Stats …

in Cosmology

J.A. Vázquez

ICF - UNAM

javazquez@icf.unam.mx

Jun 13th, 2019

$$P(\theta|D,H) = \frac{P(D|\theta,H)P(\theta|H)}{P(D|H)}.$$

LEVEL 1

I have selected a model M and prior P(**θ**|M)

Parameter inference

(assumes M is the true model)

$$P(\theta|d,M) = \frac{P(d|\theta,M)P(\theta|M)}{P(d|M)}$$

LEVEL 2

Actually, there are several possible models: M_0, M_1,...

Model comparison

What is the relative plausibility of M_0, M_1,... in light of the data?

$$\mathrm{odds} = \frac{\mathrm{P(M_0|d)}}{\mathrm{P(M_1|d)}}$$

LEVEL 3

None of the models is clearly the best

Model averaging

What is the inference on the parameters accounting for model uncertainty?

$$P(\theta|d) = \sum_i P(M_i|d)P(\theta|d,M_i)$$

Prerequisite

Ferozmente discutida desde entonces, **la teoría de Bayes** tuvo un papel decisivo en objetivos tan distintos como **descifrar los códigos alemanes** durante la Segunda Guerra Mundial, **combatir el cáncer** o contribuir al **desarrollo de los ordenadores**.

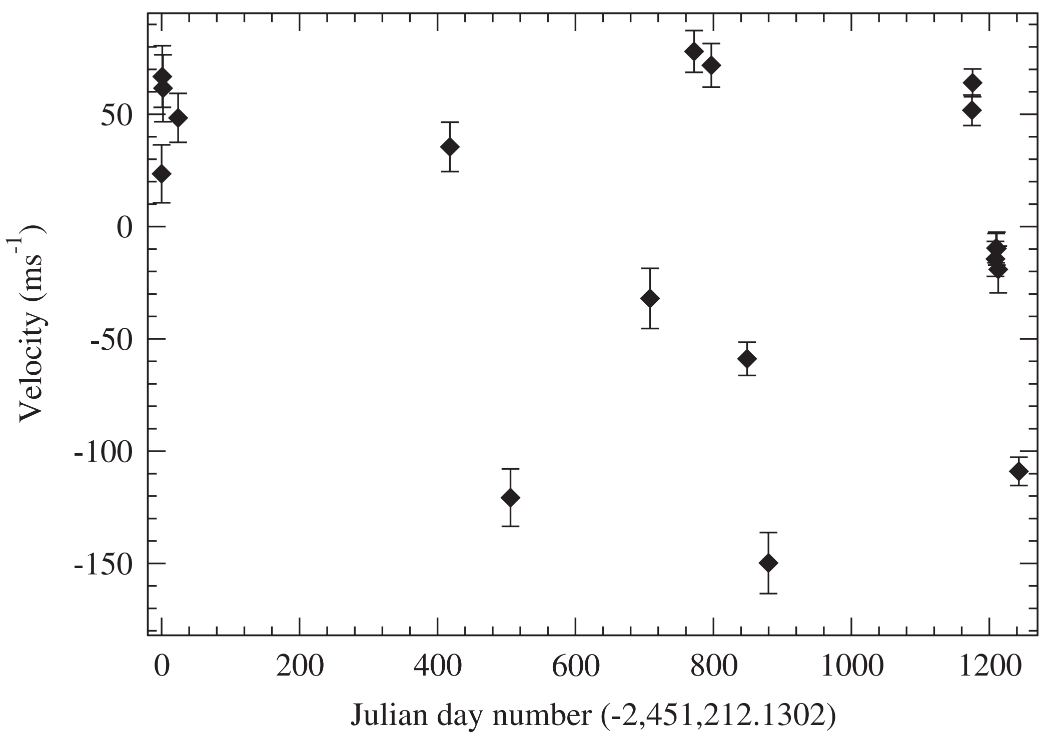
«UNA SIMPLE TEORÍA MATEMÁTICA DESCUBIERTA POR DOS CLÉRIGOS BRITÁNICOS EN EL SIGLO XVIII HA TOMADO POR ASALTO EL MUNDO MODERNO, DOMINADO POR LOS ORDENADORES»

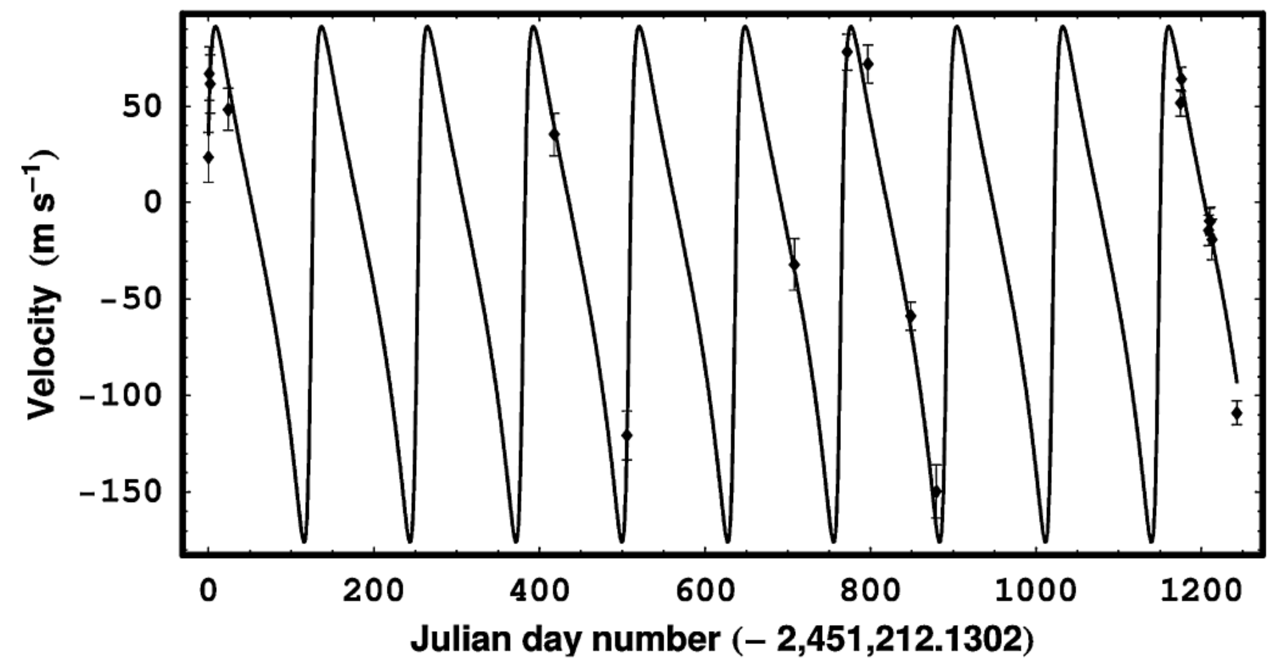
Motivation

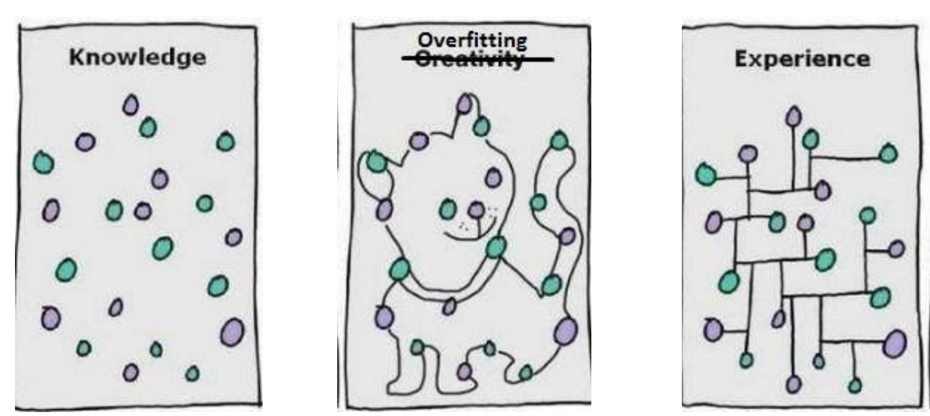
Bayesian inference in physics

Udo von Toussaint*

Radial velocity measurements of small velocity fluctuations in the movement of stars







Notas

An introduction to Markov Chain Monte Carlo

Ricardo Medel Esquivel, Isidro Gómez Vargas et. al.

A review of samplers for cosmological model comparison

Isidro Gómez Vargas,[1,a] Ricardo Medel Esquivel,[1,b] J. Alberto Vázquez,[2] and Ricardo García Salcedo[1]

Cosmological parameter inference with Bayesian statistics

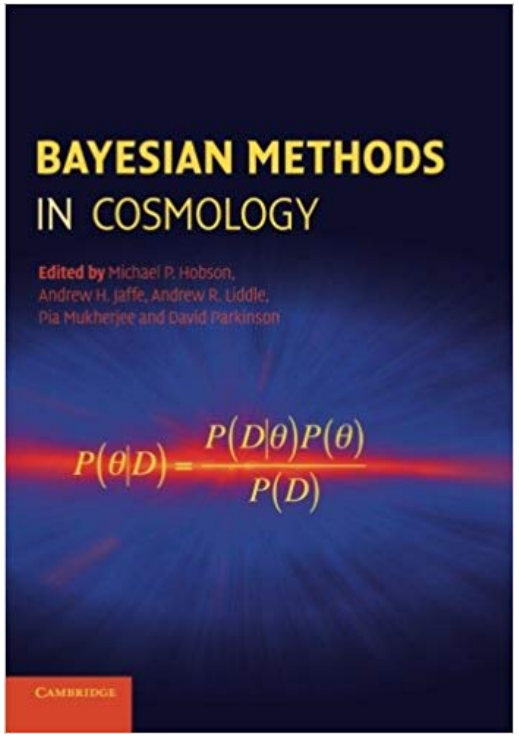
Luis Padilla-Albores,[1,2,*] Luis O. Tellez,[1] Luis A. Escamilla,[1] and J. Alberto Vazquez[3,1,†]

Una Aplicación de las Redes Neuronales Artificiales en la Cosmología

Isidro Gómez Vargas,[1,a] Ricardo Medel Esquivel,[1,b] Ricardo García Salcedo,[1] and J. Alberto Vázquez[2]

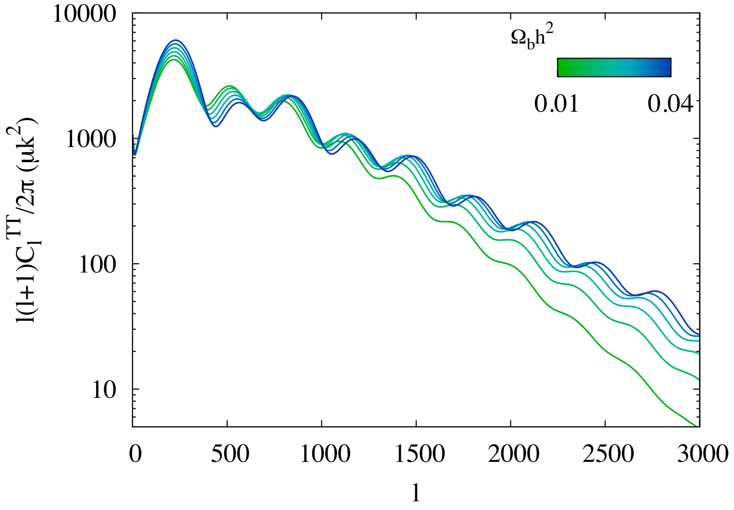
Artificial Neural Networks as optimizers of Bayesian evidence calculation

Isidro Gómez-Vargas[1a], Ricardo Medel Esquivel[1] Ricardo García Salcedo[1], and J. Alberto Vázquez[2]



Theory

$$\int d^4x\sqrt{-g}\,R$$

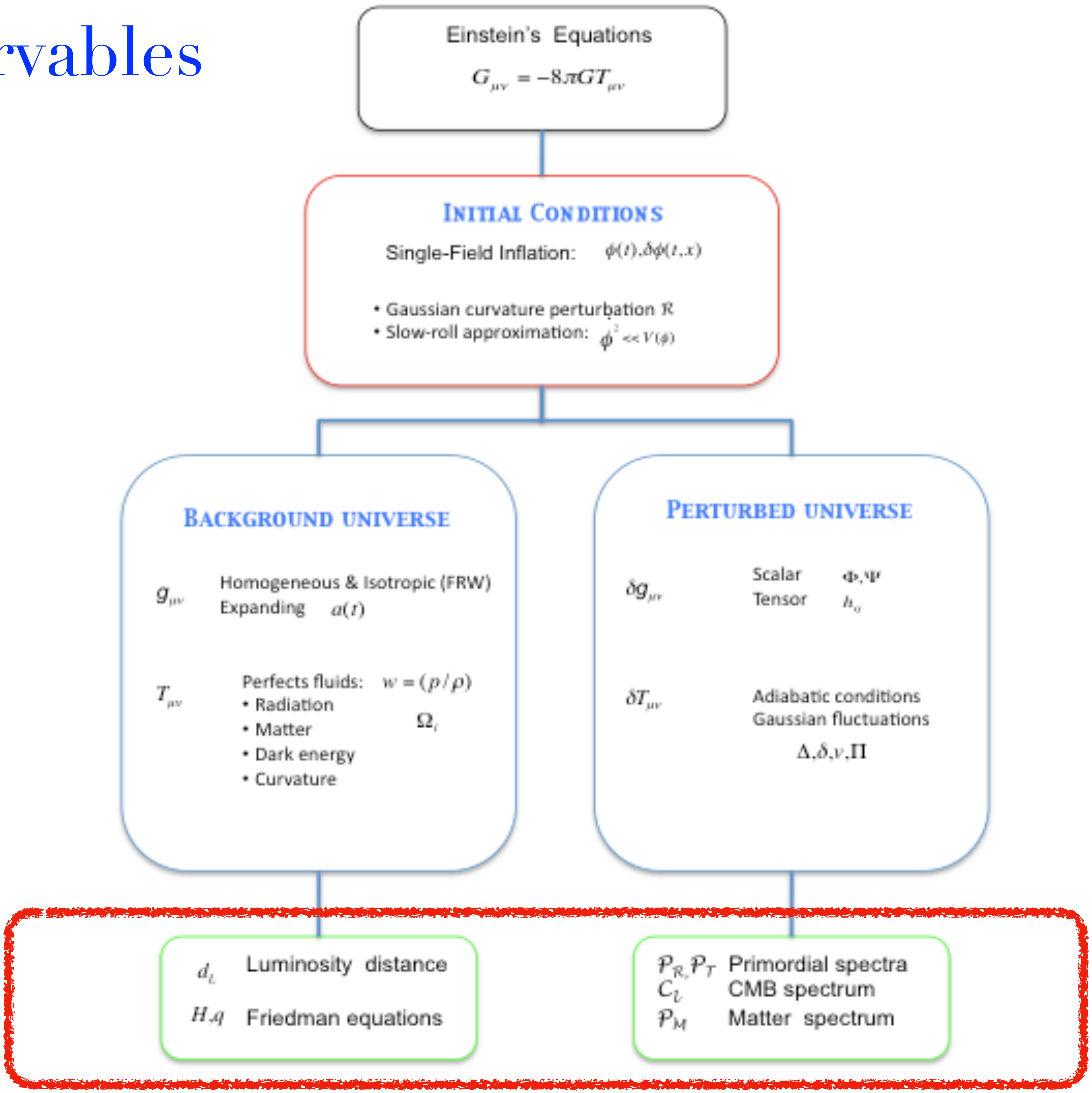


$P(\theta|H)$

Standard cosmological model

GR | $\int d^4x\sqrt{-g}R$ + FRW | ds^2 + Inflation | n_s, A_s + Baryons | Ω_b Radiation | Ω_r CDM | Ω_{dm} + Λ | $w = -1$ + ?

Observables



$\theta|$

COSMOLOGICAL PARAMETERS

Base Parameters

Derived parameters

Nuisance parameters

Candidate parameters

Base Parameters

(standard parameters)

Considered as the **principal quantities** used **describe the universe.**

They are **not predicted** by any fundamental theory, rather we have to fit them by hand in order to determine which combination best **describes current observations**

Background parameters

$$H^2 = H_0^2 \left[\left(\Omega_{\gamma,0} + \Omega_{\nu,0}\right) a^{-4} + \left(\Omega_{\rm b,0} + \Omega_{\rm dm,0}\right) a^{-3} + \Omega_{k,0} a^{-2} + \Omega_{X,0} a^{-1} + \Omega_{\Lambda,0} \right],$$

Inflationary parameters

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s} \left(\frac{k}{k_0} \right)^{n_{\rm s}-1},$$

Astrophysical parameter

$$\tau = \sigma_{\rm T} \int_{t_r}^{t_0} n_e(t) dt,$$

Base Parameters

θ

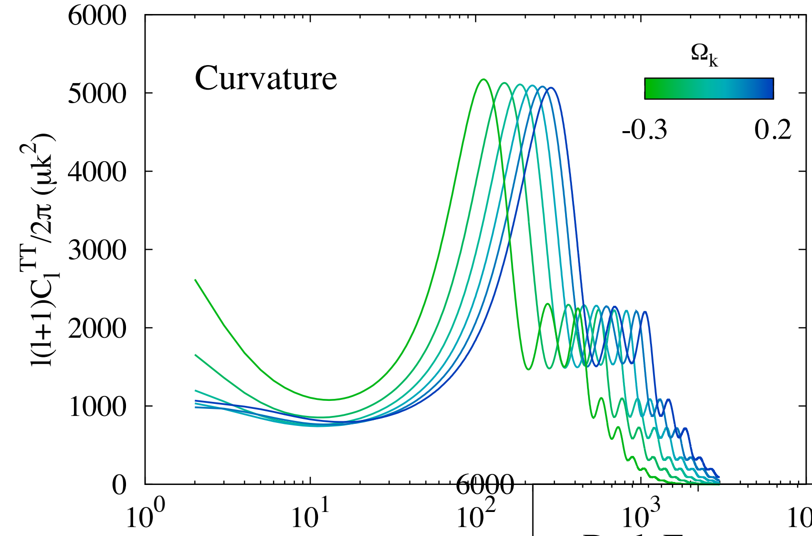
Parameter	Prior range	Baseline	Definition
$\omega_b \equiv \Omega_b h^2$	$[0.005, 0.1]$	. . .	Baryon density today
$\omega_c \equiv \Omega_c h^2$	$[0.001, 0.99]$	. . .	Cold dark matter density today
$100\theta_{MC}$	$[0.5, 10.0]$	. . .	$100 \times$ approximation to r_*/D_A (CosmoMC)
τ	$[0.01, 0.8]$	. . .	Thomson scattering optical depth due to reionization
Ω_K	$[-0.3, 0.3]$	0	Curvature parameter today with $\Omega_{tot} = 1 - \Omega_K$
$\sum m_\nu$	$[0, 5]$	0.06	The sum of neutrino masses in eV
$m^{\rm eff}_{\nu,\,\rm sterile}$	$[0, 3]$	0	Effective mass of sterile neutrino in eV
w_0	$[-3.0, -0.3]$	-1	Dark energy equation of state[a], $w(a) = w_0 + (1 - a)w_a$
w_a	$[-2, 2]$	0	As above (perturbations modelled using PPF)
$N_{\rm eff}$	$[0.05, 10.0]$	3.046	Effective number of neutrino-like relativistic degrees of freedom (see text)
Y_P	$[0.1, 0.5]$	BBN	Fraction of baryonic mass in helium
A_L	$[0, 10]$	1	Amplitude of the lensing power relative to the physical value
n_s	$[0.9, 1.1]$	. . .	Scalar spectrum power-law index ($k_0 = 0.05 \rm Mpc^{-1}$)
n_t	$n_t = -r_{0.05}/8$	Inflation	Tensor spectrum power-law index ($k_0 = 0.05 \rm Mpc^{-1}$)
$dn_s/d\ln k$	$[-1, 1]$	0	Running of the spectral index
$\ln(10^{10} A_s)$	$[2.7, 4.0]$	. . .	Log power of the primordial curvature perturbations ($k_0 = 0.05\,\rm Mpc^{-1}$)
$r_{0.05}$	$[0, 2]$	0	Ratio of tensor primordial power to curvature power at $k_0 = 0.05\,\rm Mpc^{-1}$

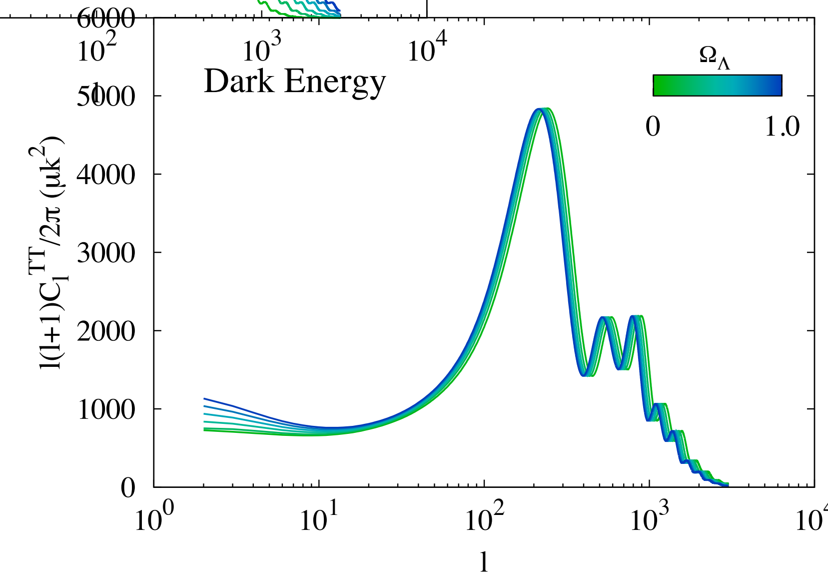
PARAMETERS

$\theta|$

$$d_{\rm A}(z) \approx 2\frac{c}{H_0}\,\frac{1}{\Omega_{{\rm m},0}\, z}\,.$$

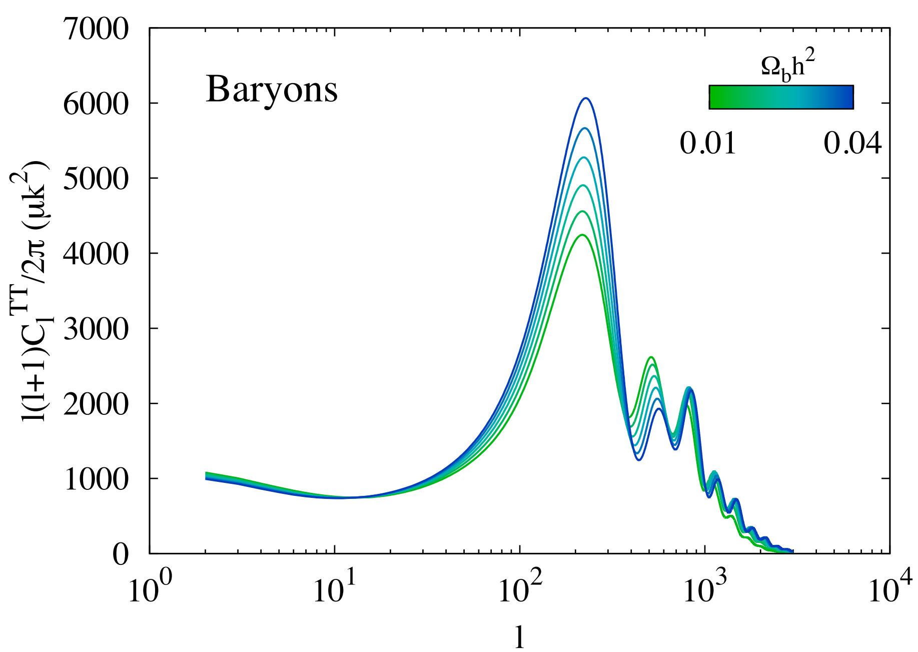
$$\theta_{\rm hor,s} \simeq \frac{1}{\sqrt{3}}\left(\frac{(1-\Omega_{{\rm k},0})}{z_{\rm dec}}\right)^{1/2} = 0.017\,{\rm radians} \sim 1^\circ$$





Both parameters principally affect the anisotropies **through dA** and so simply shift the peaks.

SN Ia



The **increase in baryon** inertia **reduces the sound speed**, **shifting the acoustic peaks** in temperature and polarization to smaller scales (larger l).

Inflation

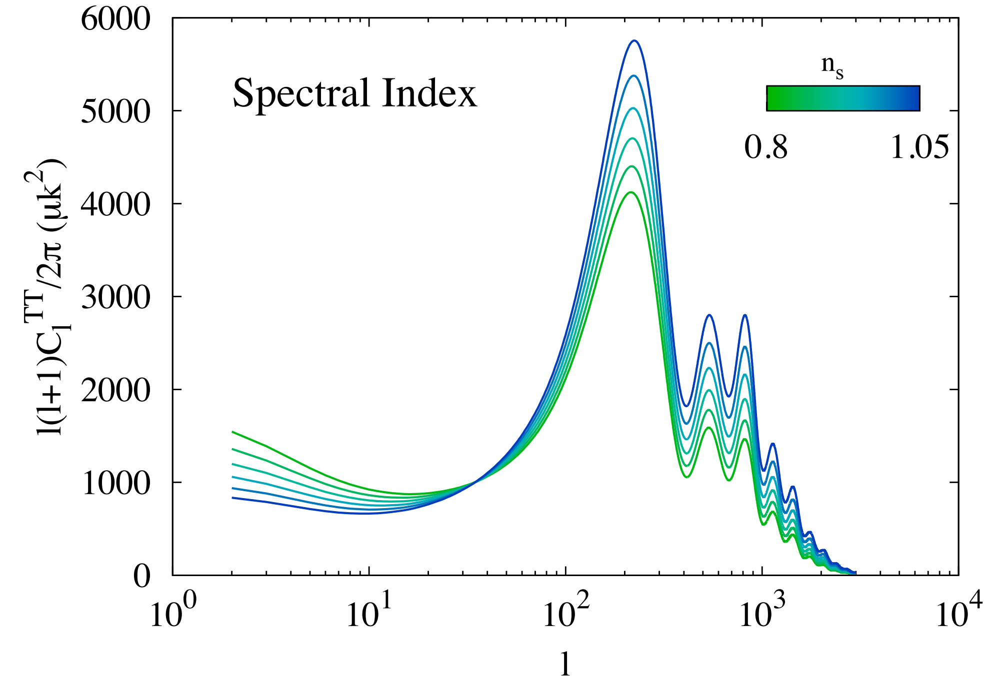
$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}-1},$$

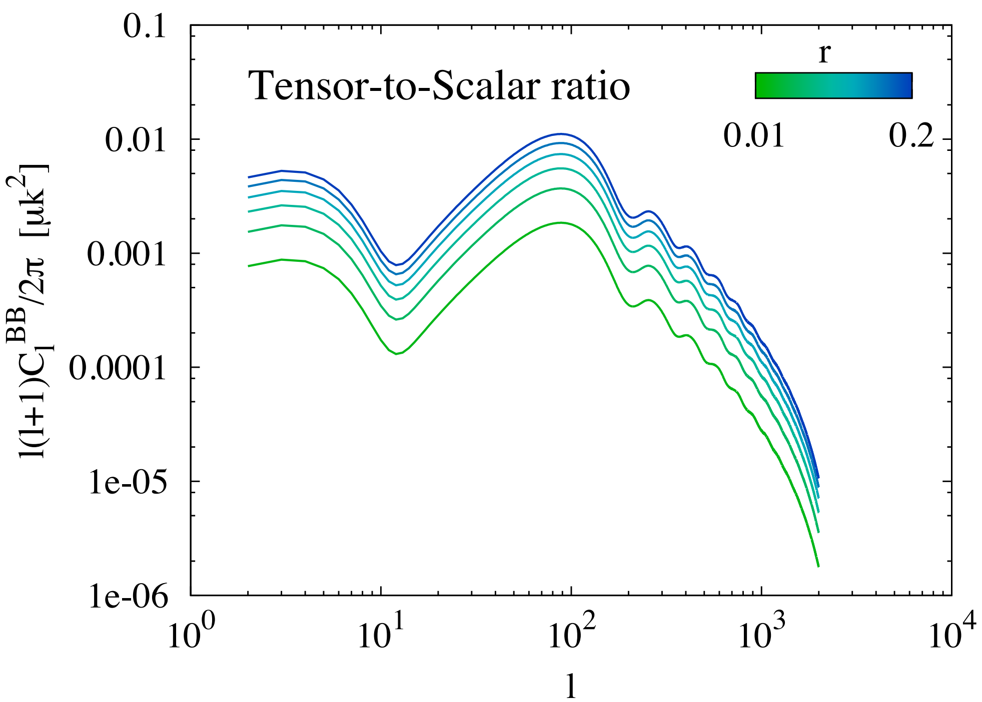
$$\mathcal{P}_{\mathcal{T}}(k) = A_{\rm t}\left(\frac{k}{k_0}\right)^{n_{\rm t}}.$$

$$n_{\rm s}-1 \simeq -6\,\epsilon_{\rm v}(\phi)+2\,\eta_{\rm v}(\phi),$$

$$n_{\rm t} \simeq -2\,\epsilon_{\rm v}(\phi),$$

$$r \simeq 16\,\epsilon_{\rm v}(\phi).$$





Nuisance parameters

$\theta|$

We do not have particular interest on these type of parameters, however **they may influence the rest** of the parameter-space constraints.

