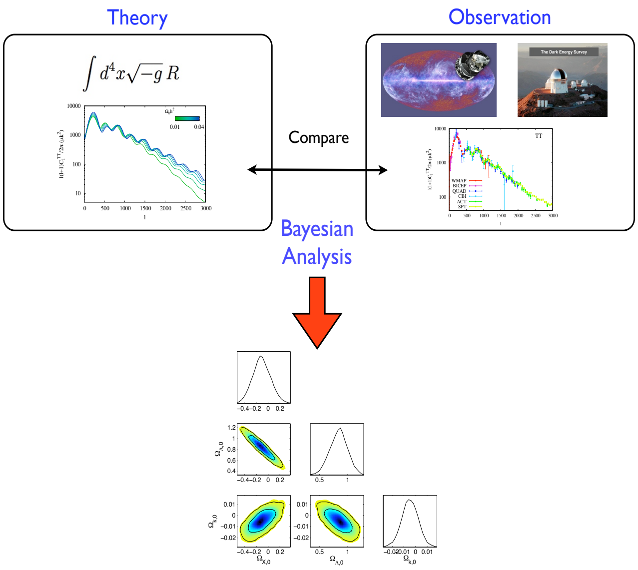
# Updated Cosmology

## with Python

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# Homework 6

**HW**: Compute $R$ and $G$

Hint:

$$R = 6\left[\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{c^2 k}{R^2}\right]$$

$$G^0_0 = -3\left[\left(\frac{\dot{R}}{R}\right)^2 + \frac{c^2 k}{R^2}\right], \tag{60}$$

$$G^i_j = -\left[2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{c^2 k}{R^2}\right]\delta^i_j. \tag{61}$$

**HW**: Compute the Kretschmann scalar for a FRW spacetime.

3.- We start by assuming the components of the Universe behave as perfect fluids and hence described by a barotropic equation of state $p_i = (\gamma_i - 1)\rho_i c^2$, where $\gamma_i$ describes each fluid: radiation ($\gamma_r = 4/3$), baryonic and dark matter ($\gamma_m = 1$), and dark energy in the form of cosmological constant ($\gamma_\Lambda = 0$). Once we introduce the dimensionless *density parameters*, defined as

$$\Omega_i = \frac{\kappa_0}{3H^2}\rho_i, \tag{62}$$

a) Show that the continuity equations (for all the fluids $i$) can be written as a dynamical system

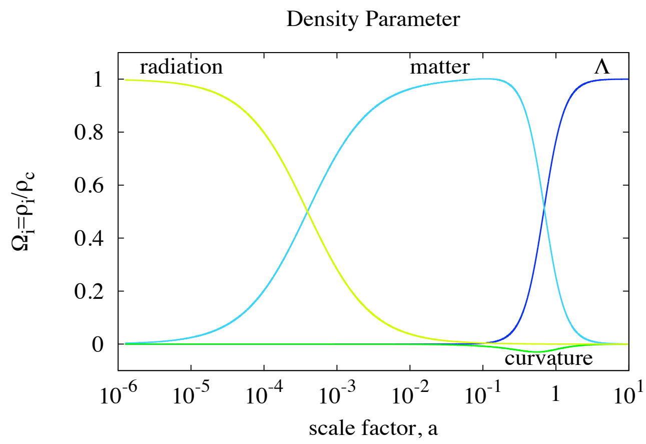


**Figure 6:** The evolution of the density parameters $\Omega_i(a)$.

with the following form:

$$\Omega_i' = 3(\Pi - \gamma_i)\Omega_i, \tag{63}$$

with $\Pi = \sum_i \gamma_i \Omega_i$, and prime notation means derivative with respect to the e-fold parameter $N = \ln(a)$.

b) Also, show that the Friedmann equation becomes a constraint for the density parameters at all time $\sum_i \Omega_i = 1$.

c) Considering the initial conditions ($a = 1$) $\Omega_{r,0} = 10^{-4}$, $\Omega_{m,0} = 0.3$, $\Omega_{k,0} = -0.01$, $H_0 = 68 \mathrm{kms}^{-1}\mathrm{Mpc}$, with cosmological constant, solve explicitly the dynamical system (63) [hint: use odeint], along with the Friedmann constraint to get the following plot [hint: use matplotlib].

d) The deceleration parameter is computed in terms of the contents of the universe, as $q = \frac{1}{2}\sum_i \Omega_i(1 + 3w_i)$. Use the solutions from above to plot $q(z)$, where $1 + z = 1/a$.

4) A step further to the standard model is to consider the dark energy being dynamic, where the evolution of its EoS is usually parameterised. A commonly used form of $w(z)$ is to take into account the next contribution of a Taylor expansion in terms of the scale factor $w(a) = w_0 + (1 - a)w_a$ or in terms of redshift $w(z) = w_0 + \frac{z}{1+z}w_a$; we refer to this model as CPL.

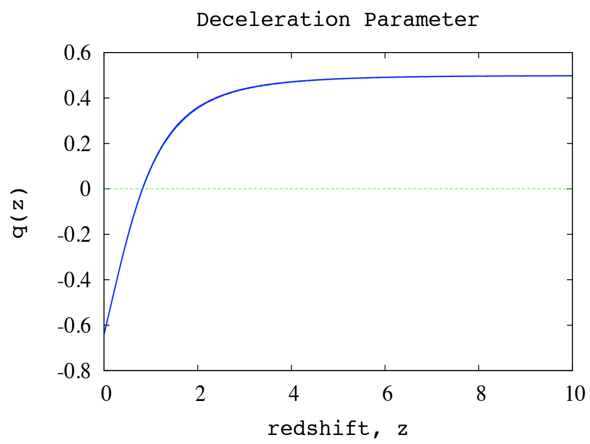


**Figure 7:** Deceleration parameter $q(z)$ as a function of redshift $z$ for a multi-fluid universe. Notice that the universe is currently accelerating ($q(z=0) < 0$).

The parameters $w_0$ and $w_a$ are real numbers such that at the present epoch $w|_{z=0} = w_0$ and $dw/dz|_{z=0} = -w_a$; we recover ΛCDM when $w_0 = -1$ and $w_a = 0$.

a) Show the Friedmann equation for the CPL parameterisation turns out to be:

$$\frac{H(z)^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + (1 - \Omega_{m,0} - \Omega_{k,0})(1+z)^{3(1+w_0+w_a)} e^{-\frac{3w_a z}{1+z}}.$$

b) Repeat the same process in a), but now use the equation of state $w(z) = w_0 + w_a ln(1+z)$