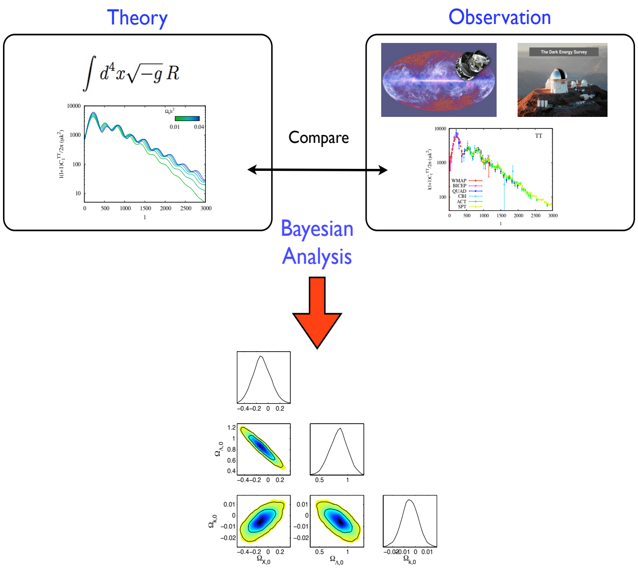
Updated Cosmology

with Python



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In progress

August 12, 2017

Homework 7

1.- Compute the following integral

$$t = \frac{1}{H_0}\int_0^a \left[\frac{x}{\sqrt{\Omega_{r,0} + (1-\Omega_{r,0})x^2}}\right] dx, \tag{77}$$

for $\Omega_{r,0} < 1$ $(k=-1)$ & $\Omega_{r,0} > 1$ $(k=1)$, to get

$$a(t) = (2H_0\Omega_{r,0}^{1/2}t)^{1/2}\left(1 + \frac{1-\Omega_{r,0}}{2\Omega_{r,0}^{1/2}}H_0 t\right)^{1/2}. \tag{78}$$

2.- Compute

$$t = \frac{1}{H_0}\int_0^a \left[\frac{x}{\sqrt{\Omega_{m,0}x + \Omega_{r,0}}}\right] dx, \tag{79}$$

to get

$$H_0 t = \frac{2}{3\Omega_{m,0}^2}\left[(\Omega_{m,0}a + \Omega_{r,0})^{1/2}(\Omega_{m,0}a - 2\Omega_{r,0}) + 2\Omega_{r,0}^{3/2}\right]. \tag{80}$$

3.- Compute

$$t = \frac{1}{H_0}\int_0^a \sqrt{\frac{x}{(1-\Omega_{\Lambda,0}) + \Omega_{\Lambda,0}x^3}}\, dx, \tag{81}$$

to get

$$H_0 t = \frac{2}{3\sqrt{|\Omega_{\Lambda,0}|}} f(x) = \begin{cases} \sinh^{-1}[\sqrt{a^3|\Omega_{\Lambda,0}|(1-\Omega_{\Lambda,0})}], & \Omega_{\Lambda,0} > 0. \\ . \\ \sin^{-1}[\sqrt{a^3|\Omega_{\Lambda,0}|(1-\Omega_{\Lambda,0})}], & \Omega_{\Lambda,0} < 0. \end{cases} \tag{82}$$