May be **related** to **insufficiently constrained** aspects of physics, or **uncertainties** in the measuring process

* the stretch α and colour β corrections on measurements of distance modulus of SNe Type Ia

$$\mu_i^{\rm obs} = \hat{m}^*_{B,i} - M + \alpha \hat{x}_{1,i} - \beta \hat{c}_i,$$

* bias factor in galaxy surveys b $P_{\rm gal}(k) = b_0^2 P_{\rm lin}(k) \frac{1 + Qk^2}{1 + Ak}.$

* calibrations and beans uncertainties, galactic foregrounds

Nuisance parameters

$\theta|$

Parameter	Prior range	Definition
$A^{\rm PS}_{100}$	$[0, 360]$	Contribution of Poisson point-source power to $\mathcal{D}^{100\times100}_{3000}$ for *Planck* (in μK^2)
$A^{\rm PS}_{143}$	$[0, 270]$	As for $A^{\rm PS}_{100}$, but at 143 GHz
$A^{\rm PS}_{217}$	$[0, 450]$	As for $A^{\rm PS}_{100}$, but at 217 GHz
$r^{\rm PS}_{143\times217}$	$[0, 1]$	Point-source correlation coefficient for *Planck* between 143 and 217 GHz
$A^{\rm CIB}_{143}$	$[0, 20]$	Contribution of CIB power to $\mathcal{D}^{143\times143}_{3000}$ at the *Planck* CMB frequency for 143 GHz (in μK^2)
$A^{\rm CIB}_{217}$	$[0, 80]$	As for $A^{\rm CIB}_{143}$, but for 217 GHz
$r^{\rm CIB}_{143\times217}$	$[0, 1]$	CIB correlation coefficient between 143 and 217 GHz
$\gamma^{\rm CIB}$	$[-2, 2]\ (0.7 \pm 0.2)$	Spectral index of the CIB angular power ($\mathcal{D}_\ell \propto \ell^{\gamma^{\rm CIB}}$)
$A^{\rm tSZ}$	$[0, 10]$	Contribution of tSZ to $\mathcal{D}^{143\times143}_{3000}$ at 143 GHz (in μK^2)
$A^{\rm kSZ}$	$[0, 10]$	Contribution of kSZ to $\mathcal{D}_{3000}$ (in μK^2)
$\xi^{\rm tSZ\times CIB}$	$[0, 1]$	Correlation coefficient between the CIB and tSZ (see text)
c_{100}	$[0.98, 1.02]\ (1.0006 \pm 0.0004)$	Relative power spectrum calibration for *Planck* between 100 GHz and 143 GHz
c_{217}	$[0.95, 1.05]\ (0.9966 \pm 0.0015)$	Relative power spectrum calibration for *Planck* between 217 GHz and 143 GHz
β^i_j	(0 ± 1)	Amplitude of the jth beam eigenmode ($j = 1$–5) for the ith cross-spectrum ($i = 1$–4)
$A^{\rm PS,\,ACT}_{148}$	$[0, 30]$	Contribution of Poisson point-source power to $\mathcal{D}^{148\times148}_{3000}$ for ACT (in μK^2)
$A^{\rm PS,\,ACT}_{218}$	$[0, 200]$	As for $A^{\rm PS,\,ACT}_{148}$, but at 218 GHz
$r^{\rm PS}_{150\times220}$	$[0, 1]$	Point-source correlation coefficient between 150 and 220 GHz (for ACT and SPT)
$A^{\rm ACTe}_{\rm dust}$	$[0, 5]\ (0.8 \pm 0.2)$	Contribution from Galactic cirrus to $\mathcal{D}_{3000}$ at 150 GHz for ACTe (in μK^2)
$A^{\rm ACTs}_{\rm dust}$	$[0, 5]\ (0.4 \pm 0.2)$	As $A^{\rm ACTe}_{\rm dust}$, but for ACTs
$y^{\rm ACTe}_{148}$	$[0.8, 1.3]$	Map-level calibration of ACTe at 148 GHz relative to *Planck* 143 GHz
$y^{\rm ACTe}_{217}$	$[0.8, 1.3]$	As $y^{\rm ACTe}_{148}$, but at 217 GHz
$y^{\rm ACTs}_{148}$	$[0.8, 1.3]$	Map-level calibration of ACTs at 148 GHz relative to *Planck* 143 GHz
$y^{\rm ACTs}_{217}$	$[0.8, 1.3]$	As $y^{\rm ACTs}_{148}$, but at 217 GHz
$A^{\rm PS,\,SPT}_{95}$	$[0, 30]$	Contribution of Poisson point-source power to $\mathcal{D}^{95\times95}_{3000}$ for SPT (in μK^2)
$A^{\rm PS,\,SPT}_{150}$	$[0, 30]$	As for $A^{\rm PS,\,SPT}_{95}$, but at 150 GHz
$A^{\rm PS,\,SPT}_{220}$	$[0, 200]$	As for $A^{\rm PS,\,SPT}_{95}$, but at 220 GHz
$r^{\rm PS}_{95\times150}$	$[0, 1]$	Point-source correlation coefficient between 95 and 150 GHz for SPT
$r^{\rm PS}_{95\times220}$	$[0, 1]$	As $r^{\rm PS}_{95\times150}$, but between 95 and 220 GHz
$y^{\rm SPT}_{95}$	$[0.8, 1.3]$	Map-level calibration of SPT at 95 GHz relative to *Planck* 143 GHz
$y^{\rm SPT}_{150}$	$[0.8, 1.3]$	As for $y^{\rm SPT}_{95}$, but at 150 GHz
$y^{\rm SPT}_{220}$	$[0.8, 1.3]$	As for $y^{\rm SPT}_{95}$, but at 220 GHz

$\theta|$

Candidate Parameters

• Alternative models

$|H$

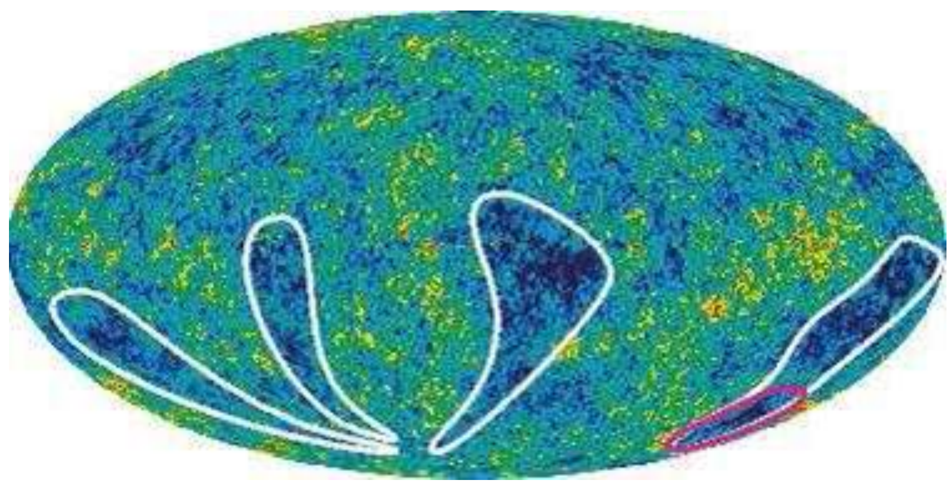
FRW

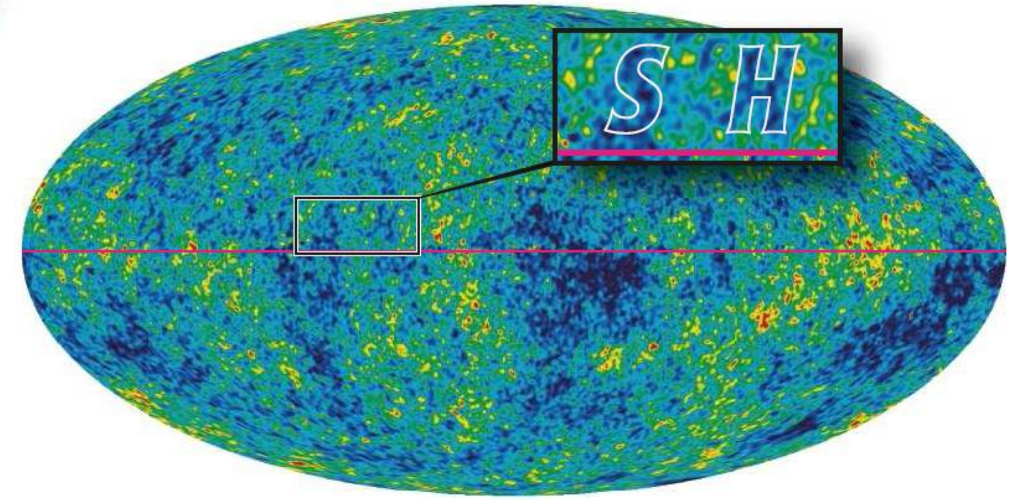
|

ds^2

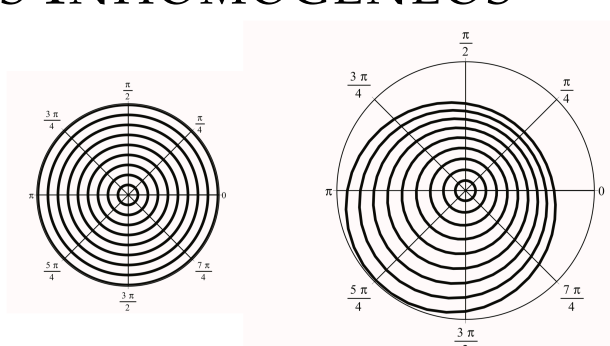
Extending ΛCDM model with restrained anisotropy

$$ds^2 = -dt^2 + S^2\left[e^{4\beta}dx^2 + e^{-2\beta}(dy^2 + dz^2)\right],$$





UNIVERSOS INHOMOGÉNEOS



$$ds^2 = dt^2 - e^{2\alpha(t,x,y,r)}dr^2 - e^{2\beta(t,x,y,r)}(dx^2 + dy^2).$$

• Alternative models

$|H$

Inflation

|

n_s, A_s

Hybrid Natural Inflation

$$V_I(\phi) = \Delta^4(1 + a\cos(\frac{\phi}{f})),$$

Cosmological inflation predicts the initial power spectrum to be close to scale-invariant with just a slight scale dependence.

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation

$$\mathcal{P}_{\mathcal{R}}(k) \quad = \quad A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}-1},$$

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}-1+(1/2)\ln(k/k_0)(dn/d\ln k)}$$

- Alternative models

H

Baryons	CDM
|	|
Ω_b	Ω_{dm}

ϕ^2 as Dark Matter

Dimensiones Extra

MOND

$\nabla^2\Phi = 4\pi G\rho$ → $\nabla\cdot\left(\mu\nabla\Phi\right) = 4\pi G\rho$

Materia Obscura Interactuante (SIDM)
Materia Obscura Difusa (FDM)
Materia Obscura Aniquiliante (SADM)
Materia Obscura que Decae (DDM)

$|H$

GR
|
$\int d^4x\sqrt{-g}R$

Cosmological constraints on $f(R)$

$$f(R) = R - \lambda R_c \left[1 - \left(1 + \alpha \frac{R}{R_c}\right)^{-n}\right],$$

Anisotropic Brans-Dicke extension of standard ΛCDM model

$$S_{\mathrm{JBD}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\varphi^2}{8}R - \omega\left(\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi + \frac{1}{2}M^2\varphi^2\right)\right] + S_{\mathrm{Matter}},$$

Λ
|
$w = -1$

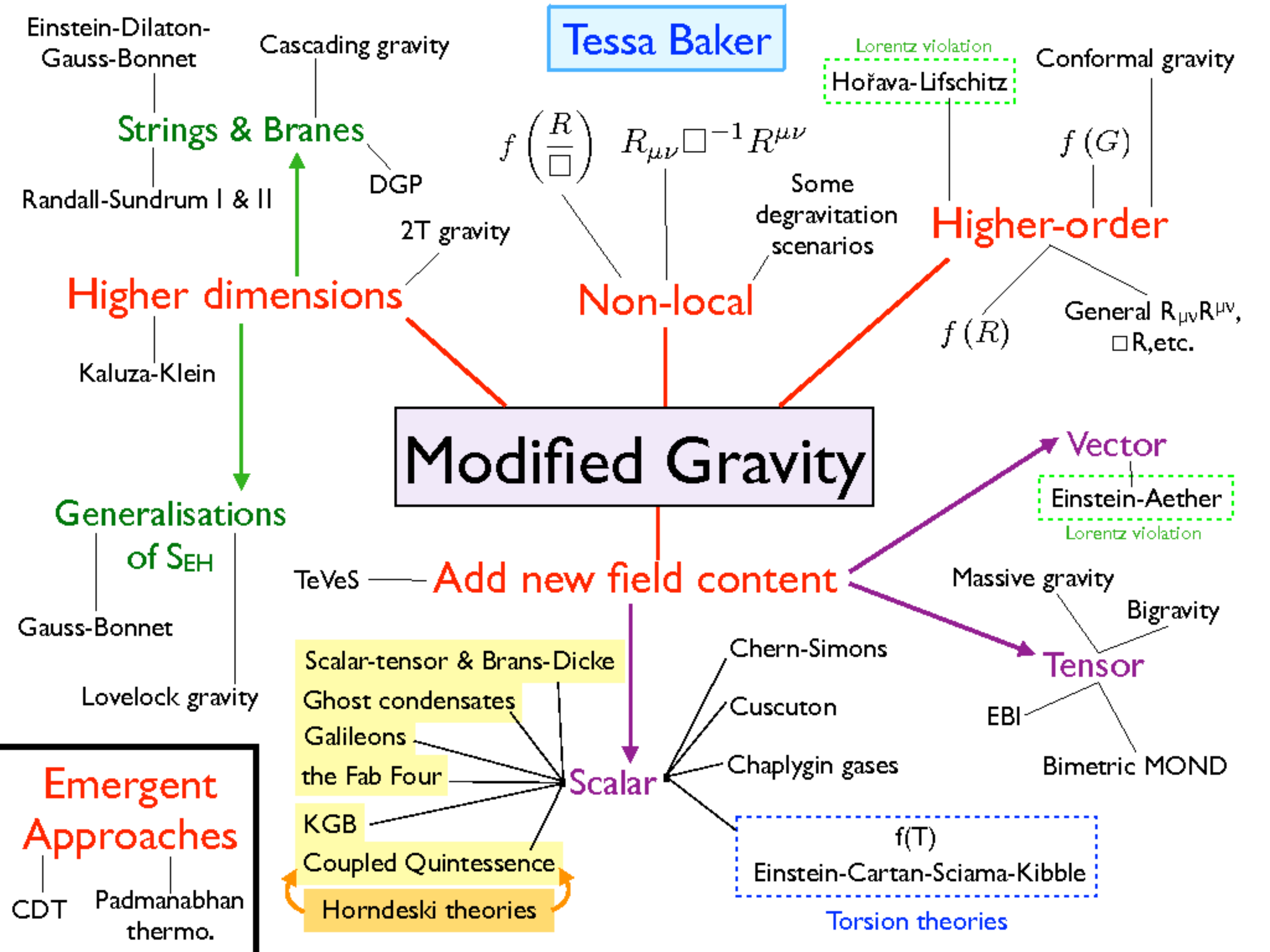
Energy-Momentum Log Gravity extension of ΛCDM model and screening Λ

$$S = \int\left[\frac{1}{2\kappa}(R - 2\Lambda) + f(T_{\mu\nu}T^{\mu\nu}) + \mathcal{L}_{\mathrm{m}}\right]\sqrt{-g}\,\mathrm{d}^4x,$$

$$f(T_{\mu\nu}T^{\mu\nu}) = \alpha\ln(T_{\mu\nu}T^{\mu\nu}),$$

Quintom Cosmology

$$\pounds = \sqrt{-g}\,(R - \mathcal{L}_\Phi - \mathcal{L}^*_\Phi - \mathcal{L}_\gamma)$$



Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
DGP
Randall-Sundrum I & II
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$,etc.
Add new field content
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
Emergent Approaches
CDT
Padmanabhan thermo.

- # Alternative models

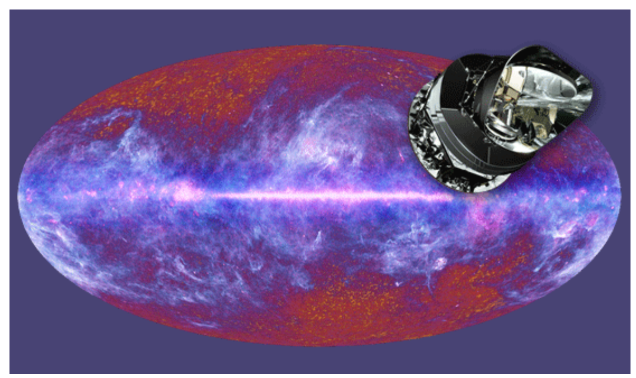
$|H$

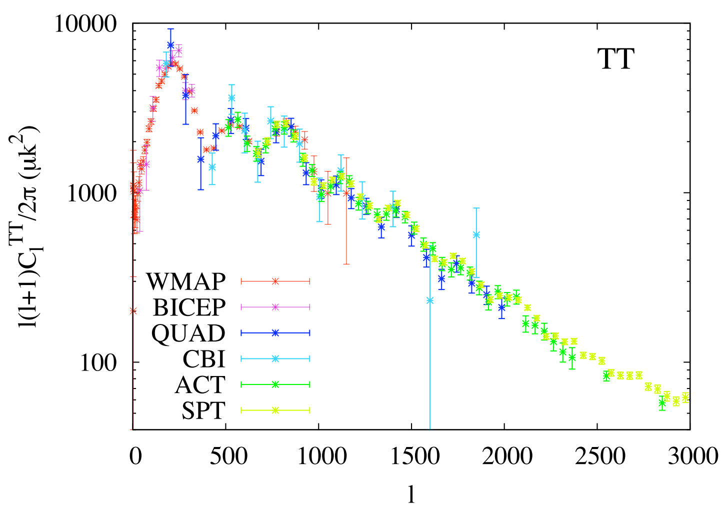
A non-exhaustive list of candidates beyond the standard cosmological model

?

αR^n	Modifications to gravity
	[or more complex theories]
$d\tilde{s}^2$	Anisotropic universe
$d\alpha/dz,\ dG/dz$	Variations of fundamental constants
$f_{\rm NL}$	Non-gaussianity
$n_{\rm run}$	Running of the scalar spectral index
$k_{\rm cut}$	Large-scale cut-off in the spectrum
	[or a more complex parameterisation of $\mathcal{P}_{\mathcal{R}}(k)$]
$r + 8n_{\rm t}$	Violation of the inflationary consistency relation
$n_{\rm t,run}$	Running of the tensor spectral index
	[or a more complex parameterisation of $\mathcal{P}_{\mathcal{T}}(k)$]
$P_{\rm iso}$	CDM isocurvature perturbations
Ω_k	Spatial curvature
Ω_X	Additional components
$m_{\rm dm}$	Warm dark matter mass
	[or scalar field dark matter]
m_{ν_i}	Neutrino mass for species 'i'
$w_{\rm DE}$	Dark energy equation-of-state
	[or a more complex parameterisation of $w(z)$]
ρ^α	Polytropic equation of state
Γ	Interacting fluids

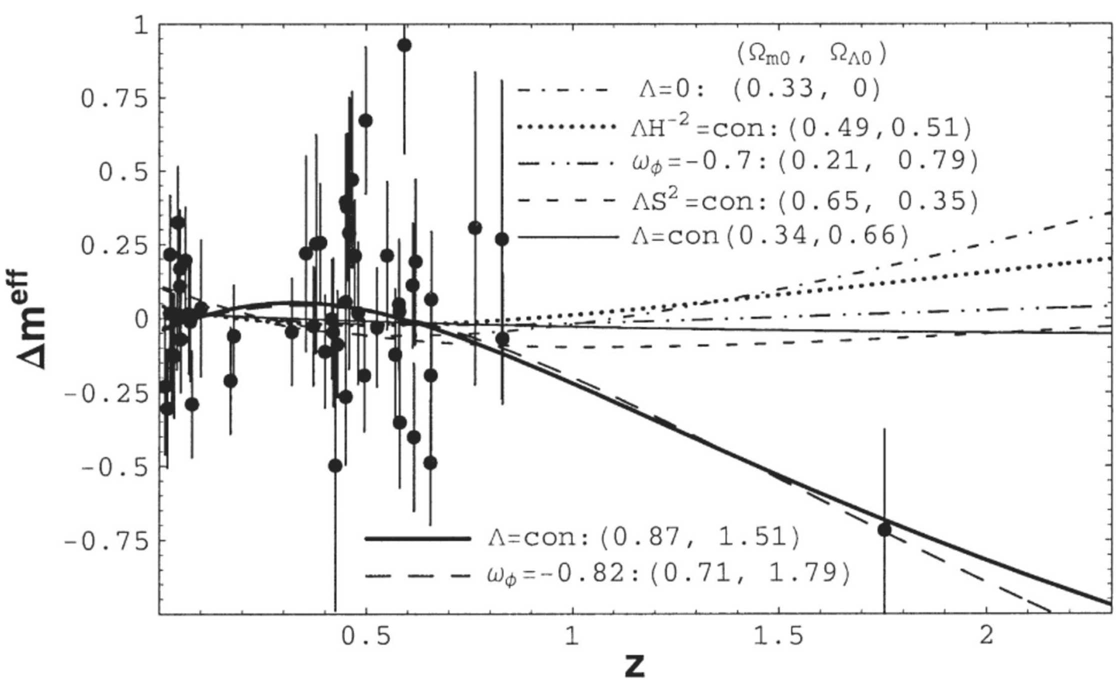
Observations

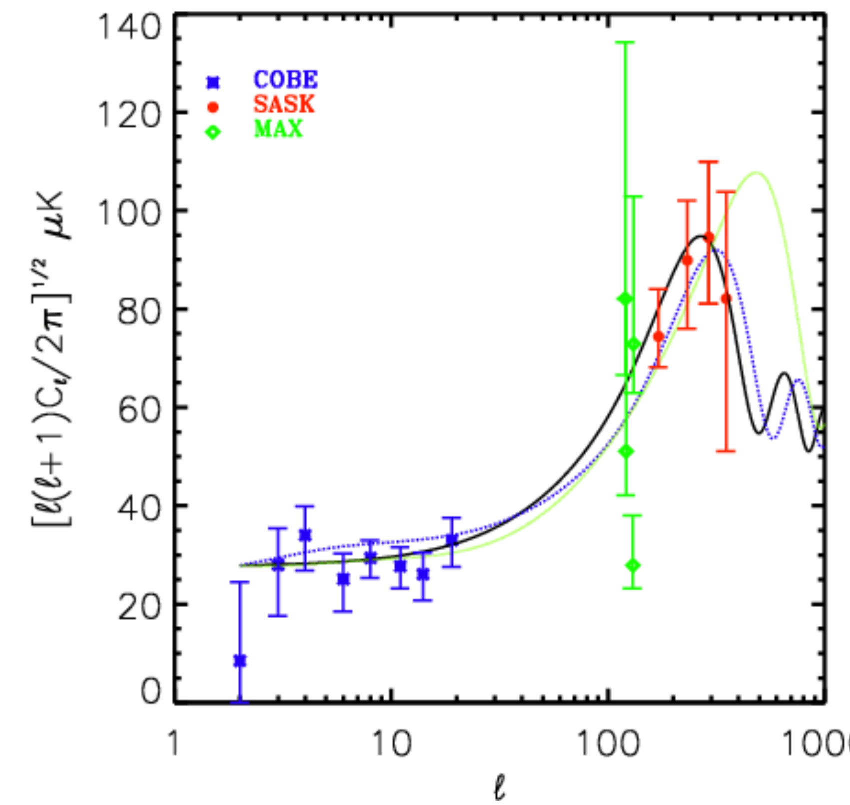


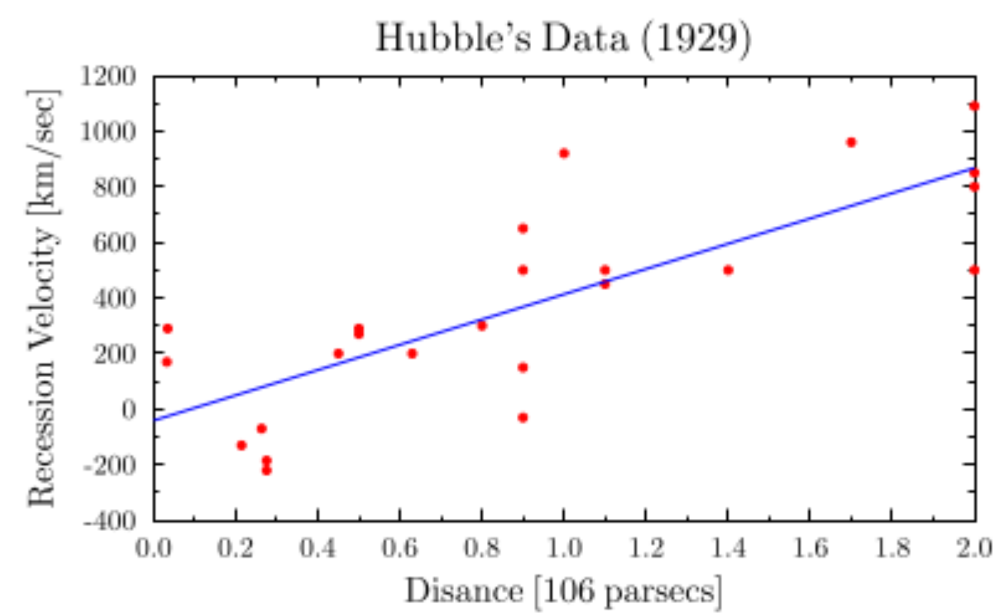


Observations

Rapid advance in the **development of powerful observational-instruments** has led to the establishment of **precision cosmology**.







