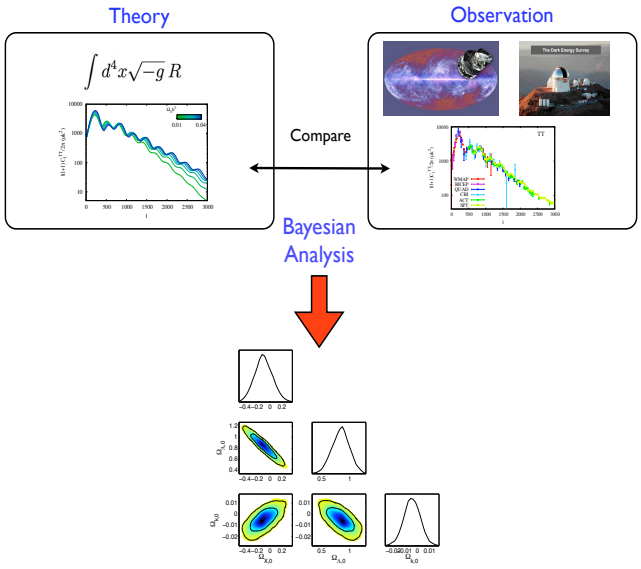
Updated Cosmology

with Python



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In progress

August 12, 2017

Homework 9

1.- Show that a flat universe is an *unstable* fixed point if the strong energy condition is satisfied. Hint: Show that the density parameter evolves with the scale factor R as:

$$\frac{d\Omega}{d\ln R} = (1+3w)\Omega(\Omega-1) \tag{1}$$

2.- Probe the equivalence amongst the relations

$$\begin{aligned} \text{INFLATION} &\iff \frac{d}{dt}\left(\frac{1}{RH}\right) < 0, && (2)\\ &\iff \ddot{R} > 0, && (3)\\ &\iff \rho + 3p < 0, && (4)\\ &\iff \epsilon \equiv -\frac{\dot{H}}{H^2} < 1. && (5) \end{aligned}$$

3.- From the scalar-field action, the Euler-Lagrange equations with a FLRW universe ($\sqrt{-g} = R^3$) lead to the Klein-Gordon equation, show this is:

$$\ddot{\phi} + 3H\dot{\phi} - \nabla^2\phi + V_{,\phi} = 0, \tag{6}$$

4.- Get ρ_ϕ and p_ϕ, from

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\partial_\sigma\phi\partial^\sigma\phi - V(\phi)\right]. \tag{7}$$

5.- Show that $\dot{H} = -4\pi G\dot{\phi}^2$.

6.- The potential that describes a massive scalar field is $V(\phi) = \frac{1}{2}m^2\phi^2$. Show that the dynamics of this type of model, in the slow-roll approximation, is described by

$$\begin{aligned} \phi(t) &= \phi_i - \frac{m}{\sqrt{12\pi G}}\,t, && (8)\\ R(t) &= R_i \exp\left[m\sqrt{\frac{4\pi G}{3}}\left(\phi_i t - \frac{m}{\sqrt{48\pi G}}t^2\right)\right], \end{aligned}$$

where ϕ_i and R_i represent the initial conditions at a given initial time $t = t_i$.