is called *slow-roll inflation* [9, 11]. The equations of motion (1.20) and (1.21), under the slow-roll approximation, then become

$$\begin{aligned} \epsilon \ll 1 \qquad &\rightarrow \qquad \frac{1}{2}\dot{\phi}^2 \ll V(\phi) \sim H^2, \\ \text{therefore} \qquad &H^2 \simeq \frac{8\pi G}{3} V(\phi). \end{aligned} \tag{1.29}$$

$$\begin{aligned} |\delta| \ll 1 \qquad &\rightarrow \qquad \left|\frac{\ddot{\phi}}{H\dot{\phi}}\right| \ll 1 \\ \text{therefore} \qquad &3H\dot{\phi} \simeq -\frac{dV}{d\phi}, \end{aligned} \tag{1.30}$$

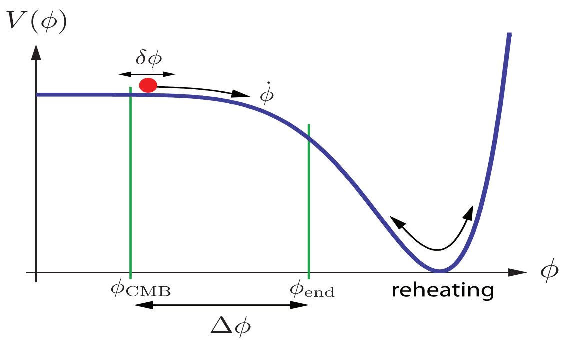

Figure 1.6: Schematic Inflationary process followed by a reheating epoch. Figure reprinted from [2].

The inflationary process can be summarised as an accelerated universe which takes place when the kinetic part of the inflaton field is subdominant over the potential $V(\phi)$ term. Then, when both quantities become comparable inflation ends giving rise to the *reheating process*. Figure 1.6 displays the schematic behaviour of the inflationary process. Also, the slow-roll approximation is consistent if the slope and curvature of the potential are small: $V_{,\phi}, V_{,\phi\phi} \ll V$. Thus, it is now useful to introduce the potential slow-roll parameters ϵ_{v} and η_{v} in the following way [9]:

Combining the above equations, (1.24 and 1.29), we have

$$\epsilon = \frac{4\pi G \dot{\phi}^2}{H^2} \simeq \frac{1}{16\pi G}\left(\frac{V_{,\phi}}{V}\right)^2 \equiv \epsilon_{\mathrm{v}}(\phi), \tag{1.31}$$

and combining equations (1.9) and (1.26), with the differential of (1.30), it yields to

$$\epsilon + \delta = -\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{H\dot{\phi}} \simeq \frac{1}{8\pi G}\frac{V_{,\phi\phi}}{V} \equiv \eta_{\mathrm{v}}(\phi),, \tag{1.32}$$

which define the *potential slow-roll parameters*; where ϵ_{v} measures the slope of the potential and η_{v} its curvature

$$\epsilon_{\mathrm{v}}(\phi) \equiv \frac{1}{16\pi G}\left(\frac{V_{,\phi}}{V}\right)^2, \qquad |\eta_{\mathrm{v}}(\phi)| \equiv \frac{1}{8\pi G}\frac{|V_{,\phi\phi}|}{V}. \tag{1.33}$$

Equations (1.29) and (1.30) are in agreement with the slow-roll approximation when the following conditions hold

$$\epsilon_{\mathrm{v}}(\phi) \ll 1, \quad \mid \eta_{\mathrm{v}}(\phi) \mid \ll 1. \tag{1.34}$$

However, these conditions are necessary but not sufficient since even if the potential is flat, it may happen that the scalar field has a large velocity. Hence, one should also consider that the condition $\dot{\phi}^2 \ll V(\phi)$ holds. Notice that ϵ and η are often called the *Hubble slow-roll parameters*, and during the slow-roll approximation these are related by

$$\epsilon_{\rm v} \simeq \epsilon, \qquad \eta_{\rm v} \simeq 2\epsilon - \frac{1}{2}\eta. \tag{1.35}$$

Within these approximations, it is straightforward to find out the scale factor R between the beginning (t_i) and end (t_e) of inflation, defined by $\epsilon(t_i) = \epsilon(t_e) \equiv 1$. Then, the e-fold number is

$$N_{tot} \equiv \int_{R_i}^{R_e} d\ln a = \int_{t_i}^{t_e} H(t)dt, \tag{1.36}$$

in slow-roll

$$Hdt = \frac{H}{\dot{\phi}}d\phi \simeq -3\frac{H^2}{V_{,\phi}}d\phi \simeq -8\pi G\frac{V}{V_{,\phi}}d\phi = \sqrt{8\pi G}\frac{d\phi}{\sqrt{2\epsilon_{\rm v}}},$$

and therefore

$$N_{tot} = \sqrt{8\pi G}\int_{\phi_i}^{\phi_e} \frac{d\phi}{\sqrt{2\epsilon_{\rm v}}}. \tag{1.37}$$

An estimate of the e-folds number $N(k)$ is given by [10]:

$$N(k) = 62 - \ln\frac{k}{a_0 H_0} + \text{corrections},$$

where the comoving wavenumber k is evaluated at the crossing Hubble radius during inflation. The last 'corrections' is a small term related with energy scales during the inflationary process. The precise value for the second quantity depends on the model as well as normalisation factors, however it does not present any significant change to the total amount of e-folds. Therefore, the value of the e-folds number is ranged to 50-70 [15].

The potential that describes a massive scalar field is given by:

$$V(\phi) = \frac{1}{2}m^2\phi^2. \tag{1.38}$$

Considering the slow-roll approximation, equations (1.29) and (1.30) become:

$$\begin{aligned} H^2 &\simeq \frac{4\pi G}{3}m^2\phi^2, \\ 3H\dot{\phi} &\simeq -m^2\phi. \end{aligned}$$

Thus, the dynamics of this type of model is described by

$$\phi(t) = \phi_i - \frac{m}{\sqrt{12\pi G}}\, t, \tag{1.39}$$
$$R(t) = R_i \exp\left[m\sqrt{\frac{4\pi G}{3}}\left(\phi_i t - \frac{m}{\sqrt{48\pi G}} t^2\right)\right],$$

where ϕ_i and R_i represent the initial conditions at a given initial time $t = t_i$. The slow-roll parameters for this particular potential are computed from equations (1.33)

$$\epsilon_{\rm v} = \eta_{\rm v} = \frac{1}{4\pi G}\frac{1}{\phi^2}, \tag{1.40}$$

that is, an inflationary epoch takes place whilst the condition $|\phi| > 1/\sqrt{4\pi G} = \sqrt{2}M_{pl} \equiv \phi_e$[1] is satisfied, and the total amount that lapses during this accelerated period is encoded on the e-folds number

$$N = 2\pi G\left[\phi^2 - \phi_e^2\right] = 2\pi G\phi^2 - \frac{1}{2}. \tag{1.41}$$

The field value N e-folds before the end of inflation, to the scales relevant for the CMB is

$$\phi_{60} \sim 15 M_{pl}. \tag{1.42}$$

The steps shown before might, in principle, apply to any inflationary single-field model. That is, the general information we need to characterised cosmological inflation is specified by its potential.

[1] where $M_{pl}^2 = \frac{1}{8\pi G}$.

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