Observations

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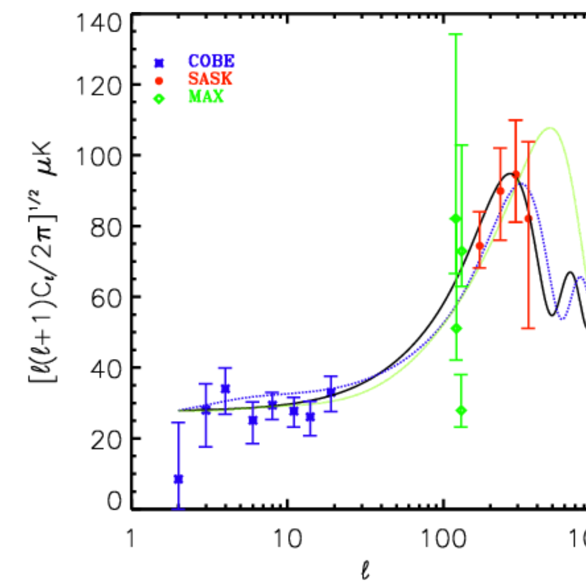
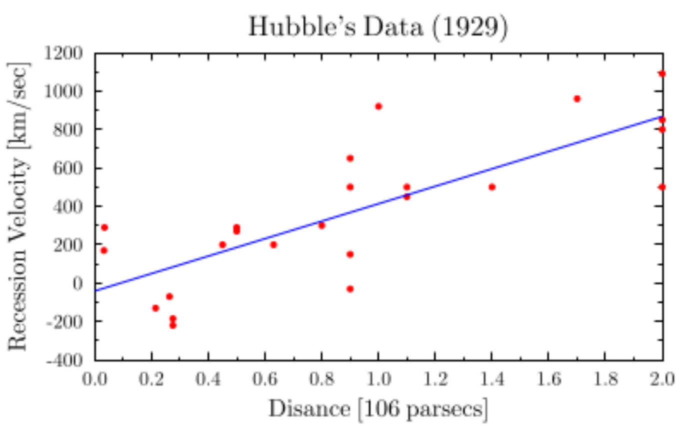
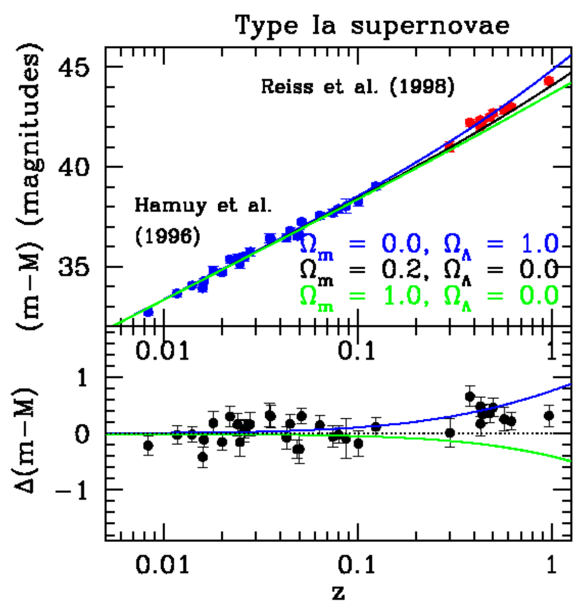
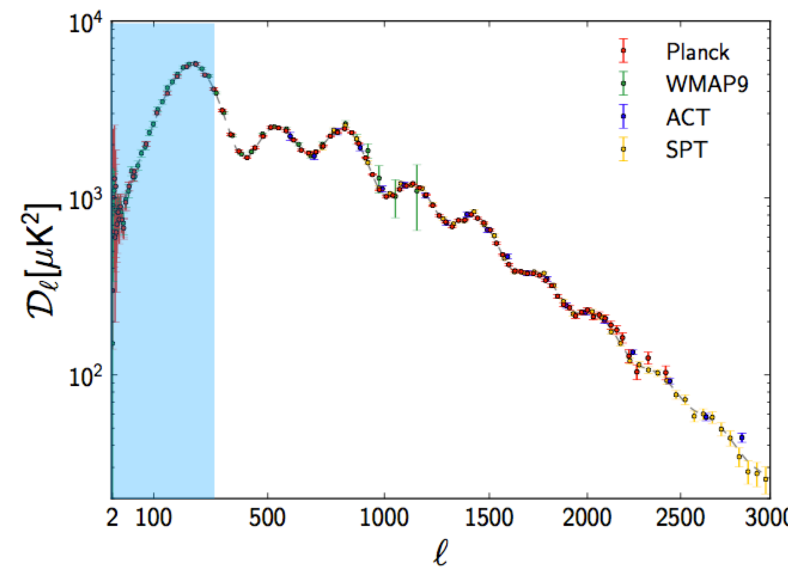
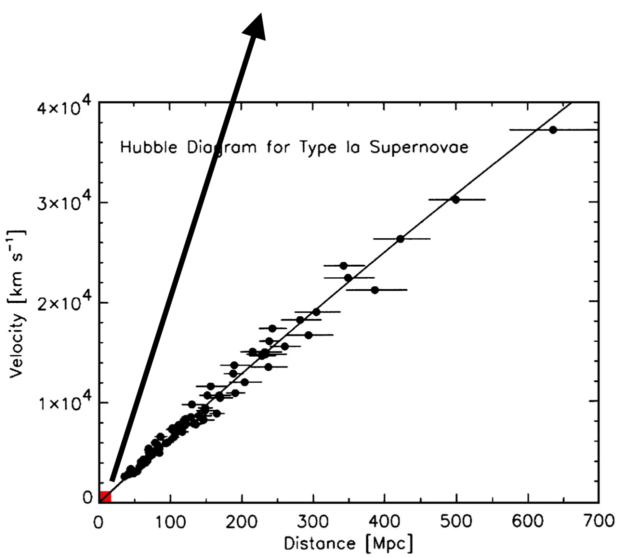
Rapid advance in the **development of powerful** observational-instruments has led to the establishment of **precision cosmology**.

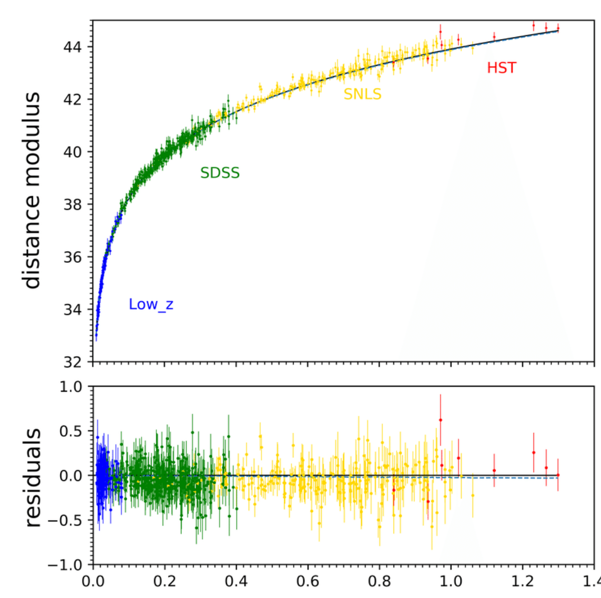


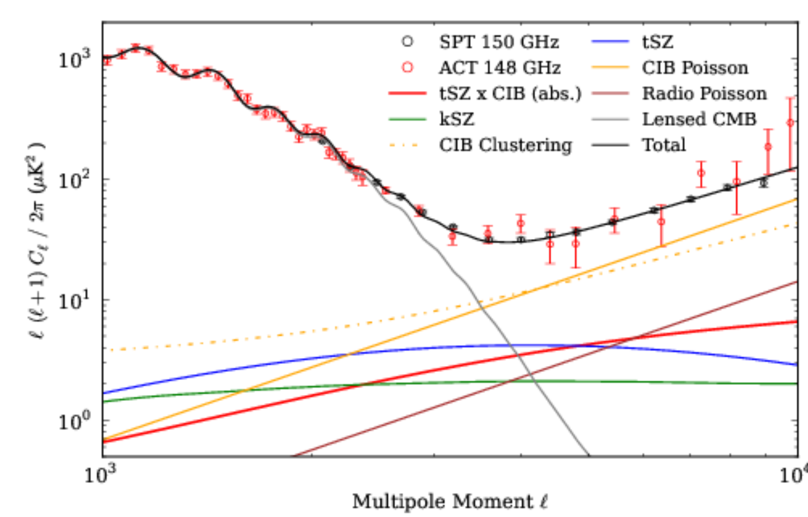










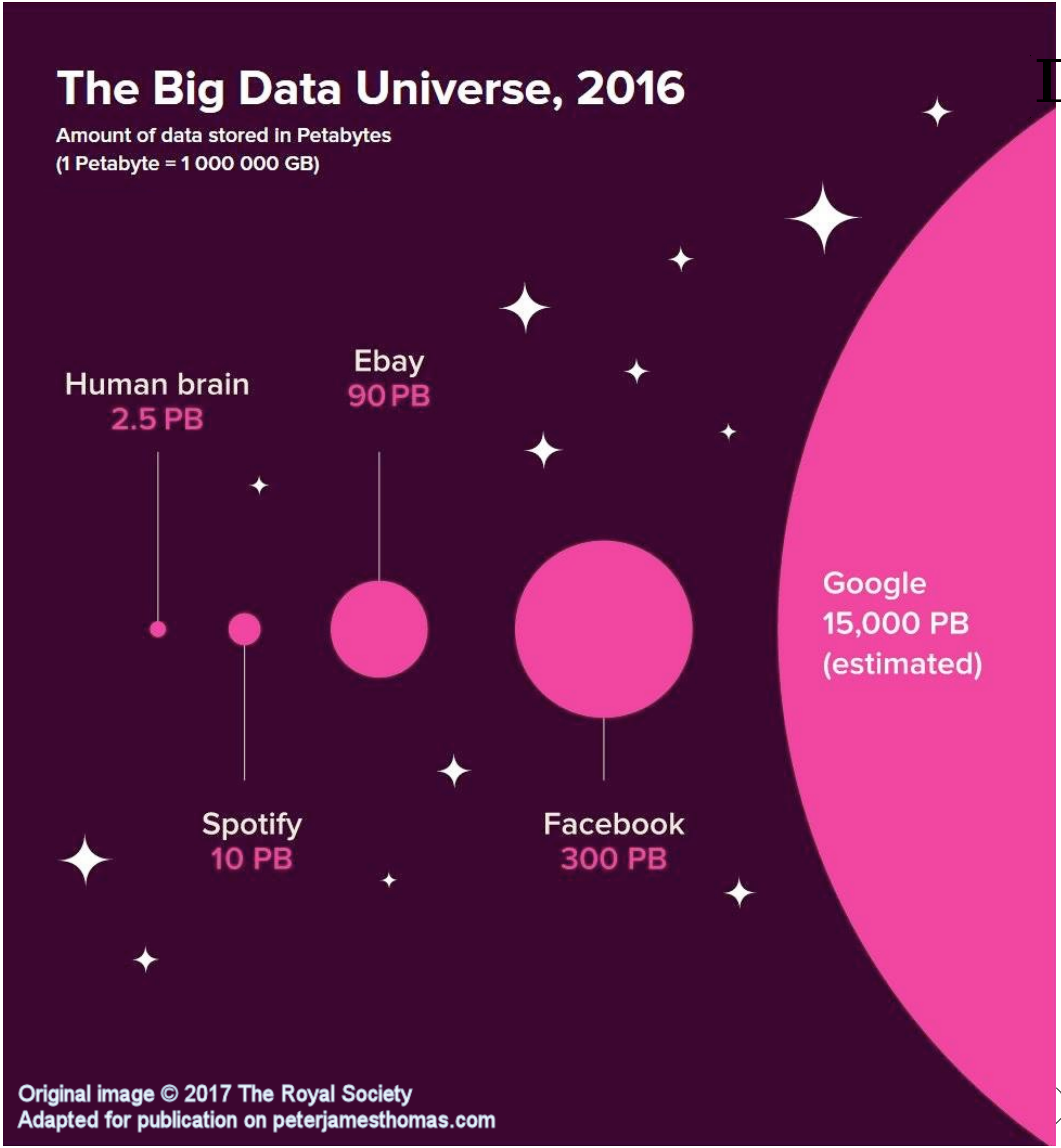


Datos

720 KB

x 1'000

x 1'000,000



D|

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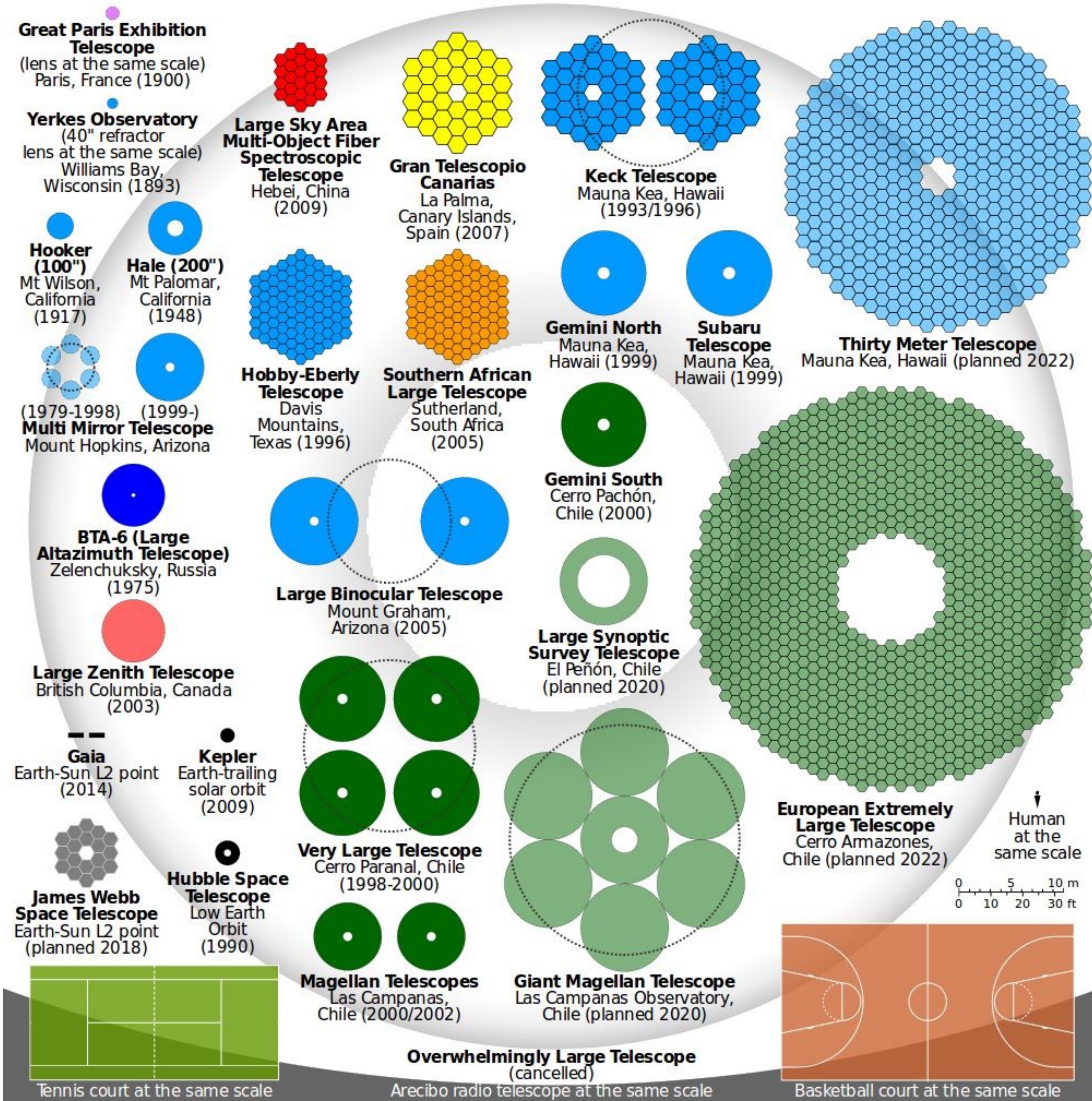
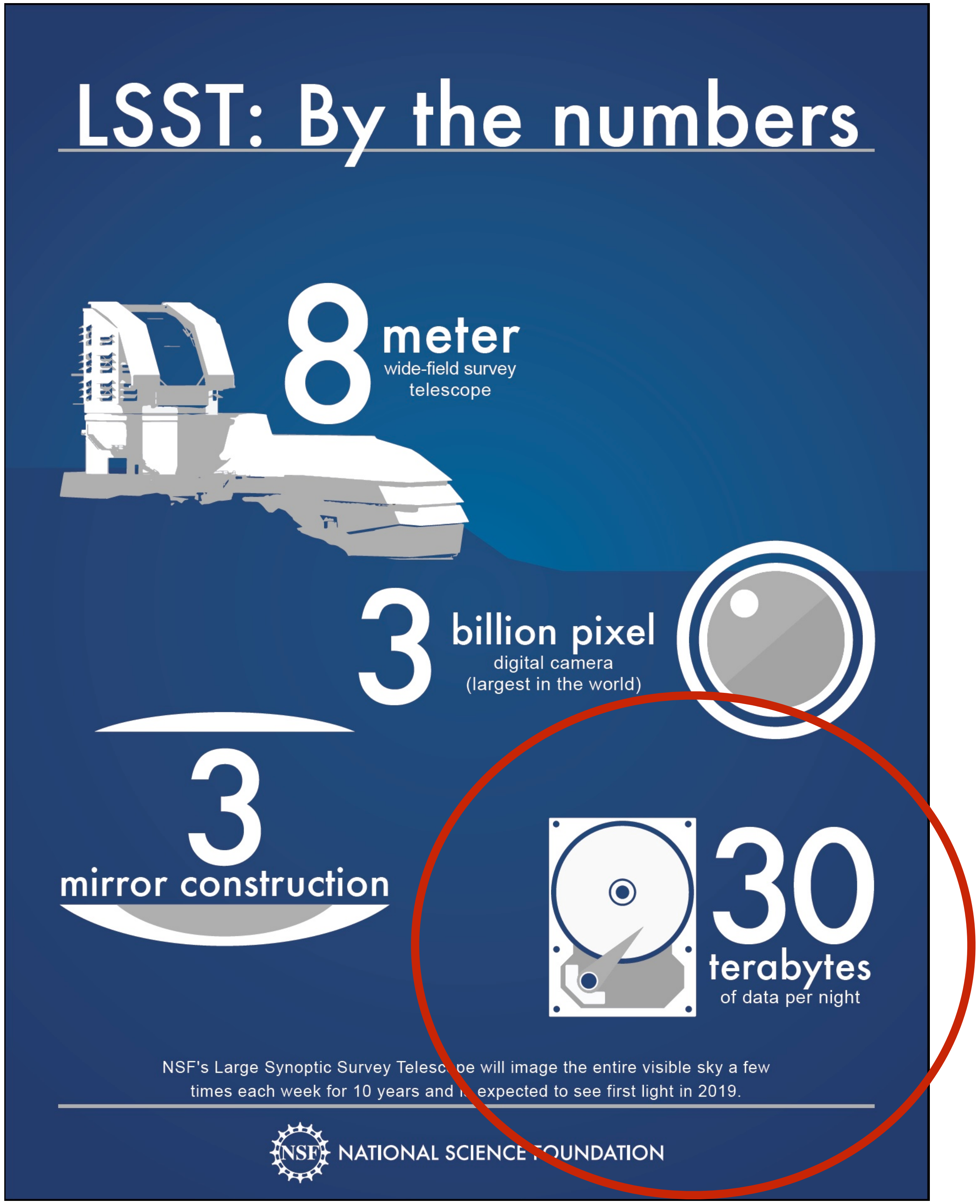
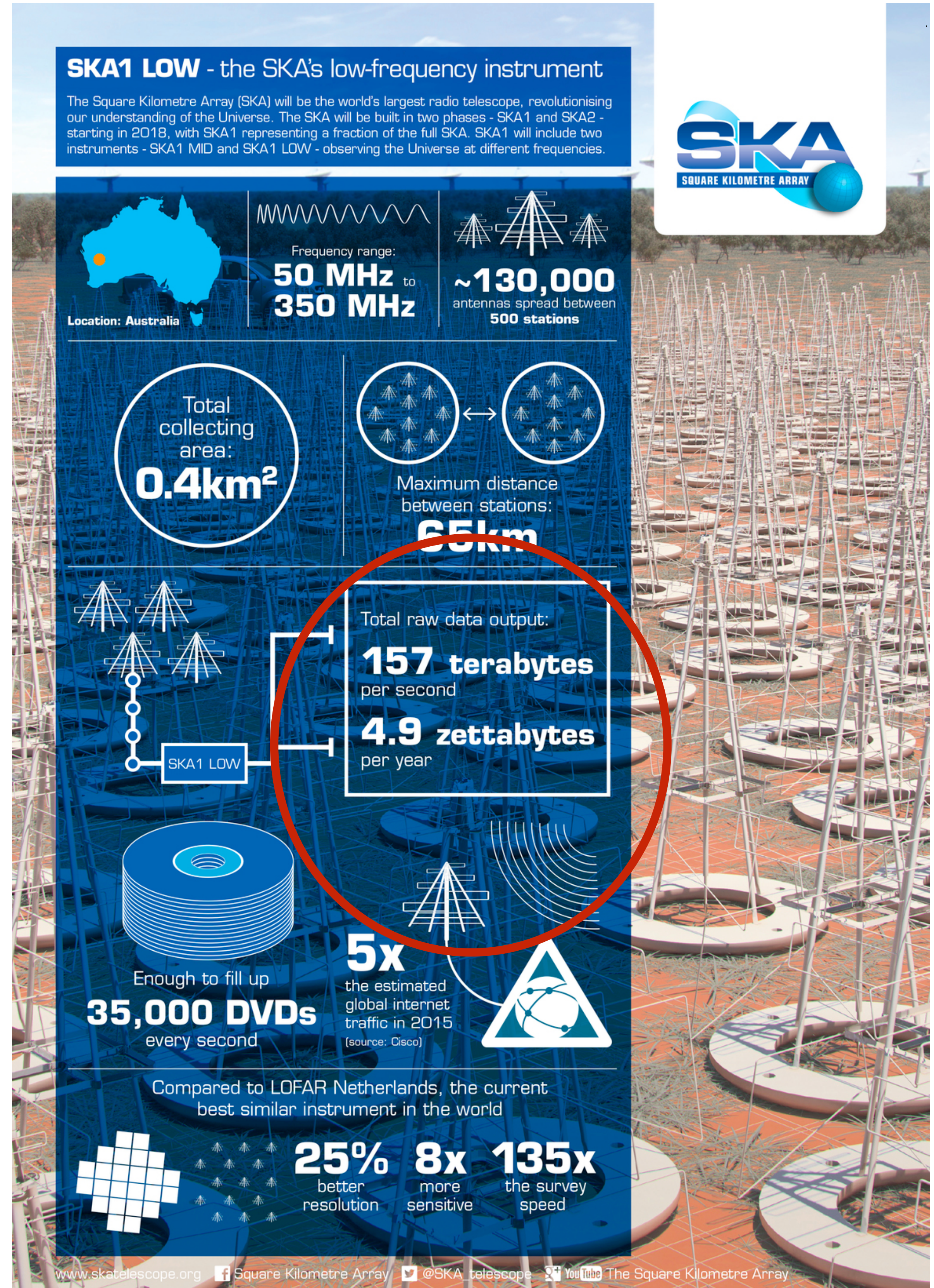
D|



Great Paris Exhibition Telescope
(lens at the same scale)
Paris, France (1900)
Yerkes Observatory
(40" refractor lens at the same scale)
Williams Bay, Wisconsin (1893)
Large Sky Area Multi-Object Fiber Spectroscopic Telescope
Hebei, China (2009)
Gran Telescopio Canarias
La Palma, Canary Islands, Spain (2007)
Keck Telescope
Mauna Kea, Hawaii (1993/1996)
Thirty Meter Telescope
Mauna Kea, Hawaii (planned 2022)
Hooker (100")
Mt Wilson, California (1917)
Hale (200")
Mt Palomar, California (1948)
Hobby-Eberly Telescope
Davis Mountains, Texas (1996)
Southern African Large Telescope
Sutherland, South Africa (2005)
Gemini North
Mauna Kea, Hawaii (1999)
Subaru Telescope
Mauna Kea, Hawaii (1999)
(1979-1998) (1999-)
Multi Mirror Telescope
Mount Hopkins, Arizona
Gemini South
Cerro Pachón, Chile (2000)
BTA-6 (Large Altazimuth Telescope)
Zelenchuksky, Russia (1975)
Large Binocular Telescope
Mount Graham, Arizona (2005)
Large Synoptic Survey Telescope
El Peñón, Chile (planned 2020)
Large Zenith Telescope
British Columbia, Canada (2003)
Gaia
Earth-Sun L2 point (2014)
Kepler
Earth-trailing solar orbit (2009)
Very Large Telescope
Cerro Paranal, Chile (1998-2000)
European Extremely Large Telescope
Cerro Armazones, Chile (planned 2022)
Human at the same scale
0 5 10 m
0 10 20 30 ft
James Webb Space Telescope
Earth-Sun L2 point (planned 2018)
Hubble Space Telescope
Low Earth Orbit (1990)
Magellan Telescopes
Las Campanas, Chile (2000/2002)
Giant Magellan Telescope
Las Campanas Observatory, Chile (planned 2020)
Overwhelmingly Large Telescope
(cancelled)
Tennis court at the same scale
Arecibo radio telescope at the same scale
Basketball court at the same scale

Motivation





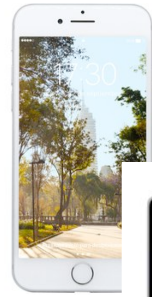
1 ZB = 10^21bytes=1trillion gigabytes.

