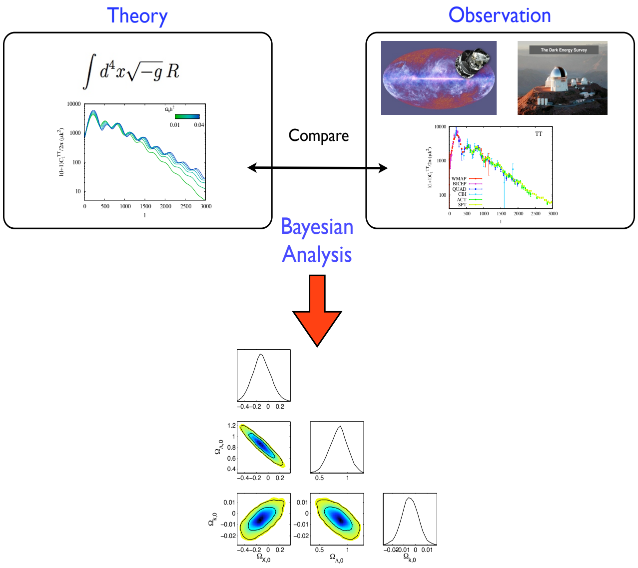
Updated Cosmology

with Python



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Homework 10

1.- The number density of particles (in real space), n, the energy density of a gas of particles and its pressure given respectively by ρ and P, are

$$n = \frac{g}{2\pi^2}\int_0^\infty dp \frac{p^2}{\exp[\sqrt{p^2+m^2/T}]\pm 1}, \quad (1.65)$$

$$\rho = \frac{g}{2\pi^2}\int_0^\infty dp \frac{p^2\sqrt{p^2+m^2}}{\exp[\sqrt{p^2+m^2/T}]\pm 1}, \quad (1.66)$$

$$P = \frac{g}{6\pi^2}\int_0^\infty dp \frac{p^4(p^2+m^2)^{-1/2}}{\exp[\sqrt{p^2+m^2/T}]\pm 1} \quad (1.67)$$

Fermi-Dirac (+) and Bose-Einstein (-) distributions a temperature T.

Show that in the relativistic limit ($T \gg m$), they become:

$$n = \frac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 & \text{bosons} \\ \frac{3}{4} & \text{fermions,} \end{cases} \quad (1.68)$$

$$\rho = \frac{\pi^2}{30} g T^4 \begin{cases} 1 & \text{bosons} \\ \frac{7}{8} & \text{fermions,} \end{cases} \quad (1.69)$$

$$P = \frac{\rho}{3}. \quad (1.70)$$

Some useful integrals:

$$\int_0^\infty d\xi \frac{\xi^n}{e^\xi - 1} = \zeta(n+1)\Gamma(n+1), \quad (1.71)$$

$$\int_0^\infty d\xi \xi^n e^{-\xi^2} = \frac{1}{2}\Gamma\left(\frac{1}{2}(n+1)\right), \quad (1.72)$$

with $\zeta(z)$ is the Riemann zeta-function, and

$$\frac{1}{e^\xi + 1} = \frac{1}{e^\xi - 1} - \frac{2}{e^{2\xi - 1}} \quad (1.73)$$

2.- Show that the net particle number for $T \gg m$ (an exact result) is

$$\begin{aligned} n - \bar{n} &= \frac{g}{2\pi^2} \int_0^\infty dp p^2 \left(\frac{1}{e^{(p-\mu)/T} + 1} - \frac{1}{e^{(p+\mu)/T} + 1} \right) \\ &= \frac{1}{6\pi^2} g T^3 \left[\pi^2 \left(\frac{\mu}{T} \right) + \left(\frac{\mu}{T^3} \right)^3 \right]. \end{aligned} \tag{1.74}$$

3.- Show that, in the non-relativistic limit ($m \gg T$)

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}. \tag{1.75}$$