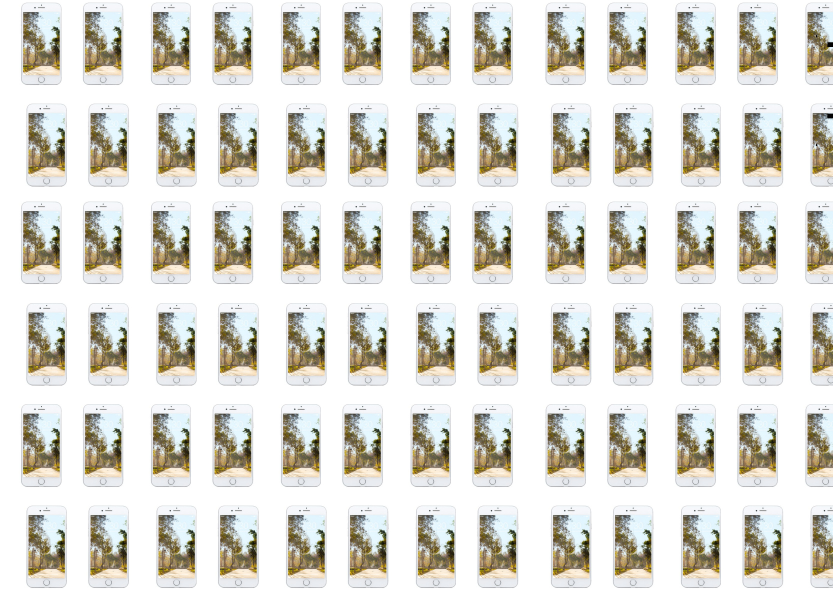
colectará el estimado de 14 exabytes por día, suficientes datos en crudo como para llenar 15 millones de iPods de 64 GB.

Motivation

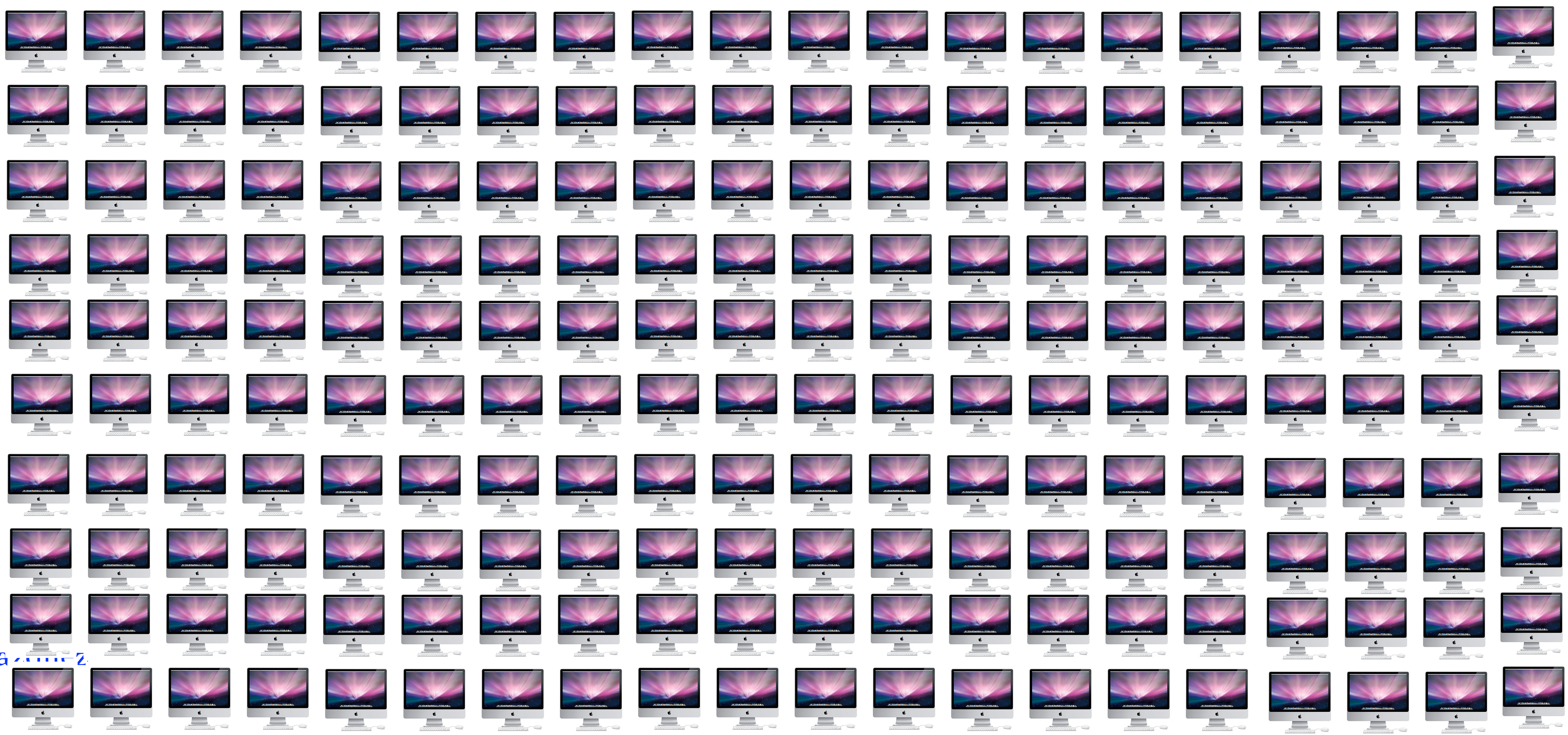
1 kilobyte	1,000
1 megabyte	1,000,000
1 gigabyte	1,000,000,000
1 terabyte	1,000,000,000,000
1 petabyte	1,000,000,000,000,000
1 exabyte	1,000,000,000,000,000,000
1 zettabyte	1,000,000,000,000,000,000,000

SOURCES: CISCO





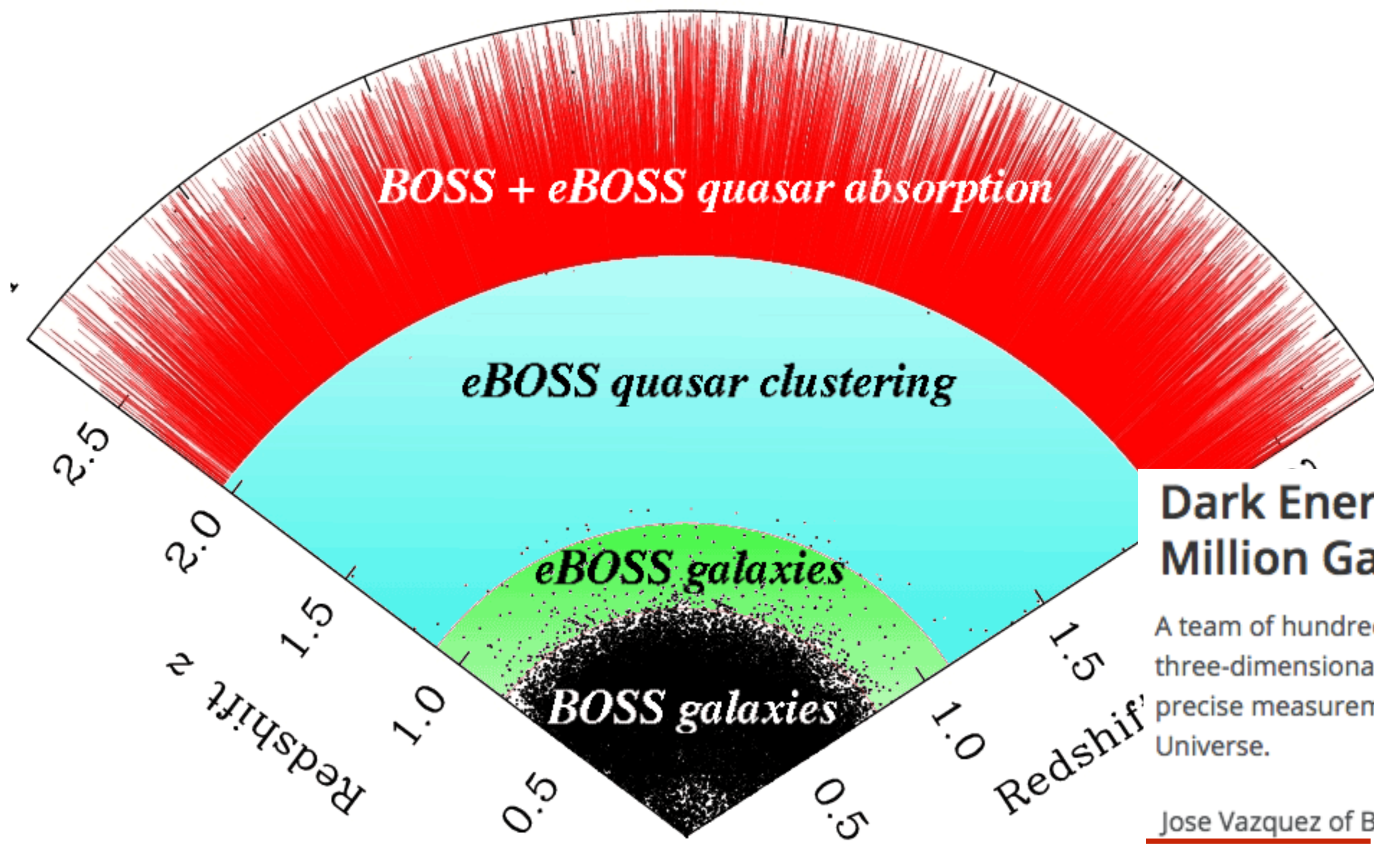
x 1,000,000,000,000



The Future

Table 1: Summary of current or planned BAO capable spectroscopic surveys. [Eisenstein, 2001][Hogg, 2005] [Drinkwater, 2010][Scrimgeour, 2012] [Eisenstein, 2011][Bolton, 2012] [Hill, 2008] [Abdalla, 2012] [Schlegel, 2011] [Ellis, 2012] [de Jong, 2012] [Amiaux, 2012]

Instrument	Telescope	Nights/ year	No. Galaxies	sq deg	Ops Start
SDSS I+II	APO 2.5m	dedicated	85K LRG	7600	2000
Wiggle-Z	AAT 3.9m	60	239K	1000	2007
BOSS	APO 2.5m	dedicated	1.4M LRG+160K Ly-α	10000	2009
HETDEX	HET 9.2m	60	1M	420	2014
eBOSS	APO 2.5m	180	600K LRG + 70K Ly-α	7000	2014
DESI	NOAO 4m	dedicated	+20M + 800k Ly-α	14000	2018
SUMIRE PFS	Subaru 8.2m	20	4M	1400	2018
4MOST	VISTA 4.1m	shared facility	6-20M bright objects	15000	2019
EUCLID	1.2m space	dedicated	52M	14700	2021

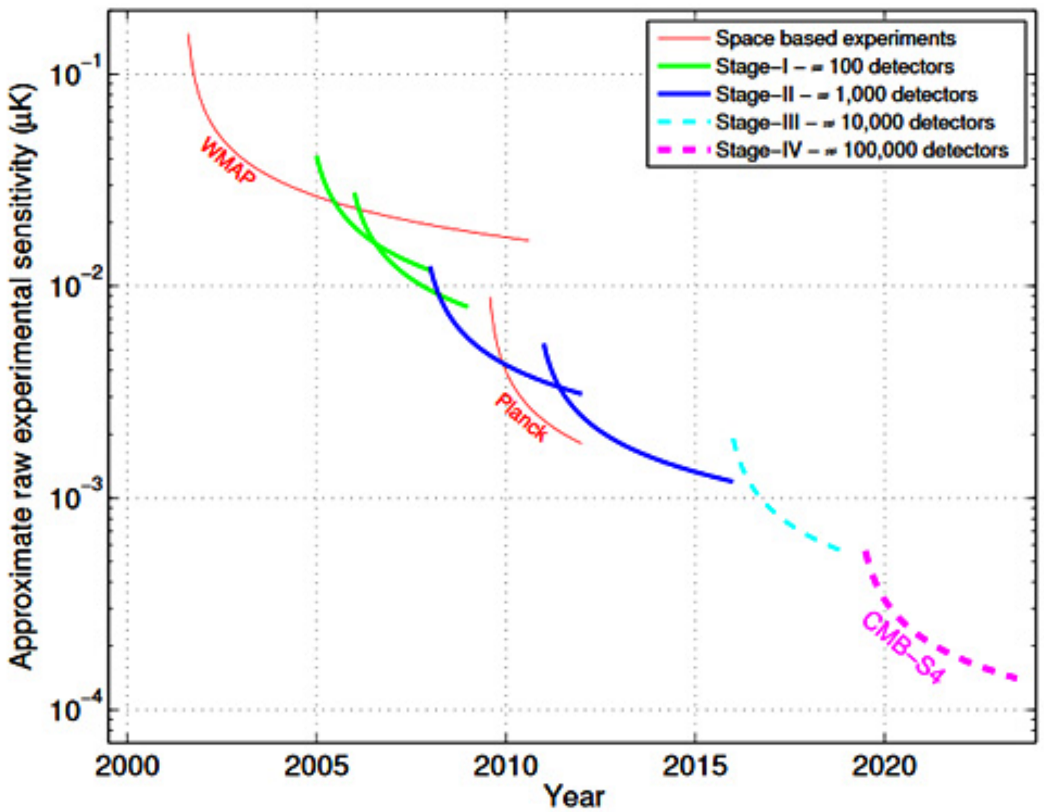


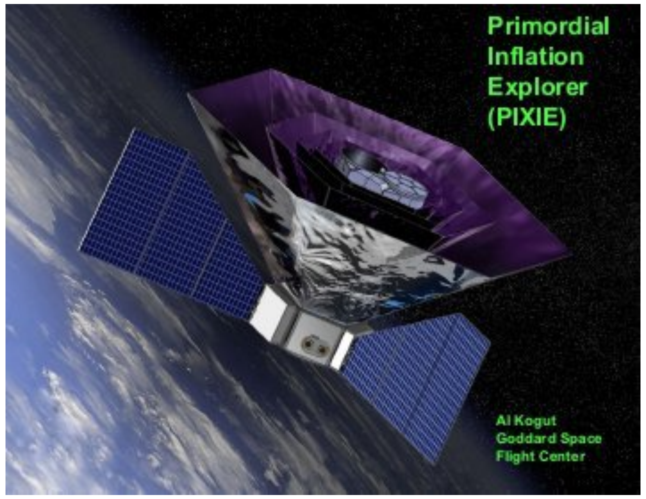
Dark Energy Measured With Record-Breaking Map of 1.2 Million Galaxies

A team of hundreds of physicists and astronomers have announced results from the largest-ever, three-dimensional map of distant galaxies. The team constructed this map to make one of the most precise measurements yet of the dark energy currently driving the accelerated expansion of the Universe.

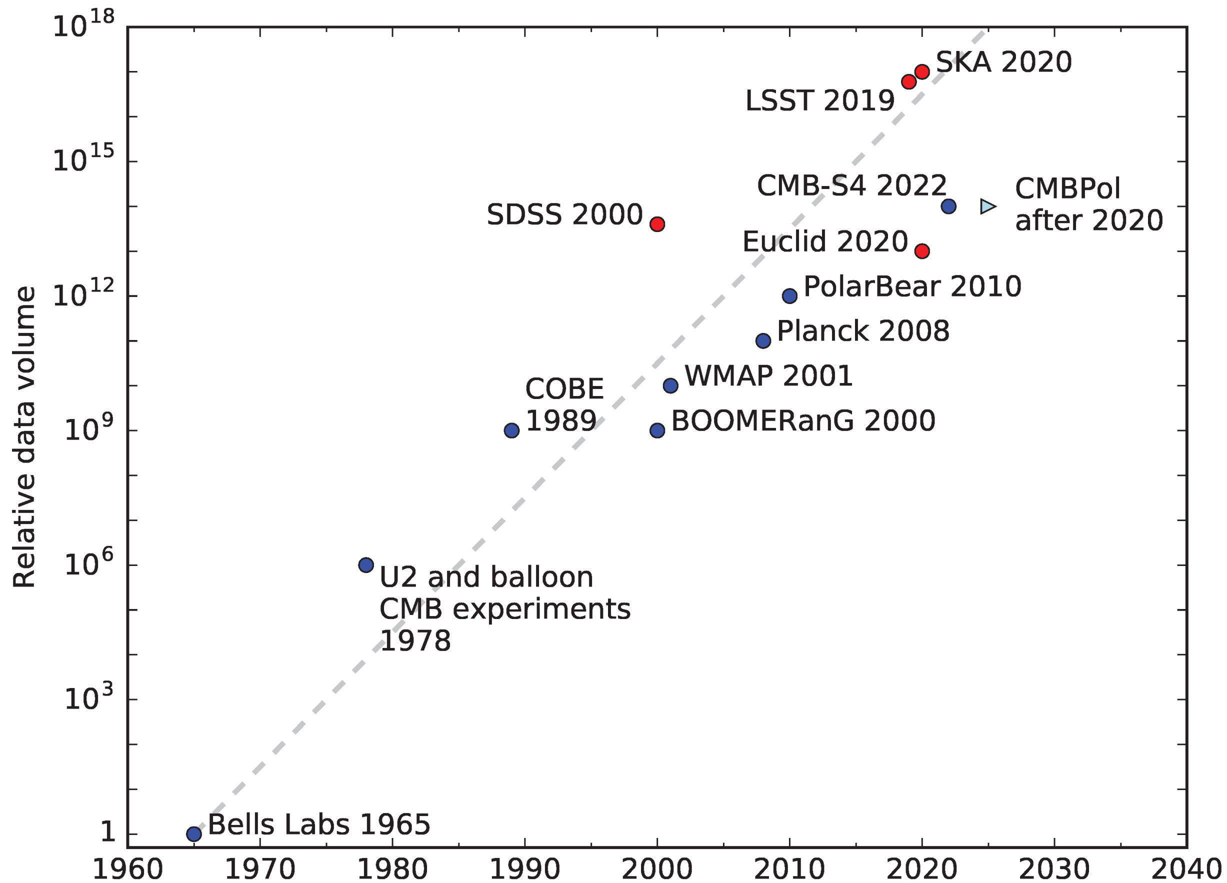
Jose Vazquez of Brookhaven National Laboratory combined the BOSS results with other surveys and searched for any evidence of unexplained physical phenomena in the results. "Our latest results tie into a clean cosmological picture, giving strength to the standard cosmological model that has emerged over the last eighteen years."

The Future





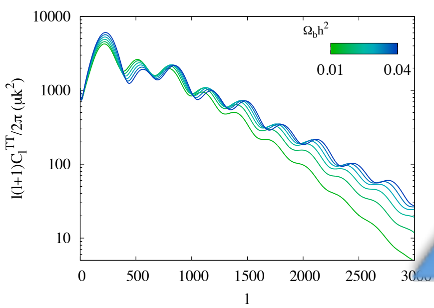
r < 10^{-3} at 5 standard deviations



Outline

Theory

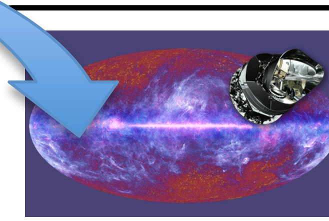
$$\int d^4x\sqrt{-g}\,R$$

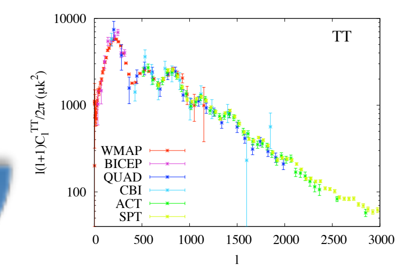


H

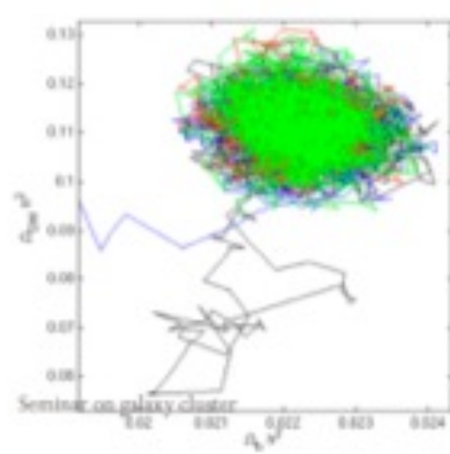
$$\mathrm{P}(\mathbf{D}|\mathbf{\Theta},\mathrm{M}) \equiv \mathcal{L}.$$

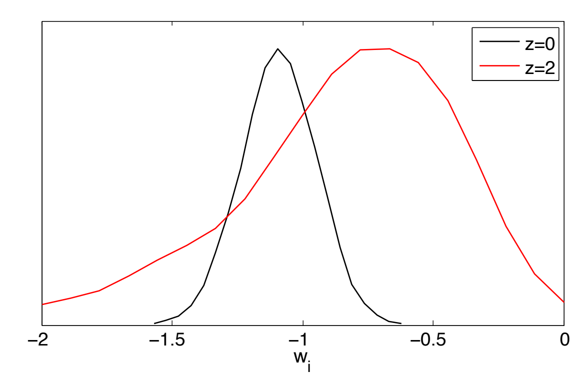
Observations





D





$$P(\mathbf{\Theta}|\mathbf{D}, M) :$$

JAVazquez

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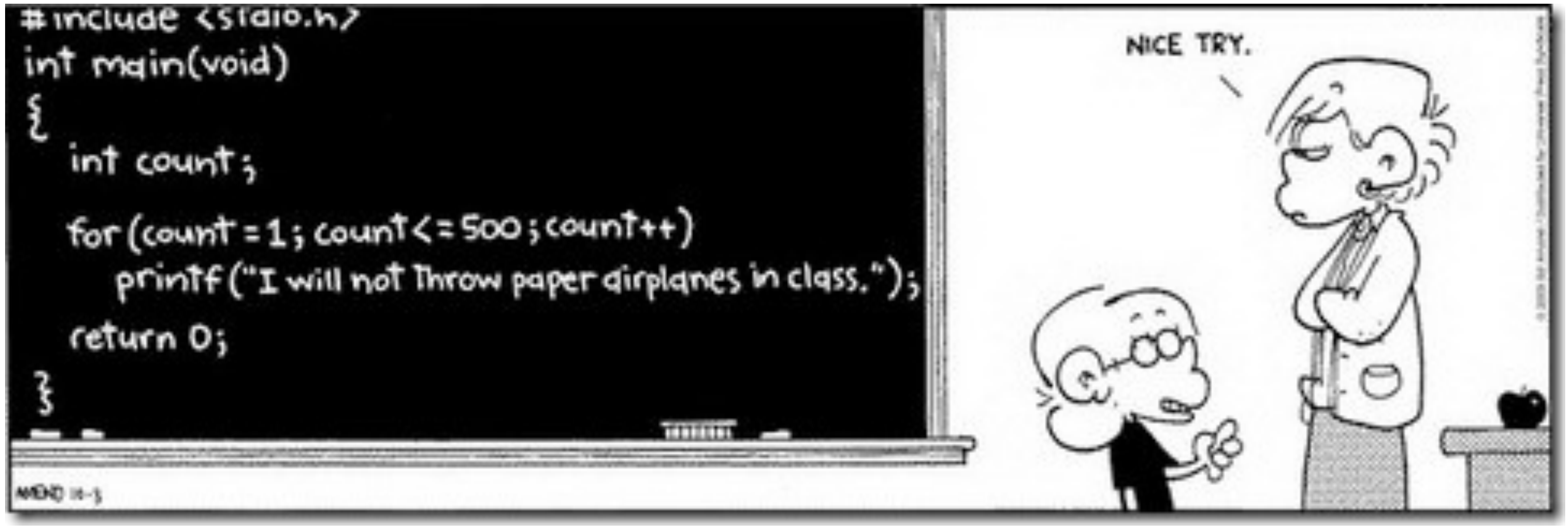
Codes & Algorithms



Posterior distributions

$$P(\mathbf{\Theta}|\mathbf{D},M) = \frac{P(\mathbf{D}|\mathbf{\Theta},M)\;P(\mathbf{\Theta}|M)}{P(\mathbf{D}|M)}.$$

$$\mathrm{P}(\mathbf{\Theta}|\mathrm{M}) \equiv \pi$$

$$\mathrm{P}(\mathbf{D}|\mathbf{\Theta},\mathrm{M}) \equiv \mathcal{L}.$$

$$\mathrm{P}(\mathbf{D}|\mathrm{M}) \equiv \mathcal{Z},$$

$$P(\theta|D,H) = \frac{P(D|\theta,H)P(\theta|H)}{P(D|H)}.$$

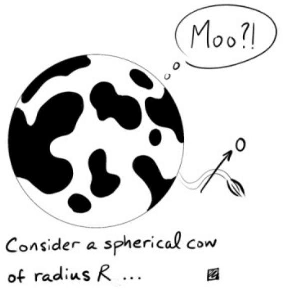
Bayes' theorem → **how one learns from experience!**

$P(\mathbf{D}|M)$ Normalisation constant

Commonly ignored in parameter estimation but it takes the **central role for model comparison**

$P(\mathbf{\Theta}|M)$ Adopt the principle of indifference and assume that all values of the parameters are equally likely, and take p(θlH)=constant

Thus for flat priors, we have simply $P(\mathbf{\Theta}|\mathbf{D},M) \;\propto\; \mathrm{P}(\mathbf{D}|\mathbf{\Theta},\mathrm{M}) \equiv \mathcal{L}$



3.4. Letting aside the priors

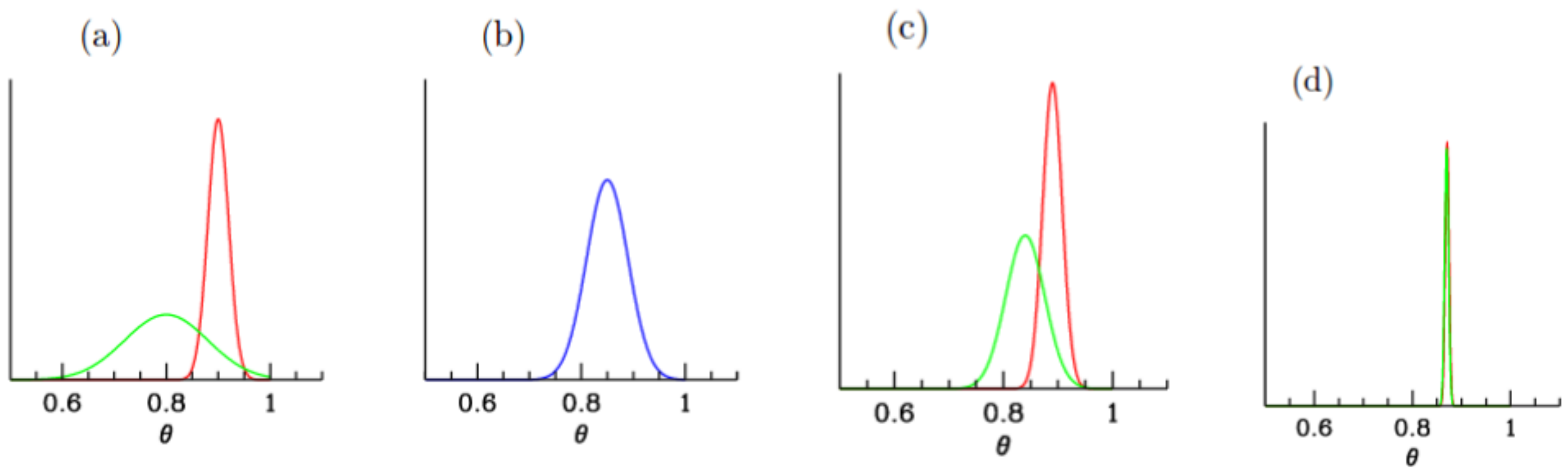


FIG. 3: Converging views in Bayesian inference (taken from [5]). A and B have different priors $P(\theta|I_i)$ for a value θ (panel (a)). Then, they observe one datum with an apparatus subject to a Gaussian noise and they obtained a likelihood $L(\theta; HI)$ (panel (b)), after which their posteriors $P(\theta|m_1)$ are obtained (panel (c)). Then, after observing 100 data, it can be seen how both posteriors are practically indistinguishable (panel (d)).

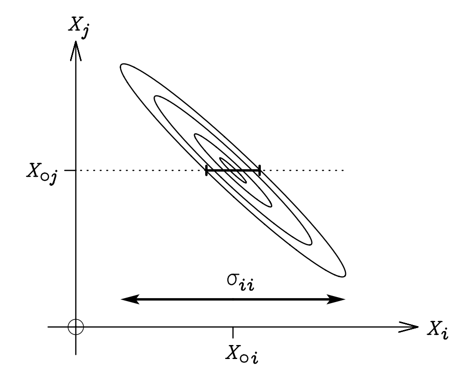
If the data are Gaussianly distributed the likelihood is given by a multi-variate Gaussian:

$$\mathscr{L} = \frac{1}{(2\pi)^{n/2}|detC|^{1/2}} \exp\left[-\frac{1}{2}\sum_{ij}(D-y)_i C_{ij}^{-1}(D-y)_j\right]$$

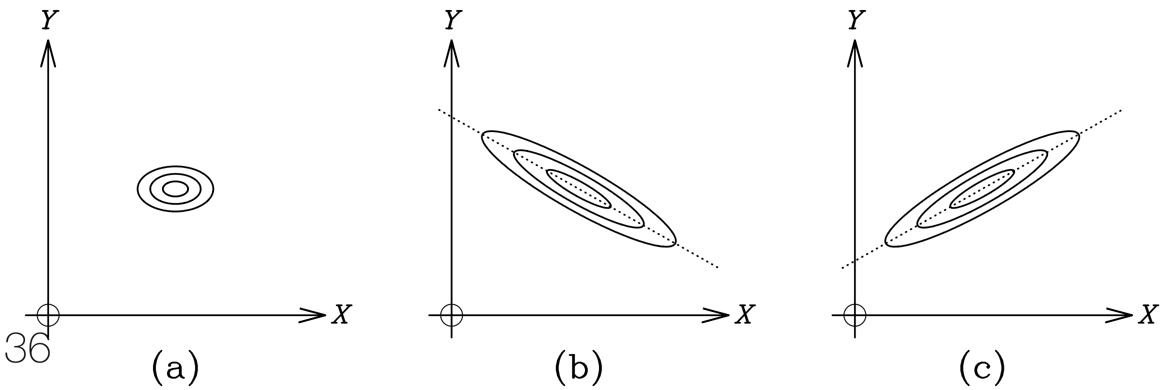
The covariance matrix

Is given by minus the inverse of $\nabla\nabla \ln L$ evaluated at y $\quad [\sigma^2]_{ij} = C_{ij} = -\left[(\nabla\nabla \ln L)^{-1}\right]_{ij}$

The square root of the diagonal elements (i = j) corresponds to the (marginal) error-bars for the associated parameters.



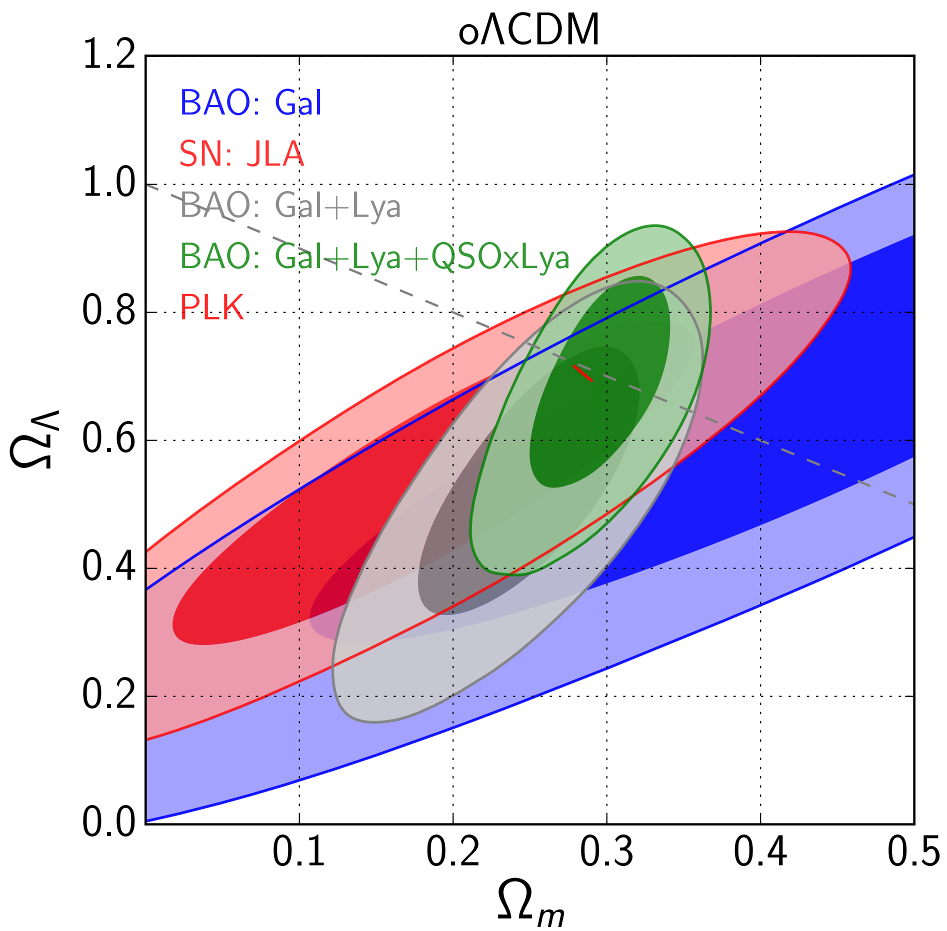
The off-diagonal components (i$\neq$ j) tell us about the correlations between the inferred values of y_i and y_j



For Gaussian distributions, the relation between χ^2 and likelihood

$$\mathscr{L} \propto \exp[-1/2\chi^2]$$

Maximizing the likelihood is equivalent at minimizing the χ^2



The Likelihoods

$P(\mathbf{D}|\mathbf{\Theta},M)$

CMB

$$\chi^2 = \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}} (2\ell+1) f_{\rm sky} \left[\frac{\hat{\mathcal{C}}_\ell^{BB}}{\mathcal{C}_\ell^{BB}} - 3 + \ln\left(\frac{\mathcal{C}_\ell^{BB}}{\hat{\mathcal{C}}_\ell^{BB}}\right) + \frac{\hat{\mathcal{C}}_\ell^{TT}\mathcal{C}_\ell^{EE} + \hat{\mathcal{C}}_\ell^{EE}\mathcal{C}_\ell^{TT} - 2\hat{\mathcal{C}}_\ell^{TE}\mathcal{C}_\ell^{TE}}{\mathcal{C}_\ell^{TT}\mathcal{C}_\ell^{EE} - (\mathcal{C}_\ell^{TE})^2} + \ln\left(\frac{\mathcal{C}_\ell^{TT}\mathcal{C}_\ell^{EE} - (\mathcal{C}_\ell^{TE})^2}{\hat{\mathcal{C}}_\ell^{TT}\hat{\mathcal{C}}_\ell^{EE} - (\hat{\mathcal{C}}_\ell^{TE})^2}\right) \right]$$

Supernovae

$$\chi^2_{\rm SN}(\mu_0, \boldsymbol{\theta}) = \sum_{j=1} \frac{(\mu_{\rm th}(z_j;\mu_0,\boldsymbol{\theta}) - \mu_{\rm obs}(z_j))^2}{\sigma^2_{\mu,j}},$$

$$\mu(z_j) = 5\log_{10}[d_L(z_j,\boldsymbol{\theta})] + \mu_0,$$

BAO

$$\chi^2_{\rm BAO} = (v_i - v_i^{\rm BAO})(C^{-1})_{ij}^{\rm BAO}(v_j - v_j^{\rm BAO})$$

$$\mathbf{v} = \left\{ \frac{r_s(z_{\rm drag},\Omega_m,\Omega_b;\boldsymbol{\theta})}{D_V(0.2,\Omega_m;\boldsymbol{\theta})}, \frac{r_s(z_{\rm drag},\Omega_m,\Omega_b;\boldsymbol{\theta})}{D_V(0.35,\Omega_m;\boldsymbol{\theta})} \right\}$$

$$\mathbf{v}^{\rm BAO} = (0.1905, 0.1097)$$

Hubble

$$\chi^2_{Hub}(H_0) = \frac{[H_0 - 74.2]^2}{3.6^2}$$

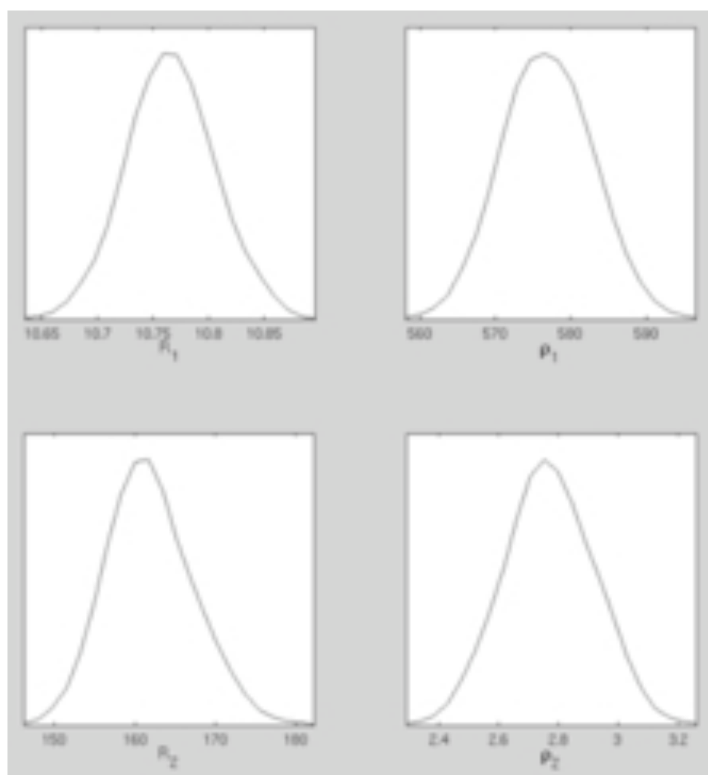
$$\begin{aligned} L &= L_{\rm SNe} \times L_{\rm BAO} \times L_{\rm CMB} \times L_{\rm Hub} \\ &= \exp[-(\chi^2_{\rm SNe} + \chi^2_{\rm BAO} + \chi^2_{\rm CMB} + \chi^2_{\rm Hub})/2]. \end{aligned}$$

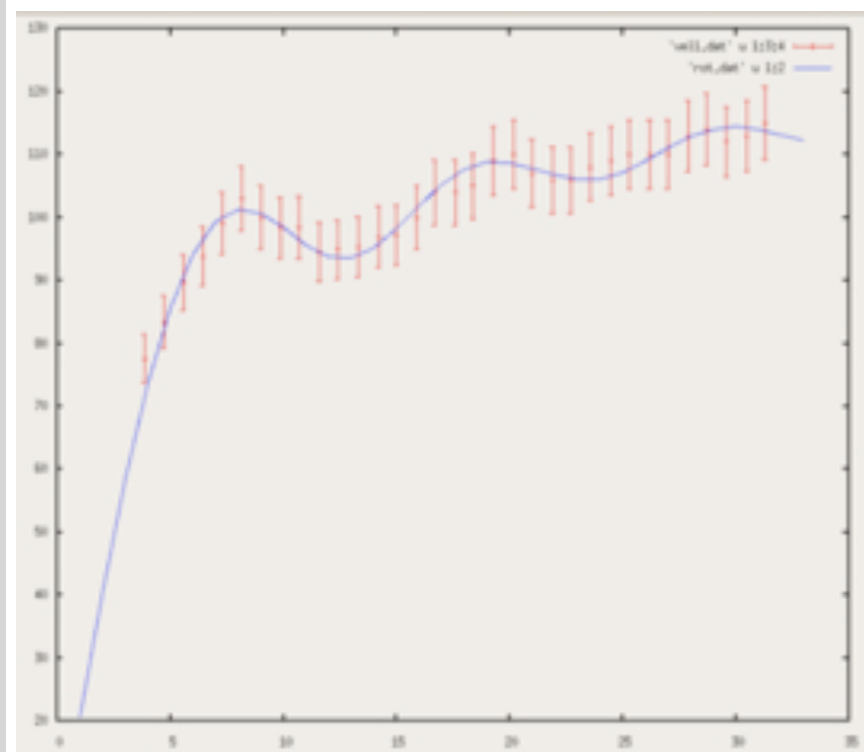
$P(\mathbf{D}|\mathbf{\Theta},M)$

Rotation curves

BEC

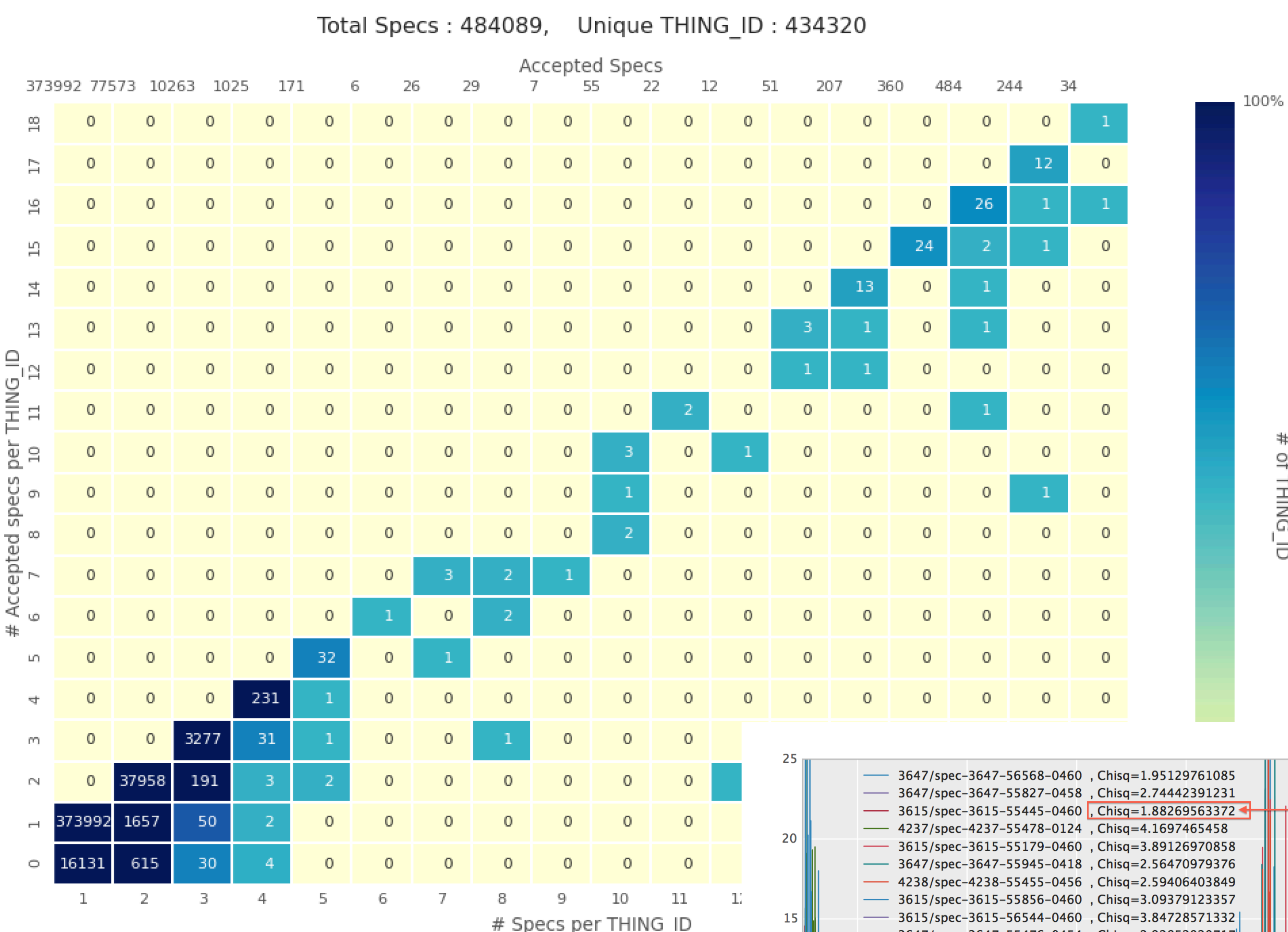
$$\rho_{\rm tot} = \sum_j \rho_0^j \frac{\sin^2(j\pi r/\mathrm{R})}{(j\pi r/\mathrm{R})^2},$$





Model selection:

$$\rho_{NFW}(r) = \frac{\rho_i}{(r/Rs)(1 + r/R_s)^2} \qquad \rho_{PI} = \frac{\rho_0^{PI}}{1 + (r/R_c)^2},$$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Accepted Specs
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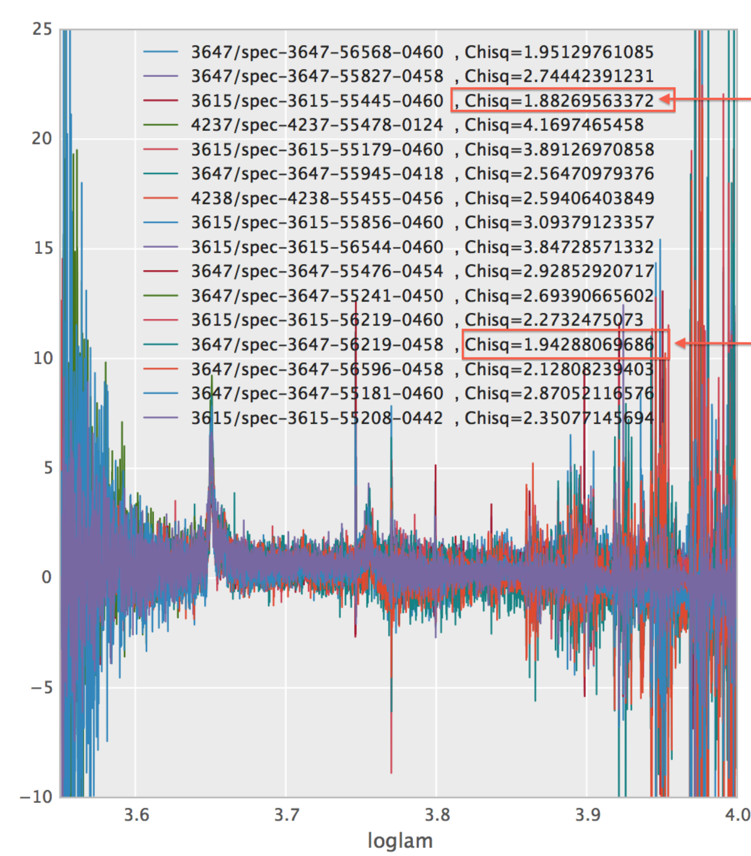
Accepted specs per THING_ID
Specs per THING_ID
100%
# of THING_ID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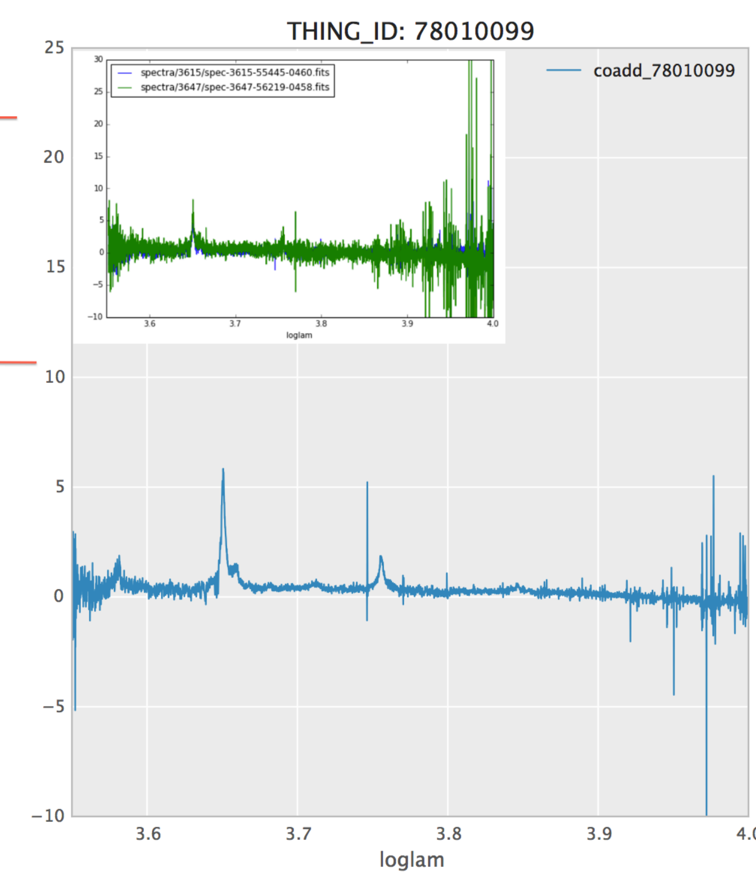
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loglam

Ly-a

The Likelihood exploration M-H

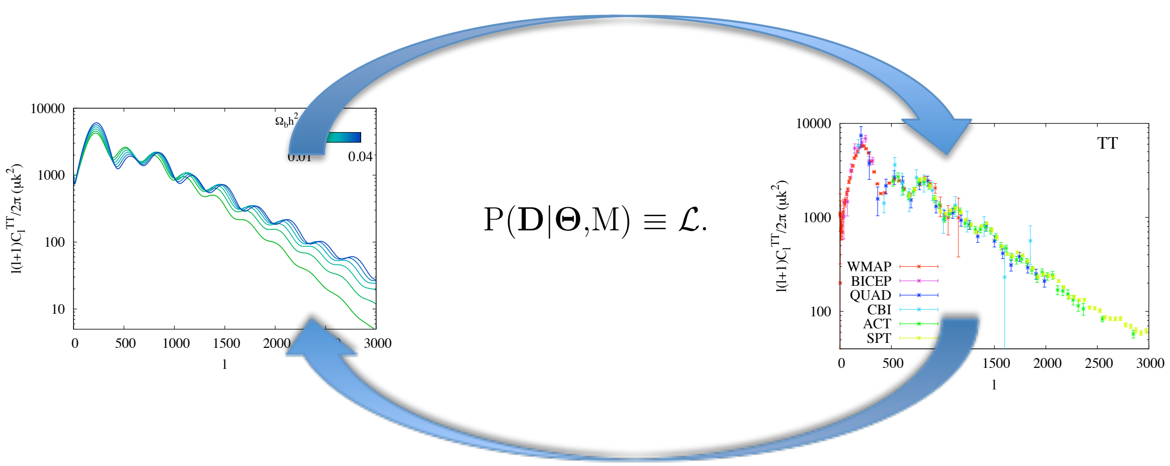
1. Start with a set of cosmological parameters $\{\theta_1\}$, compute the C_l^1 and the likelihood $\mathcal{L}_1$

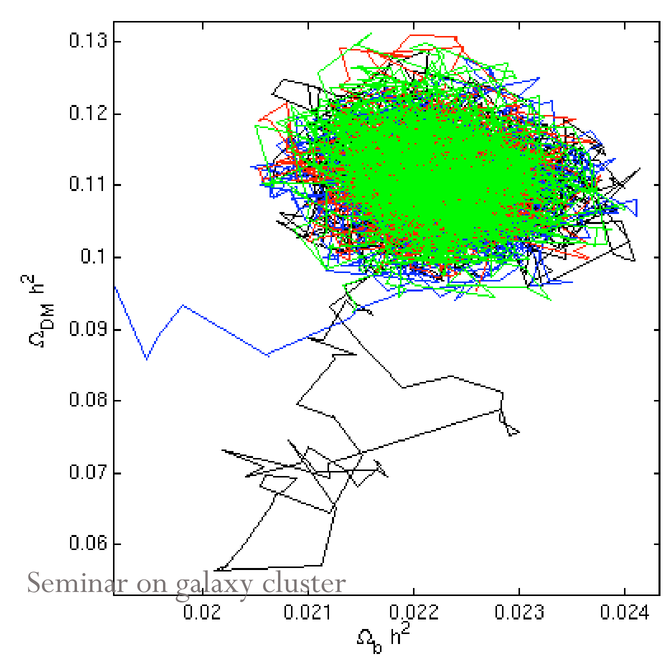
2. Take a random step in parameter space to obtain a new set of cosmological parameters $\{\theta_2\}$ and their likelihood $\mathcal{L}_2$

3. If $\mathcal{L}_2/\mathcal{L}_1 \geq$, "take the step" i.e. save the new set of cosmological parameters $\{\theta_2\}$, then go to step 2 after the substitution $\{\theta_2\} \rightarrow \{\theta_1\}$

4. If the point is not accepted, the previous point is repeated in the chain

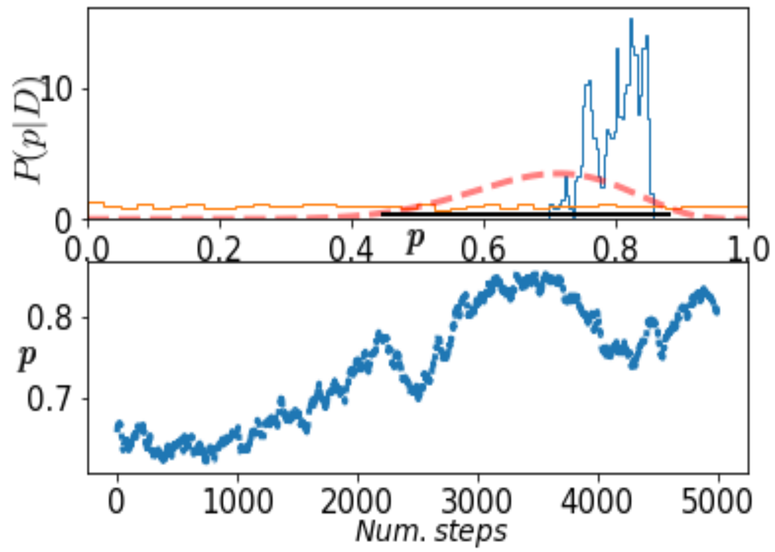
5. For each cosmological model run several chains starting at randomly chosen, well-separated points in parameter space. When the convergence criterion is satisfied stop the chains.

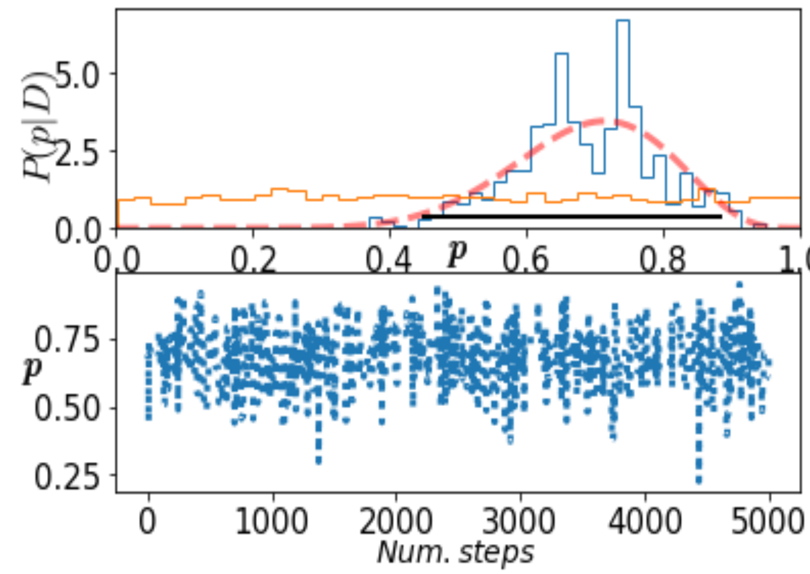




* The choice of a proposal distribution (step size) is crucial to improve the chain efficiency and speed up convergence.

Steps too big or too small will lead to slow convergence





→ to have a proposal density that is of similar shape to the posterior.

(Fortunately with cosmological data we have a reasonable idea of what the posterior might look like)

It is vitally important to have a convergence test

The Gelman-Rubin convergence criterion

$$\hat{R} = \frac{\frac{N-1}{N}W + B(1+\frac{1}{M})}{W},$$

$P(\theta|D,H)$

Cosmo Codes

TABLE I. Comparison between CMB Codes [a]

	CAMB	CLASS	CMBEASY	CMBquick	CosmoLib [b]
Language	F90	C	C++	Mathematica	F90[c]
gauge [d]	syn.	syn./Newt. [e]	syn./gauge-inv.	Newt.	Newt.
open/close universe	Yes	No	No	No	No
massive neutrinos	Yes	Yes	Yes	Yes	No
tensor perturb.	Yes	Yes	Yes	Yes	Yes
CDM isocurvature mode	Yes	Yes	Yes	Yes	Yes
dark energy perturb.	Yes	Yes	Yes	No	Yes
nonzero $c_{s,b}^2$	Yes	Yes	Yes	No	Yes
dark energy EOS.	constant	$w_0 + w_a(1-a)$	arbitrary	-1	arbitrary
non-smooth primordial power	No	No	No	No	Yes
MCMC driver	Yes	No	Yes	No	Yes
periodic proposal density	No	YES	No	No	Yes
data simulation	No	No	No	No	Yes
second-order perturb. [f]	No	No	No	Yes	No [g]

[a] Here we do not include CMBFast, which is no longer supported by its authors or available for download.

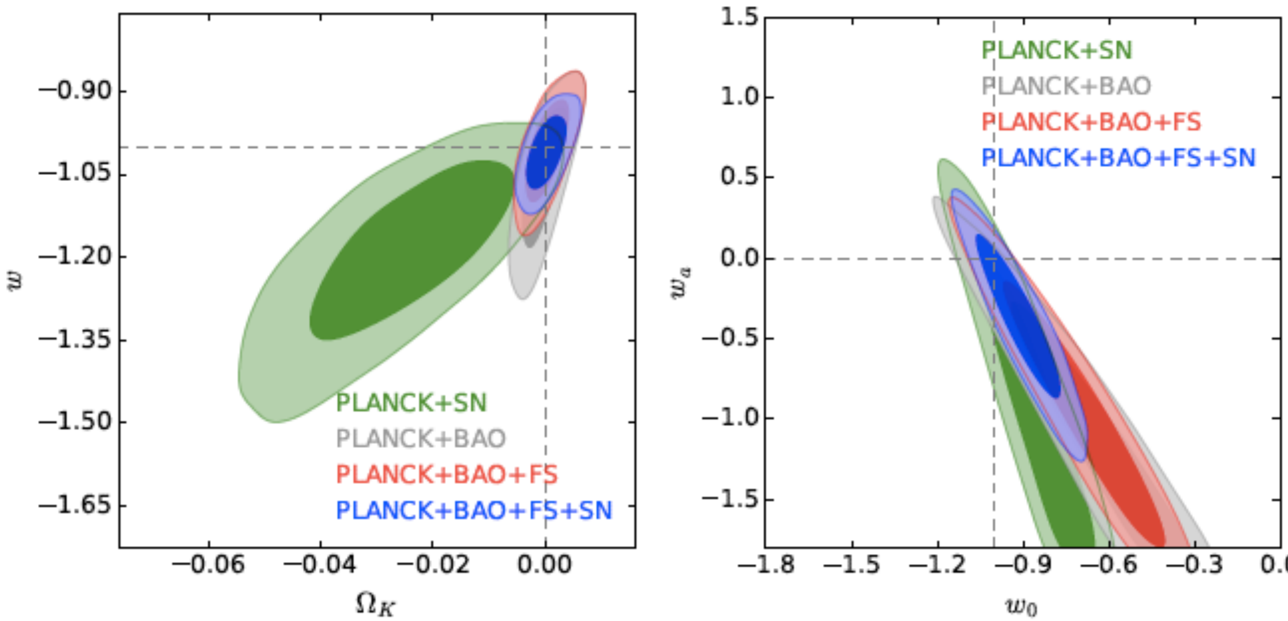
[c] CosmoLib is a mixture of Fortran and C codes. The main part is written in Fortran.

PICO (Parameters for the Impatient COsmologist) Hiranya Peiris[‡]

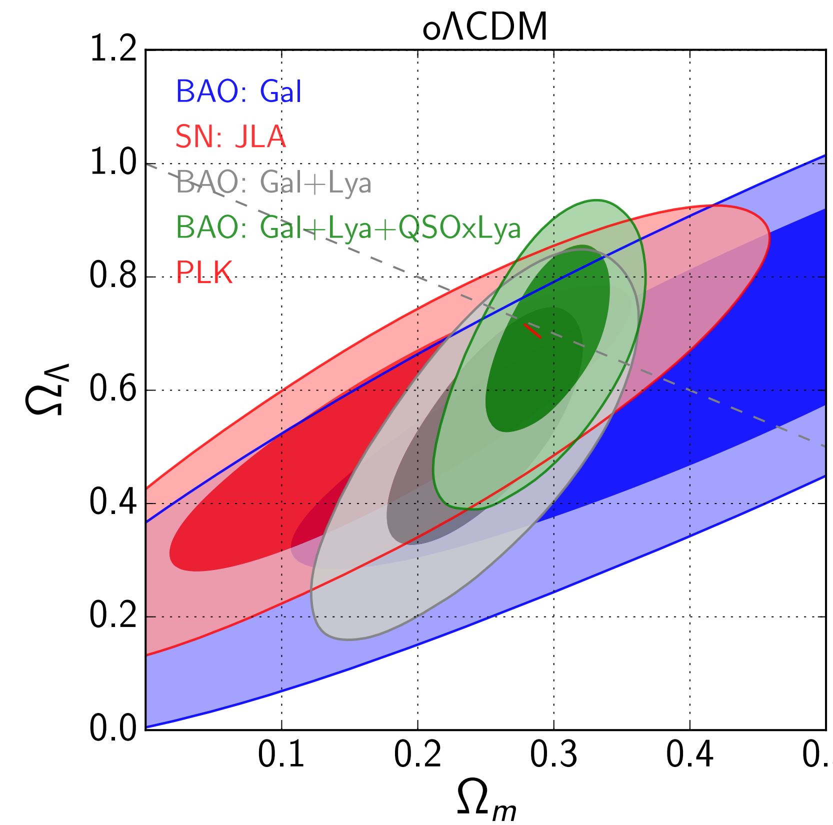
cosmoabc: Likelihood-free inference via Population Monte Carlo Approximate Bayesian Computation

E. E. O. Ishida[1], S. D. P. Vitenti[2], M. Penna-Lima[3,4],

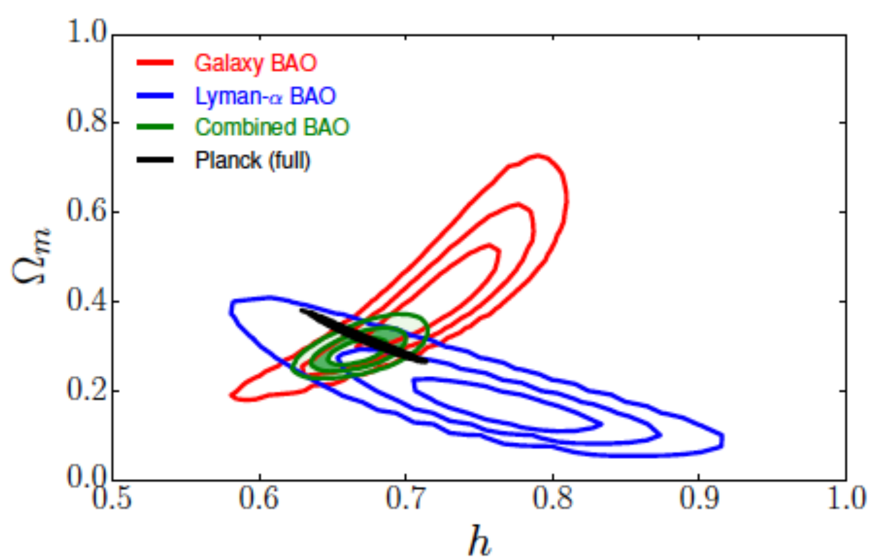
SimpleMC



DR12- Lyman-a



DR12 - arXiv:1607.03155



DR11

To perform the analysis we built a simple and fast MCMC code: **Simple MC**

with A. Slosar

JAVazquez

Posteriors

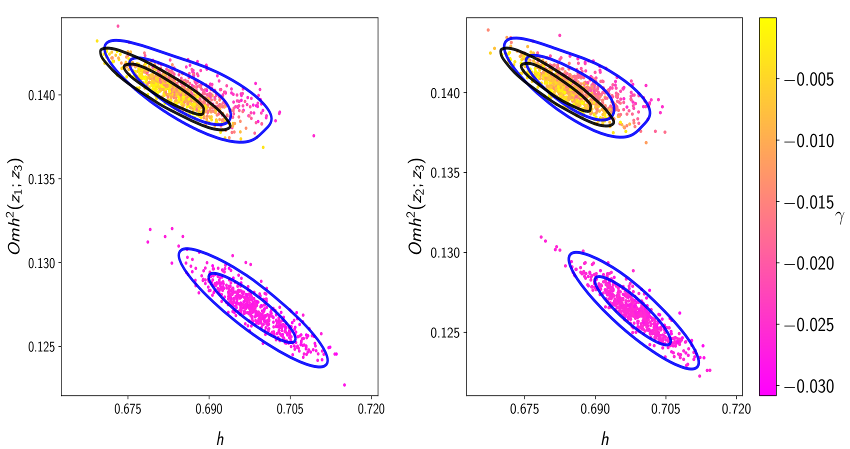
Özgür Akarsu,[1,*] John D. Barrow,[2,†]
Charles V. R. Board,[2,‡] and J. Alberto Vazquez[3,§]

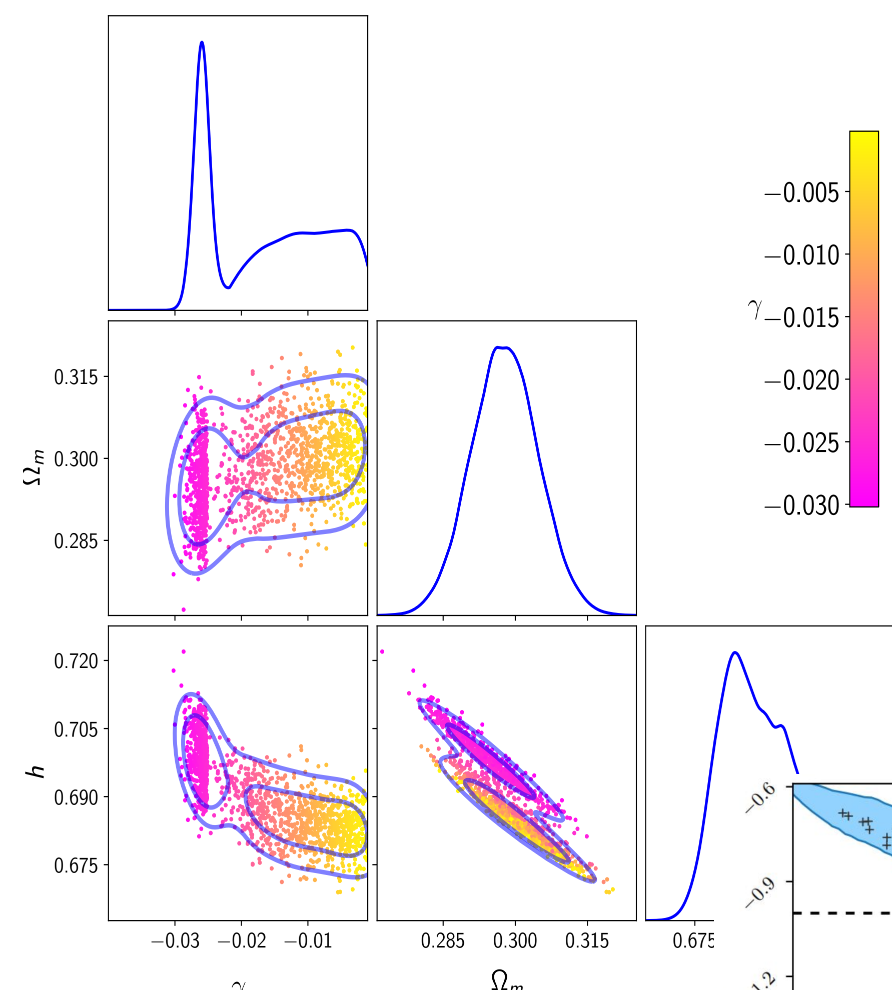
GRADUATED INFLATIONARY UNIVERSES

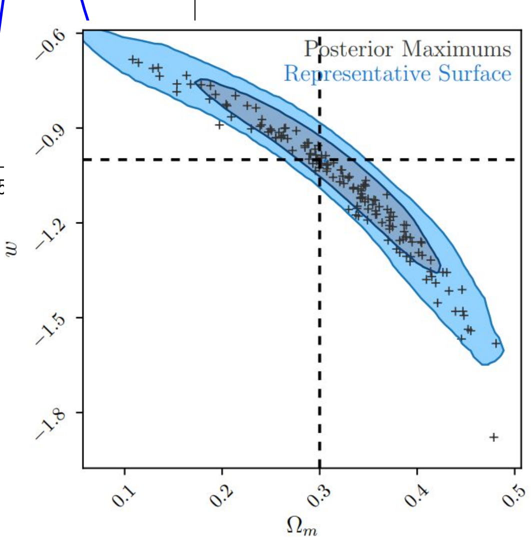
John D. BARROW

$$\rho + p = \gamma \rho^{\lambda}, \quad \gamma \neq 0, \lambda \text{ constants.}$$

$$w = -1 + \frac{\gamma}{1 + 3\gamma(\lambda - 1)\ln a}.$$







III. MCMC SAMPLERS

Gibbs sampling.

Metropolis Coupled Markov Chain Monte Carlo (MC^3).

$$P(\theta, T|D, H) \propto L(\theta, D)^{1/T} P(\theta, H),$$

a. Simulated tempering

Affine Invariant MCMC Ensemble Sampler.

$$R(AX(t) + b, \psi(t), p_{A,b}) = AR(X(t), \psi(t), p) + b.$$

Hamiltoninan Monte Carlo (see e.g. [31, 32]) or the Adaptative Metropolis-Hastings (AMH)

EMCEE

B. Approximation methods

1. Laplace approximation **2. Variational methods** (VIBES) **C. Quadrature** GSL, and NAG

CosmoSis

Classic:

- metropolis sampler Classic Metropolis-Hastings sampling
- importance sampler Importance sampling
- fisher sampler Fisher Matrices

Max-Like:

- maxlike sampler Find the maximum likelihood using various methods in scipy
- gridmax sampler Naive grid maximum-posterior
- minuit sampler MPI-aware maxlike sampler from the ROOT package.

Ensemble:

- emcee sampler Ensemble walker sampling
- kombine sampler Clustered KDE
- multinest sampler Nested sampling
- pmc sampler Adaptive Importance Sampling

Grid:

- grid sampler Regular posterior grid
- snake sampler Intelligent Grid exploration
- star sampler [In development] Single-component variation sub-grid

GM Sampler

with A. Slosar

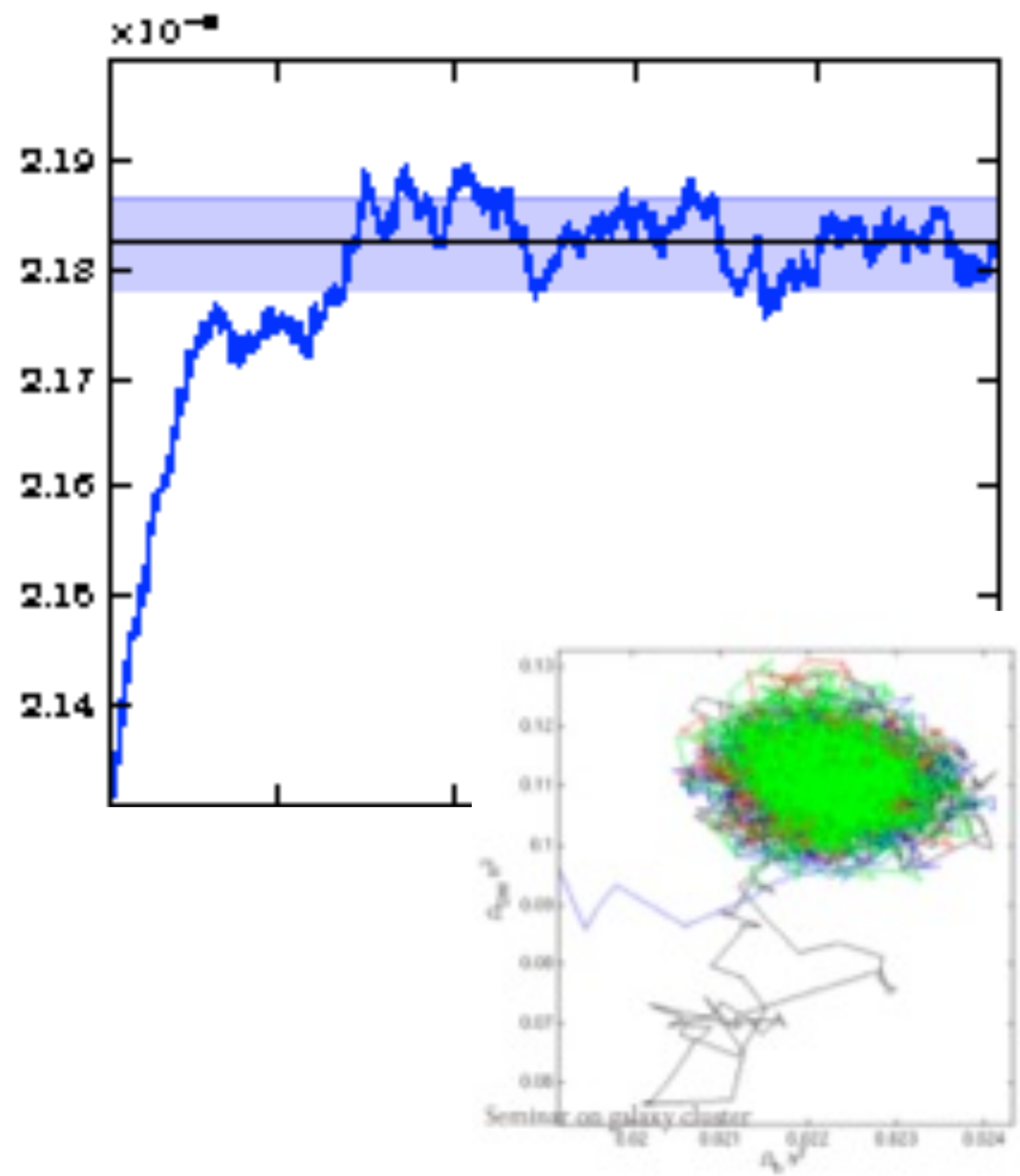
https://github.com/ja-vazquez/GM_Sampler

Gaussian Embedding – massively parallelizable sampling algorithm

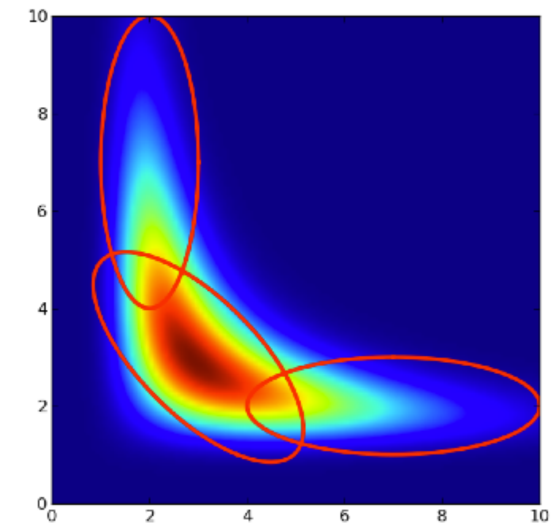
MCMC is an algorithm that walks around the likelihood and produces samples

- Scales perfectly for small number of chains, but not on modern architectures with 1000s of cores one always needs to throw away some ~thousands steps, because of the **burn in period**.

  (the initial state is “forgotten")



Game Sampler



- Populate a lists of Gaussians with a single Gaussian centered at a chosen point with a suitable covariance

- Take N samples from the most recently added Gaussian

- Calculate importance sample weights $w_i = A \frac{L_t(\mathbf{x}_i)}{\sum_{j=1...M} G_j(\mathbf{x}_i - \mu_j, \mathbf{C}_j)}$

  If Ls is sampling the target distribution well the weights will be around unity, << 1 we are oversampling the parameters space and >> 1 where we are undersampling parameter space

- Add a new Gaussian at the position of the largest importance weight

- Repeat step 2, until convergence

  - The effective number of samples $N_{\rm eff} = \frac{\sum w_i}{max(w_i)}$

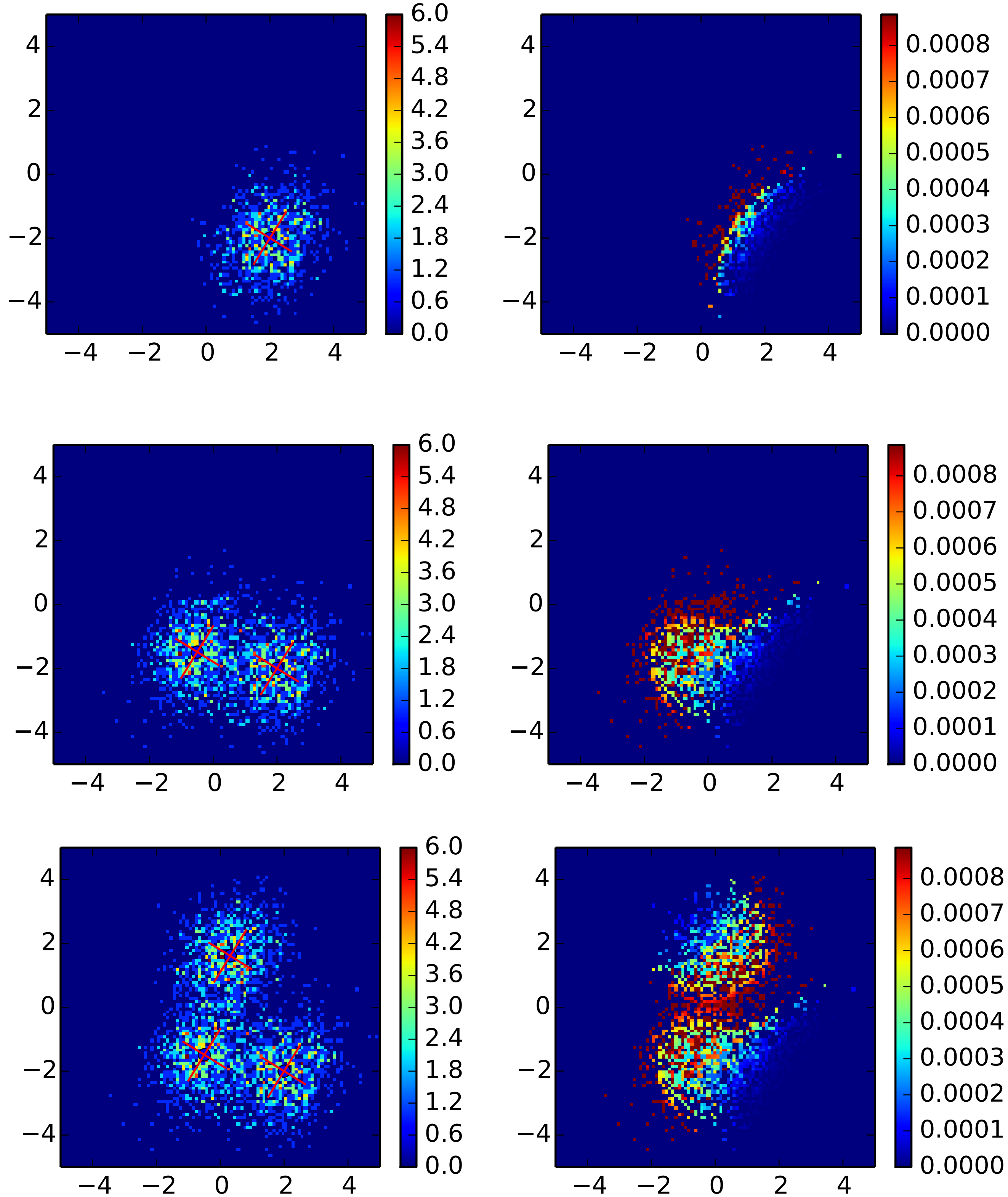
  Demanding large Neff shown to be robust. If part of posterior is not covered, weights will blow up in that region, reducing then the number of effective samples

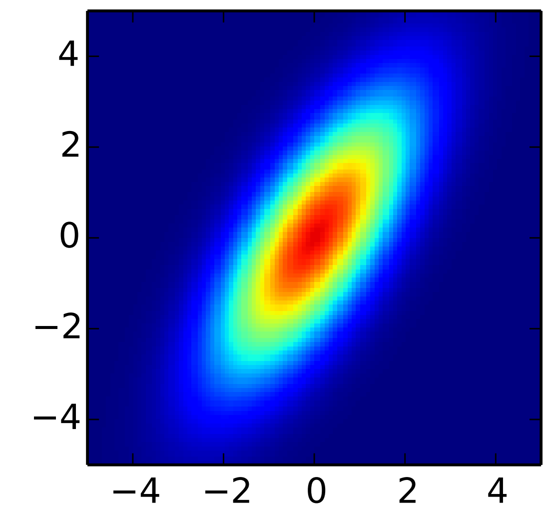
Test 1: Gaussian

1-G

2-G

3-G





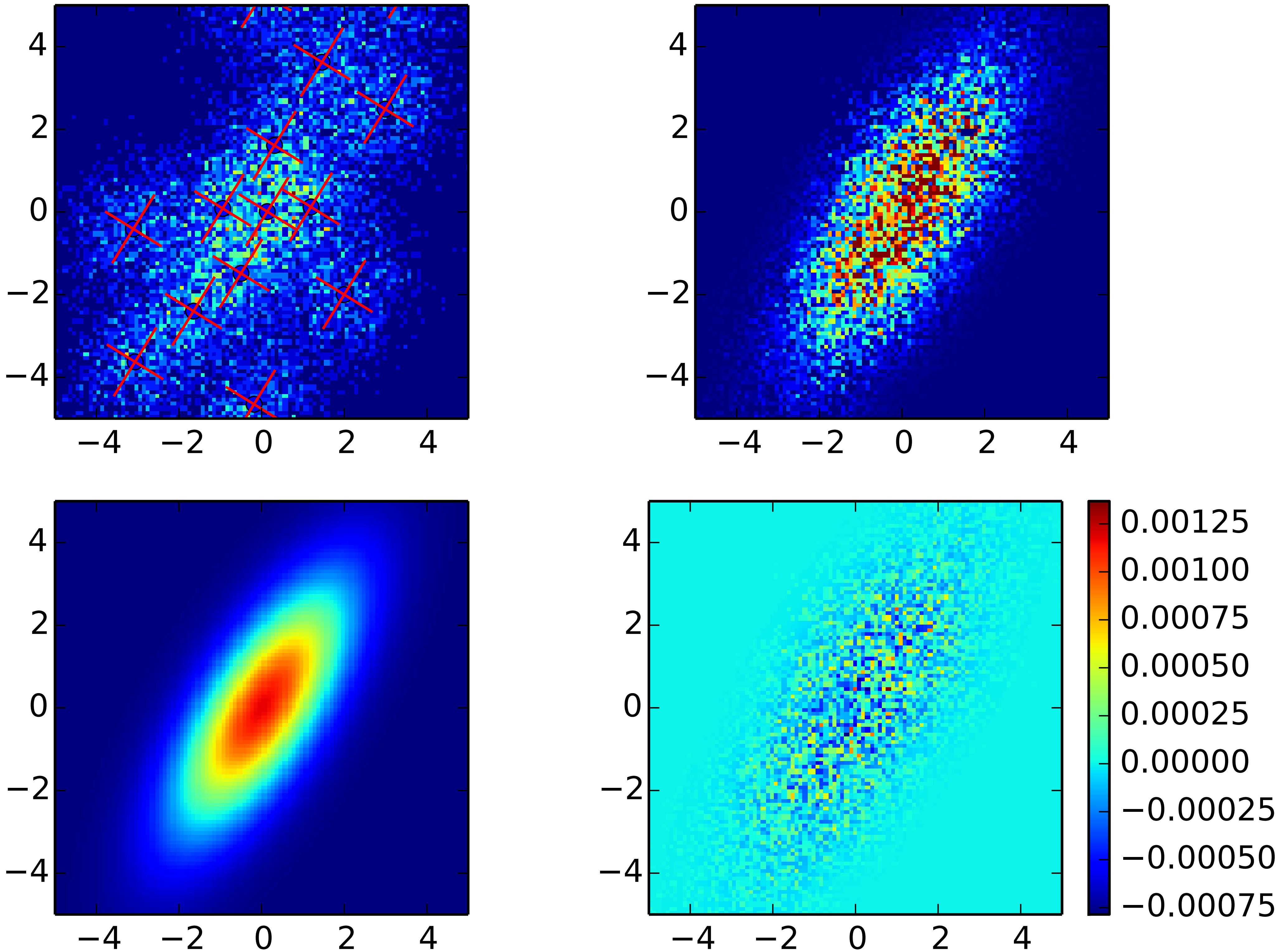
Target distribution

Source Gaussians (indicated by crosses) and the density of unweighted samples.

Density of weighted samples that sample the target distribution.

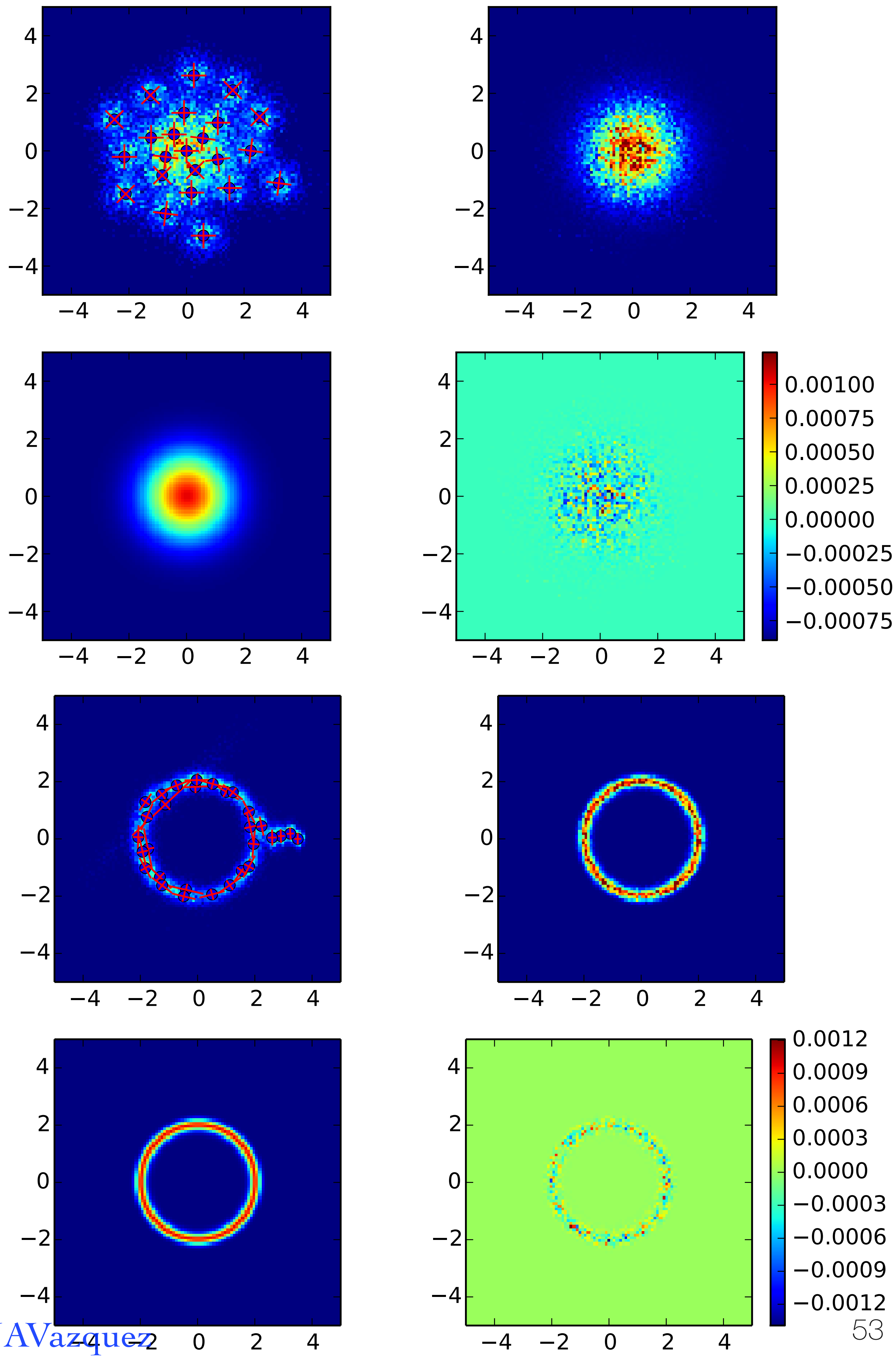
$$w_i = A \frac{L_t(\mathbf{x}_i)}{\sum_{j=1\ldots M} G_j(\mathbf{x}_i - \mu_j, \mathbf{C}_j)}$$

Test 1: Gaussian

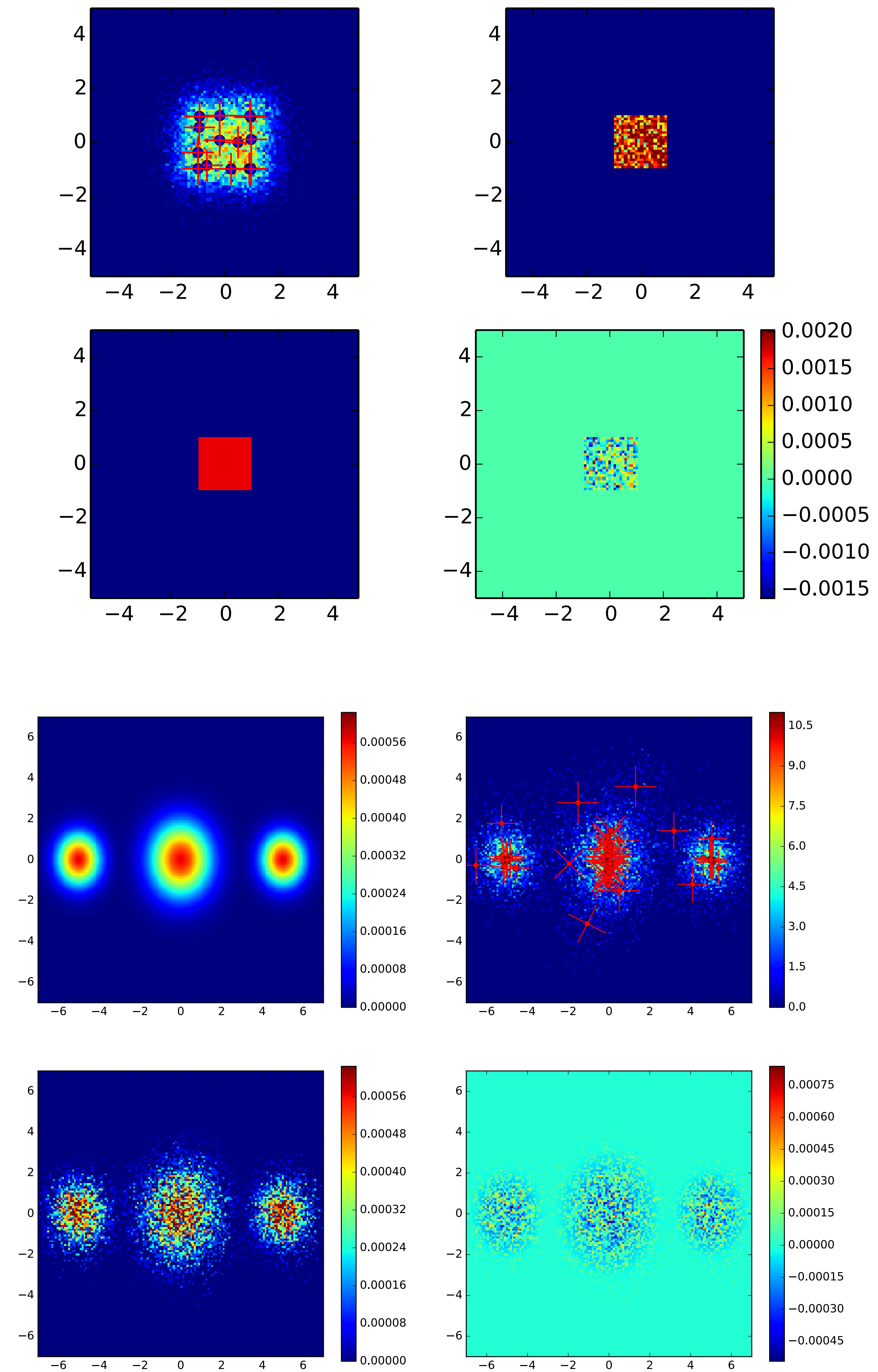


$$w_i = A \frac{L_{\rm t}(\mathbf{x}_i)}{\sum_{j=1\ldots M} G_j(\mathbf{x}_i - \mu_j, \mathbf{C}_j)}$$

after 15 Gaussians



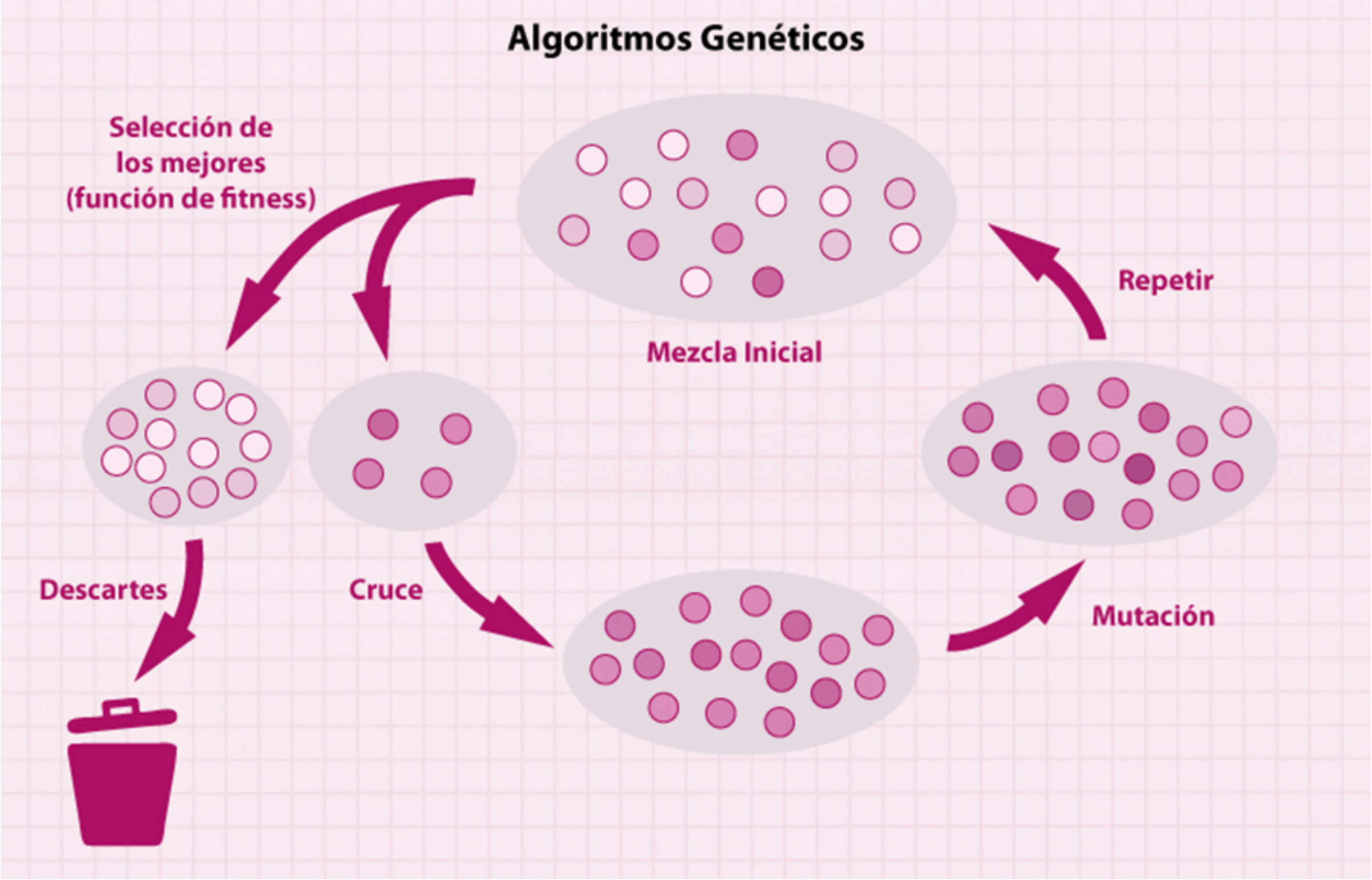
−4 −2 0 2 4
0.00100
0.00075
0.00050
0.00025
0.00000
−0.00025
−0.00050
−0.00075
0.0012
0.0009
0.0006
0.0003
0.0000
−0.0003
−0.0006
−0.0009
−0.0012



−4 −2 0 2 4
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−6 −4 −2 0 2 4 6
0.00056
0.00048
0.00040
0.00032
0.00024
0.00016
0.00008
0.00000
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0
0.00075
0.00060
0.00045
0.00030
0.00015
0.00000
−0.00015
−0.00030
−0.00045

$P(\theta|D, H)$

- Algoritmos evolutivos.



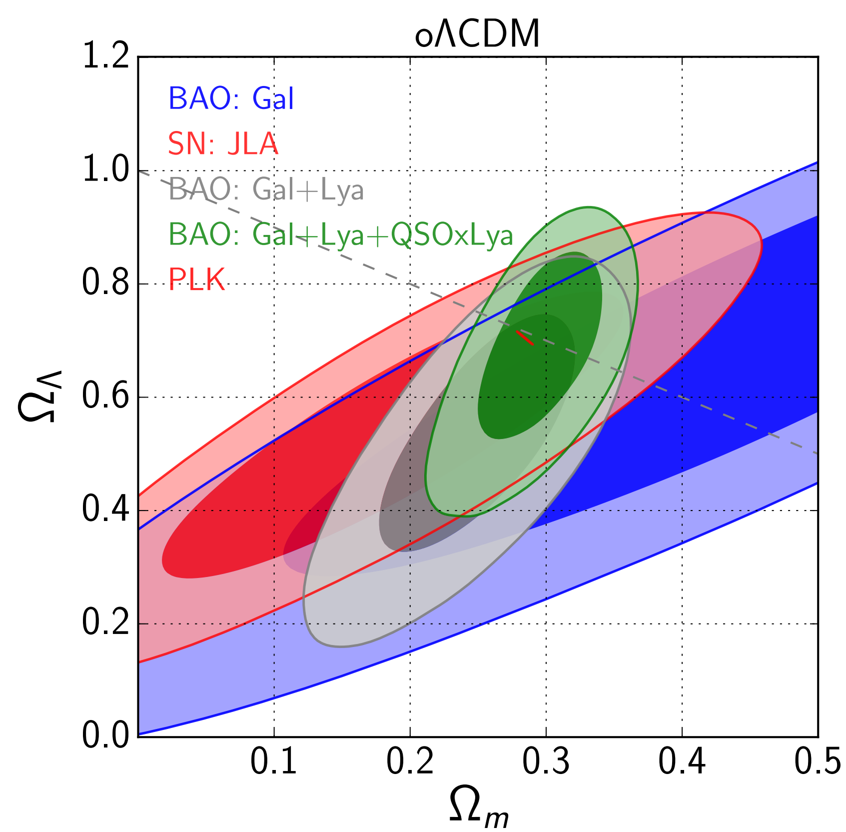
- Evolution Strategies for Cosmology: A Comparison of Nested Sampling Methods. M. Axiak, et. al.: arXiv:1101.0717v2 [astro-ph.CO]

... and finally $P(\mathbf{\Theta}|\mathbf{D}, M)$

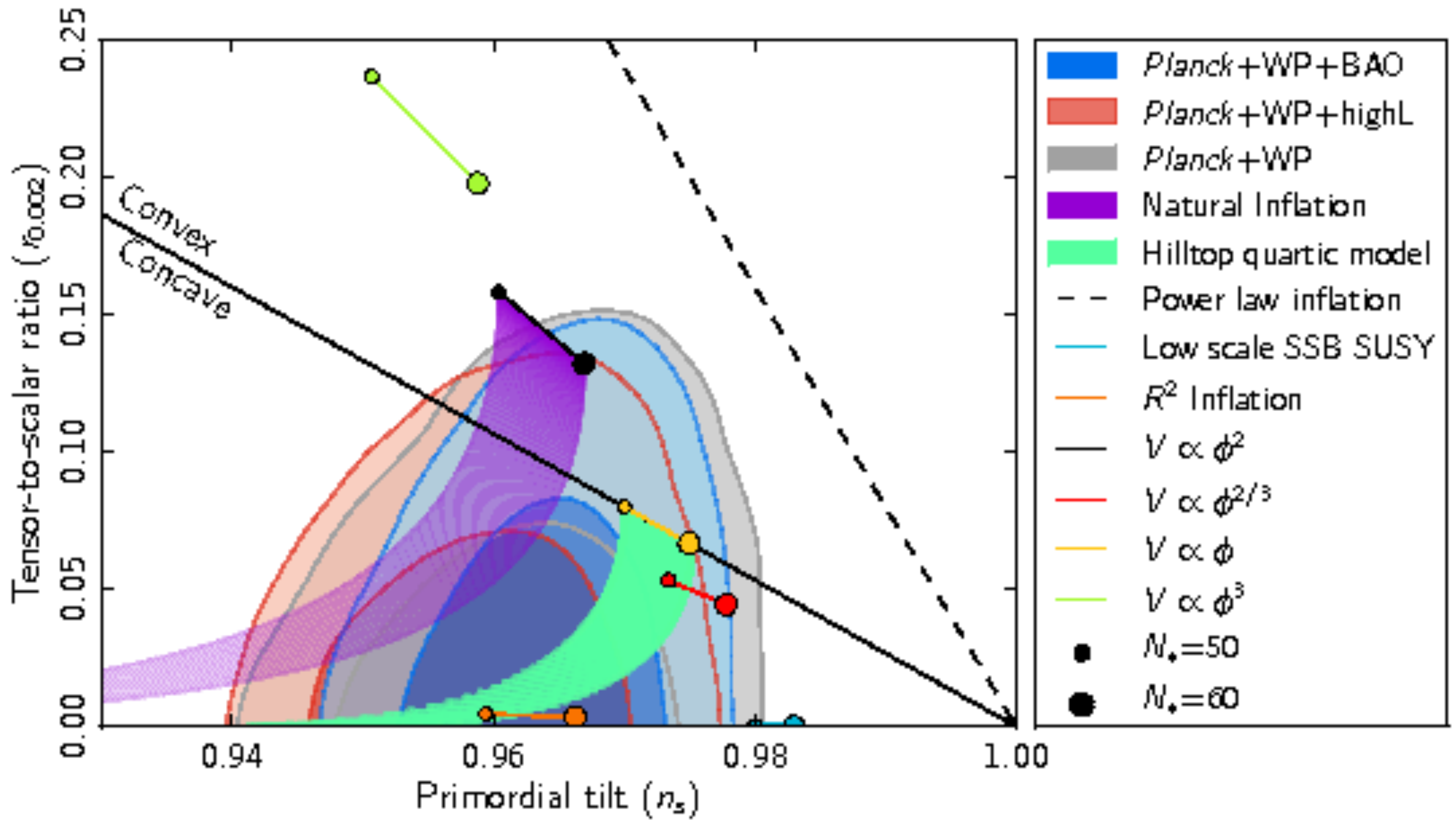
* Confidence regions are regions R in model space such that $\int_R \mathscr{P}(\theta|D)d\theta = p$

where *p* is the confidence level we request (e.g., 68.3% 95.4% etc.)

* Mean values $\hat{\theta} = \int d\theta\, \theta\, \mathscr{P}(\theta|D)$



Constraints on inflationary models



When you force your data to fit the constraints of your model

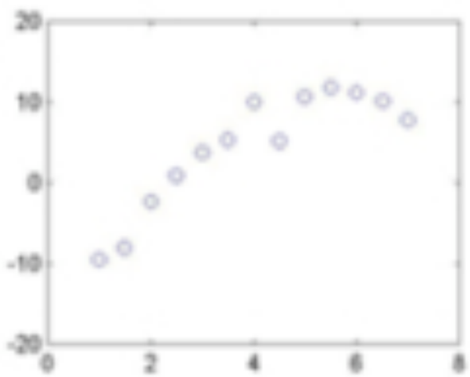
Model Selection

Two different and **competing models** of the Universe may **explain** the data equally well,...

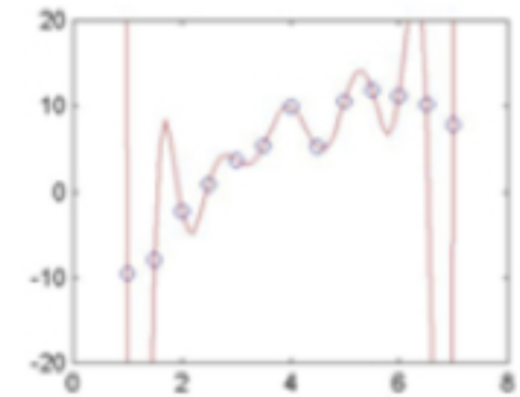
so, **how do we choose between them?**

Answer : Occam's razor (~1328)

"when you have two competing theories that **make the same predictions**, the simpler one is the better."

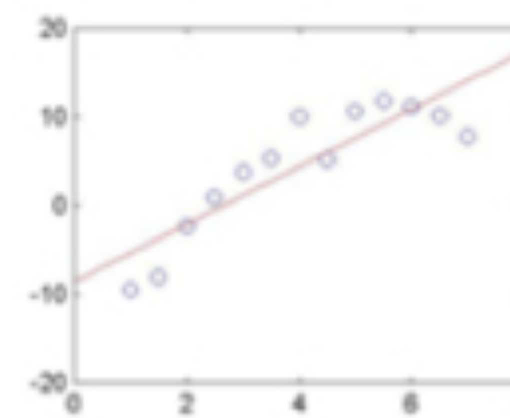


i.e. data



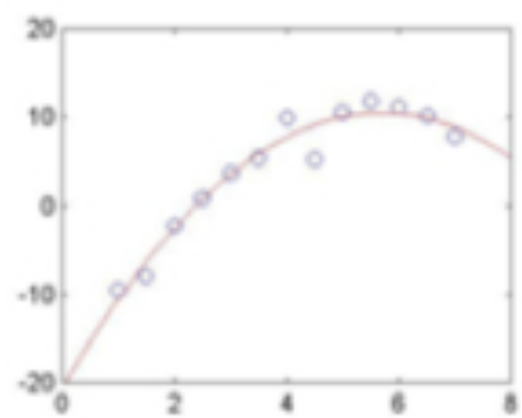
A complex model that explains the data slightly better should be penalised by the inclusion of extra information

reflects a lack of predictability



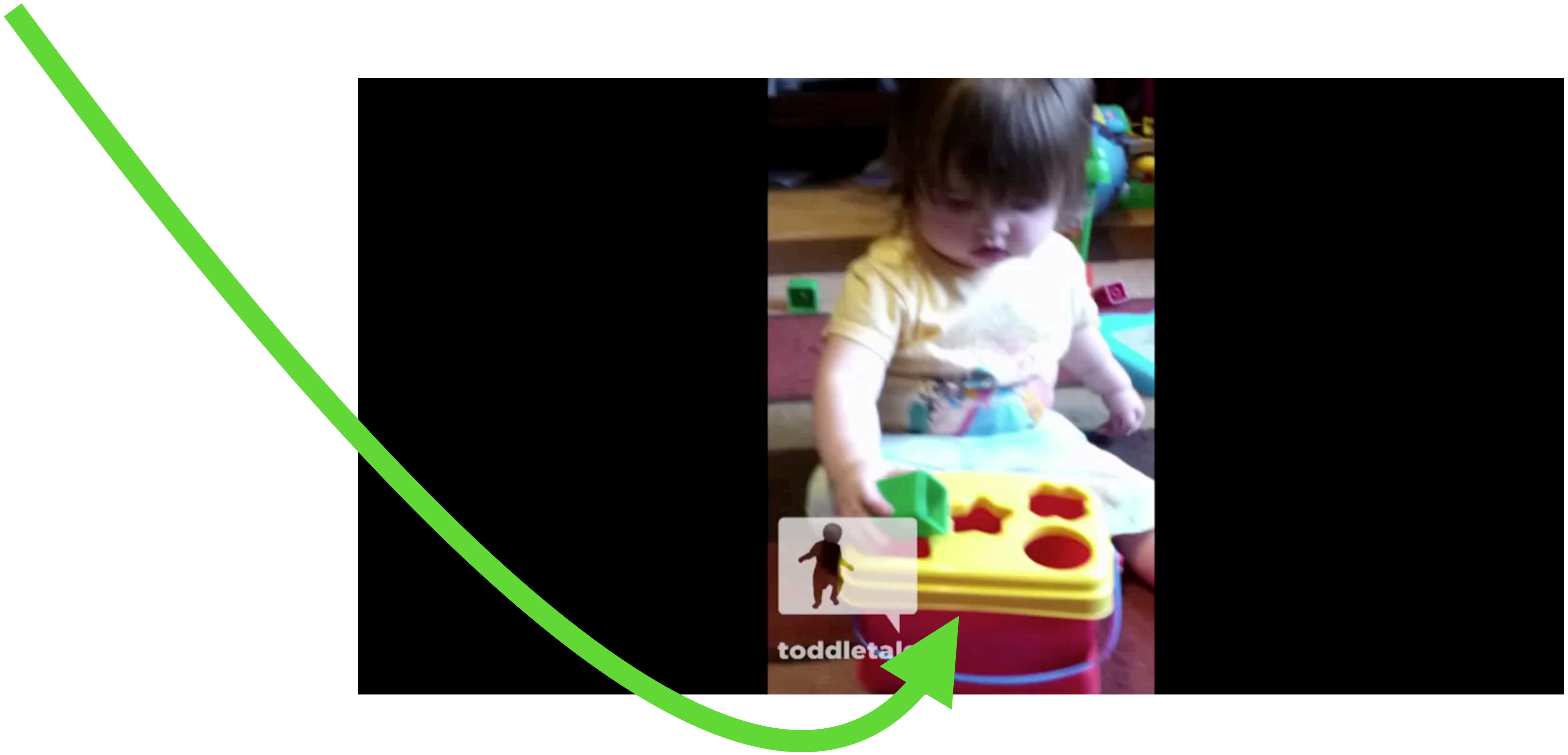
If the model is too simple , it might not fit data equally well,

then it can be discarded



"Everything should be made as simple as possible, but not simpler."

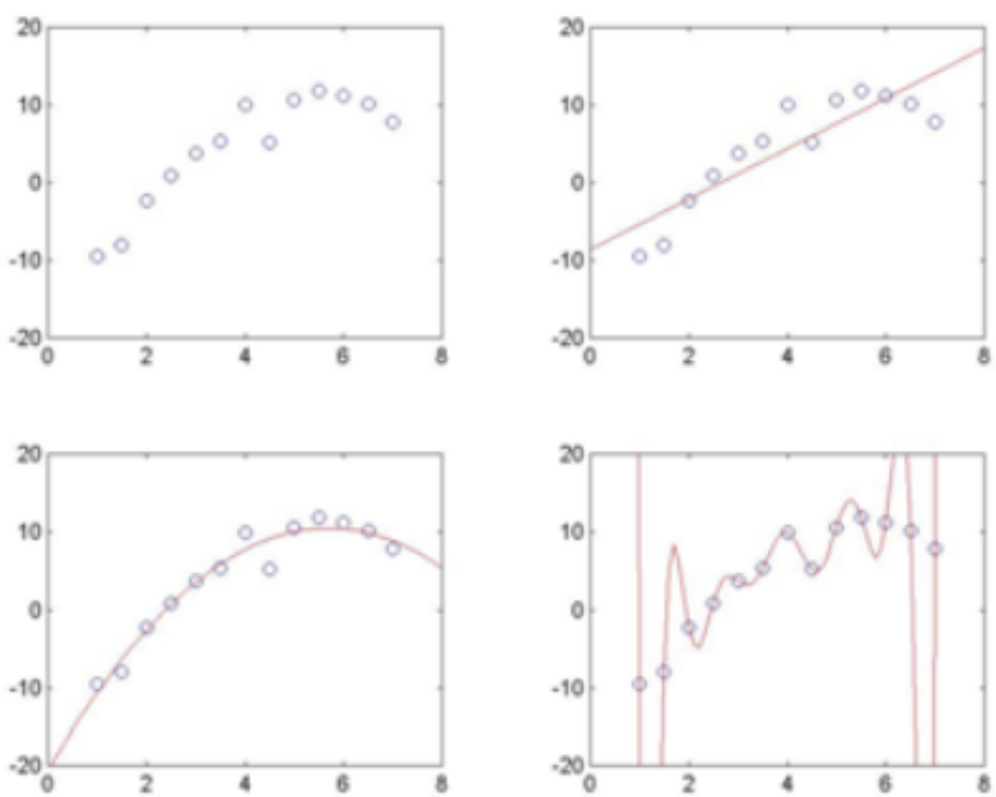
Data



Model Selection

The key quantity to bear in mind → Bayesian evidence $\Pr(\mathbf{D}|M)$.

$$P(M|\mathbf{D}) = \frac{P(\mathbf{D}|M)P(M)}{P(\mathbf{D})}.$$



$$AIC \equiv -2\ln\mathcal{L}_{max} + 2k,$$

$$BIC \equiv -2\ln\mathcal{L}_{max} + k\ln N,$$

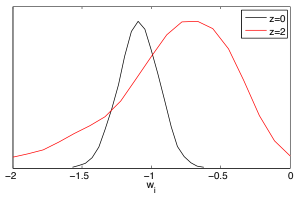
goodness of fit

~Occam's factor

Model Comparison

remember?

$$P(\mathbf{\Theta}|\mathbf{D}, M) = \frac{P(\mathbf{D}|\mathbf{\Theta}, M)\ P(\mathbf{\Theta}|M)}{P(\mathbf{D}|M)}.$$



same type of analysis, but now at the level of models rather than parameters

$$P(M|\mathbf{D}) = \frac{P(\mathbf{D}|M)P(M)}{P(\mathbf{D})}.$$

the probability of the model given the data

exactly what we are looking for model selection !!

$$P(\mathbf{\Theta}|\mathbf{D}, M) = \frac{P(\mathbf{D}|\mathbf{\Theta}, M)\; P(\mathbf{\Theta}|M)}{P(\mathbf{D}|M)}.$$

$$\mathrm{P}(\mathbf{\Theta}|\mathrm{M}) \equiv \pi$$
$$\mathrm{P}(\mathbf{D}|\mathbf{\Theta},\mathrm{M}) \equiv \mathcal{L}.$$
$$\mathrm{P}(\mathbf{D}|\mathrm{M}) \equiv \mathcal{Z},$$

The Bayesian evidence is simply the normalisation constant

$$\mathcal{Z} = \int \mathcal{L}(D|\Theta)\pi(\Theta)d^N\Theta.$$

It is the average likelihood weighted by the prior for a specific model choice

$$Evidence = \int (Likelihood \times Prior) d^N\Theta.$$

A model containing a **higher likelihood** along with a **smaller prior** volume will have a higher evidence and vice versa

It does provide a natural mechanism to balance the complexity of models and then, elegantly incorporates Occam's razor

Comparing two models

$$P(M|\mathbf{D}) = \frac{P(\mathbf{D}|M)P(M)}{P(\mathbf{D})}.$$

$$\frac{P(M_i|\mathbf{D})}{P(M_j|\mathbf{D})} = \frac{\mathcal{Z}_i}{\mathcal{Z}_j}\frac{P(M_i)}{P(M_j)},$$

the relative probability of how well model i may fit the data when is compared to model j

principle of indifference

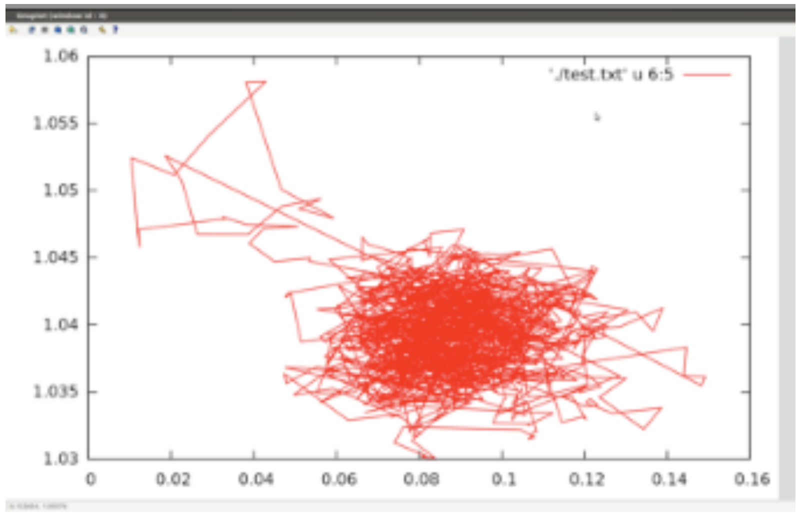
Bayes factor

$$\mathcal{B}_{i,j} = \ln \frac{\mathcal{Z}_i}{\mathcal{Z}_j}.$$

$\lvert\mathcal{B}_{i,j}\rvert$	Odds	Probability	Strength
< 1.0	$< 3:1$	< 0.750	Inconclusive
1.0-2.5	$\sim 12:1$	0.923	Significant
2.5-5.0	$\sim 150:1$	0.993	Strong
> 5.0	$> 150:1$	> 0.993	Decisive

The calculation of the integral

$$\mathcal{Z} = \int \mathcal{L}(D|\Theta)\pi(\Theta)d^N\Theta.$$



is a very computationally demanding process, since it requires a multidimensional integration over the likelihood and prior

Algorithms such as simulating annealing or thermodynamic integration, required around 10^7 likelihood evaluations

$$Z(\beta) = \int d\boldsymbol{\theta}\, p(\mathbf{d}|\boldsymbol{\theta}, M, I)^{\beta}\, p(\boldsymbol{\theta}|M, I)$$

When your MCMC converges

Nested Sampling

Skilling: nested sampling algorithm, ten times more efficient

$$\mathcal{Z} = \int \mathcal{L}(D|\Theta)\pi(\Theta)d^N\Theta.$$

$dX = \pi(\boldsymbol{\Theta})d^D\boldsymbol{\Theta}$, so that

$$X(\lambda) = \int_{\mathcal{L}(\boldsymbol{\Theta})>\lambda} \pi(\boldsymbol{\Theta})d^D\boldsymbol{\Theta},$$

$$\mathcal{Z} = \int_0^1 \mathcal{L}(X)dX,$$

iso-likelihood contour $\mathcal{L}(\boldsymbol{\Theta}) = \lambda$.

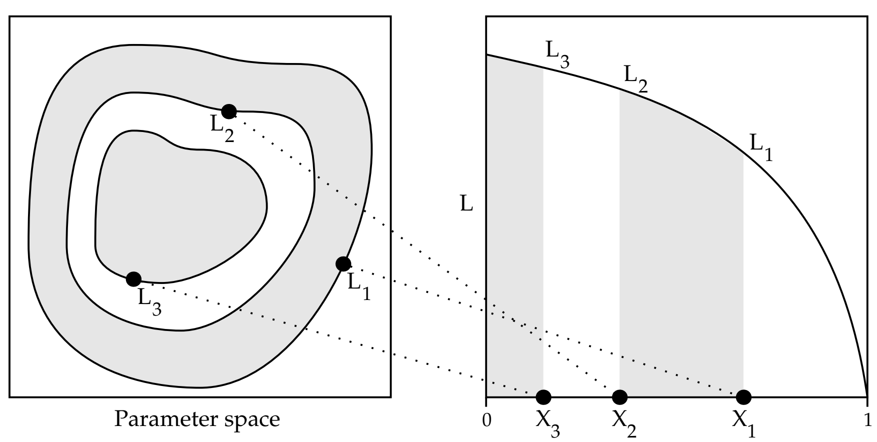
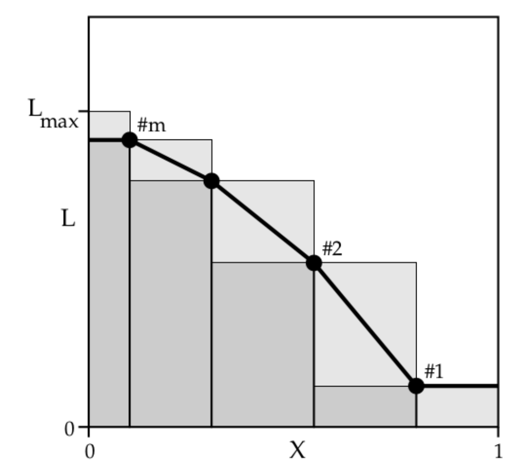
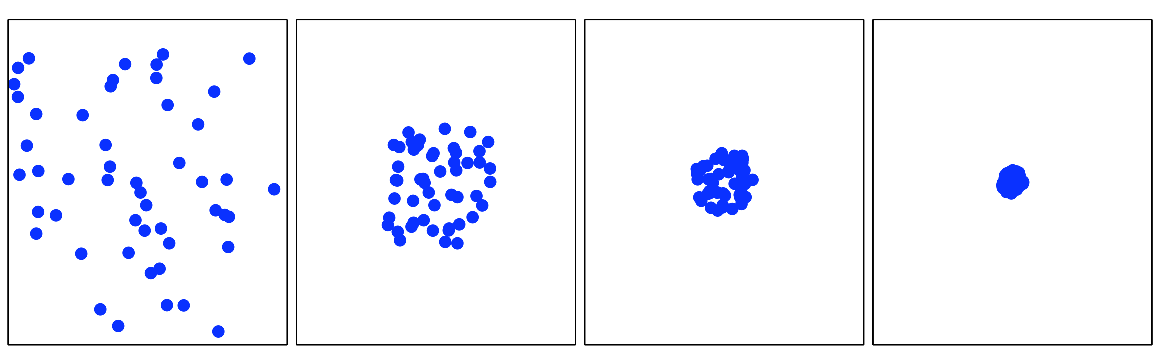


Figure 3: Nested likelihood contours are sorted to enclosed prior mass X.





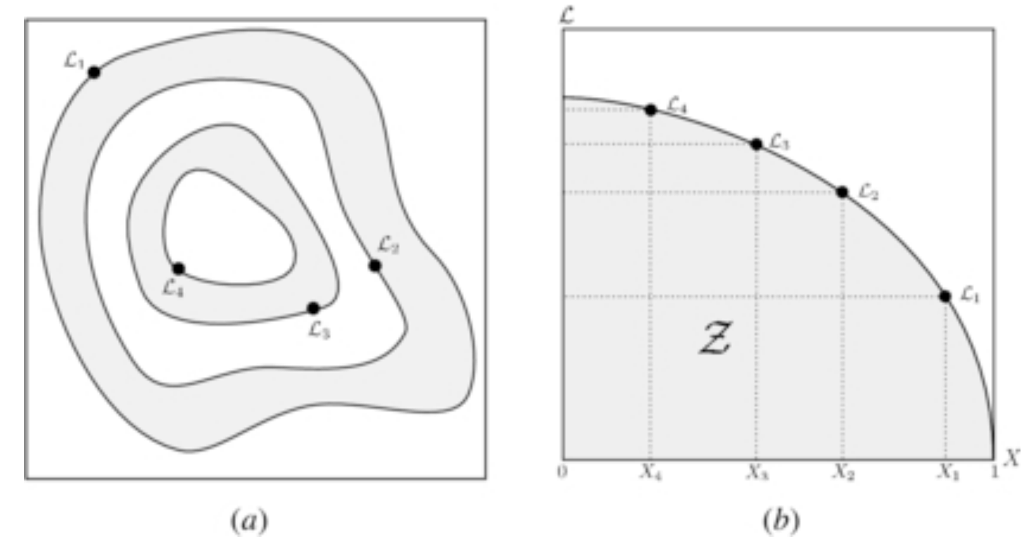
~48,000 likelihood evaluations

CosmoNest

Mukherjee, Parkinson & Liddle, astro-ph/0508461

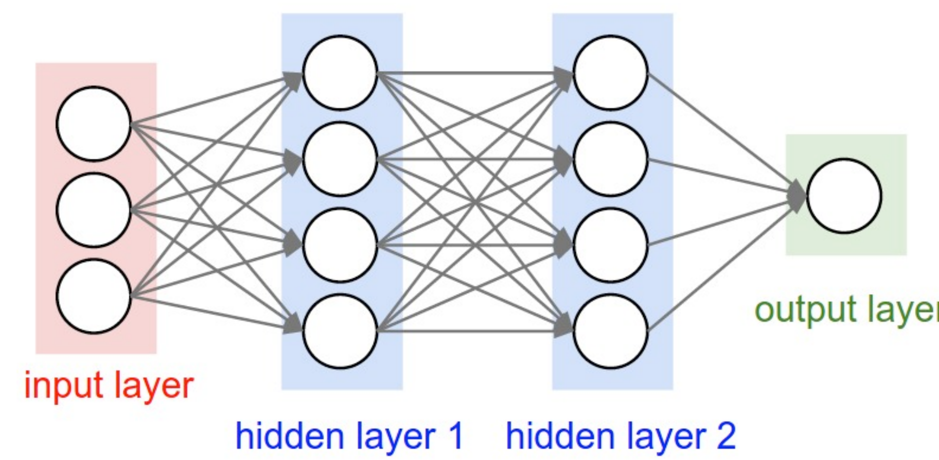
MultiNest

MultiNest is a **Bayesian inference tool** which calculates the evidence and explores the parameter space



CosmoNet Software

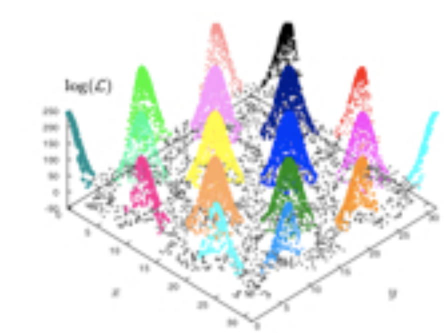
CosmoNet is an algorithm for accelerating cosmological parameter estimation.
It uses multilayer perceptron neural networks



BAMBI

(Blind Accelerated Multimodal Bayesian Inference)

Nest + Nets



Una Aplicación de las Redes Neuronales Artificiales en la Cosmología

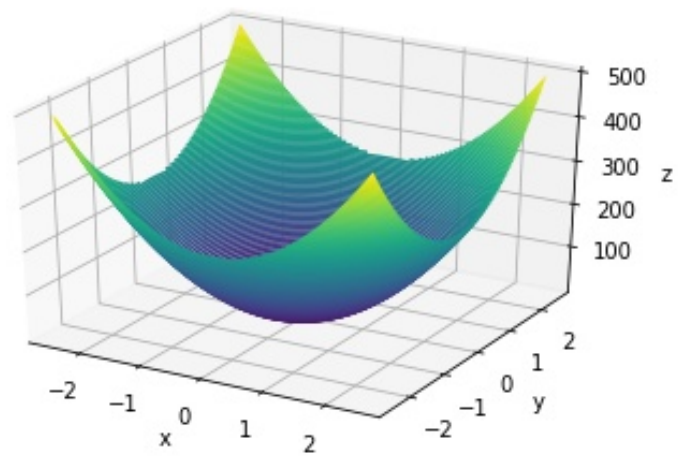
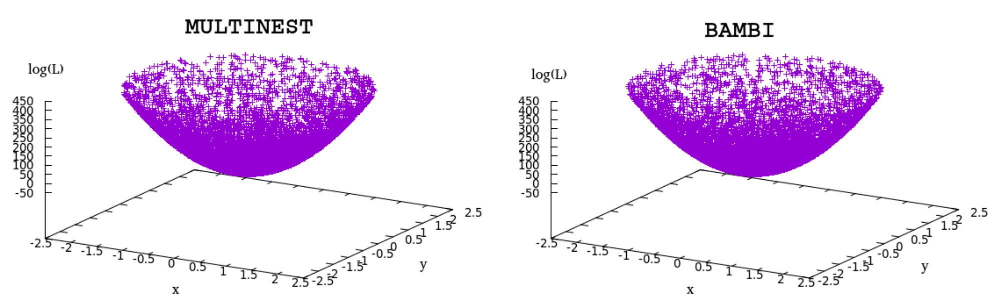


Figura 5: Paraboloide (función objetivo).



No. Eval.	Tiempo MULTINEST	Tiempo BAMBI
1 000	0.0482 s	0.495 s
10 000	0.416 s	0.771 s
100 000	4.318 s	3.719 s
1 000 000	46.187 s	32.716 s

	MULTINEST	BAMBI
Log(Evidencia)	$-0{,}782 \pm -0{,}06$	$-0{,}783 \pm -0{,}06$

Tabla II: Cálculos finales para la *Evidencia*

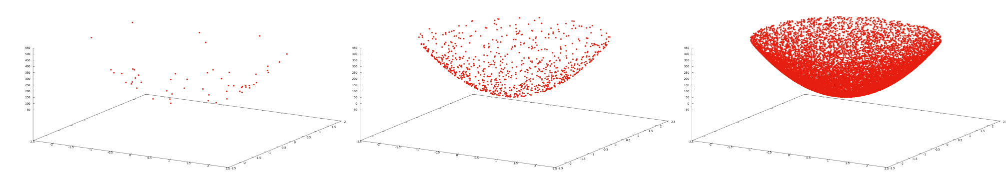


Figura 7: Tres diferentes etapas del muestreo con BAMBI.

Revista de Divulgación de la Sociedad Mexicana de Inteligencia Artificial
ISSN 2007-0691

Isidro Gómez Vargas,[1, a] Ricardo Medel Esquivel,[1, b] Ricardo García Salcedo,[1] and J. Alberto Vázquez[2]

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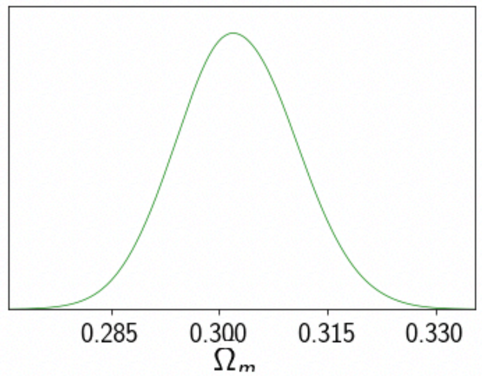
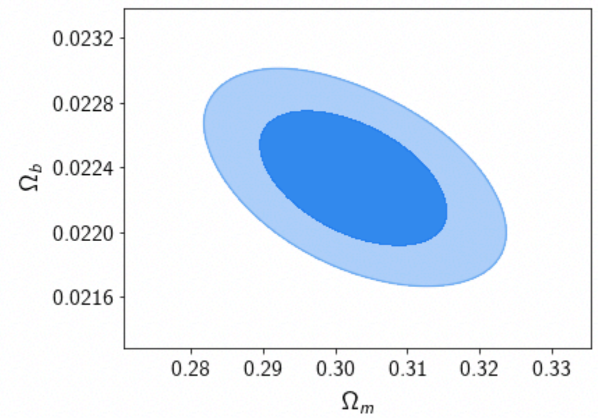
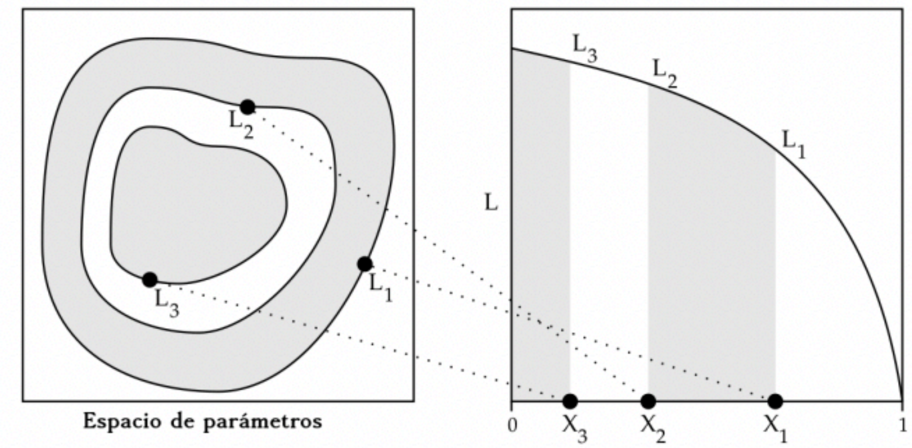
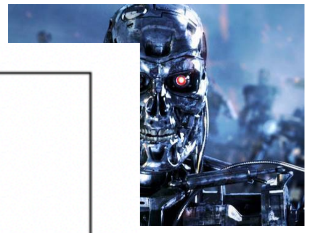
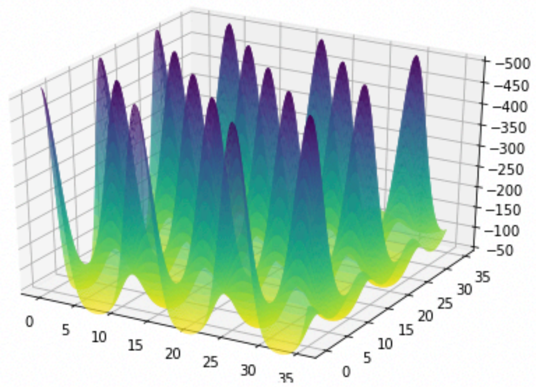
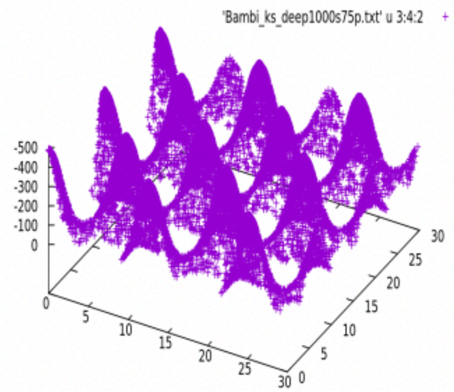


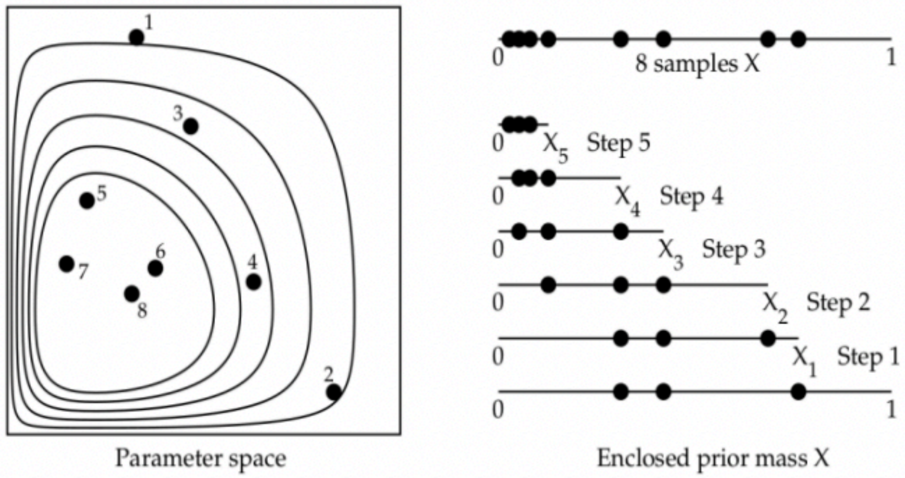


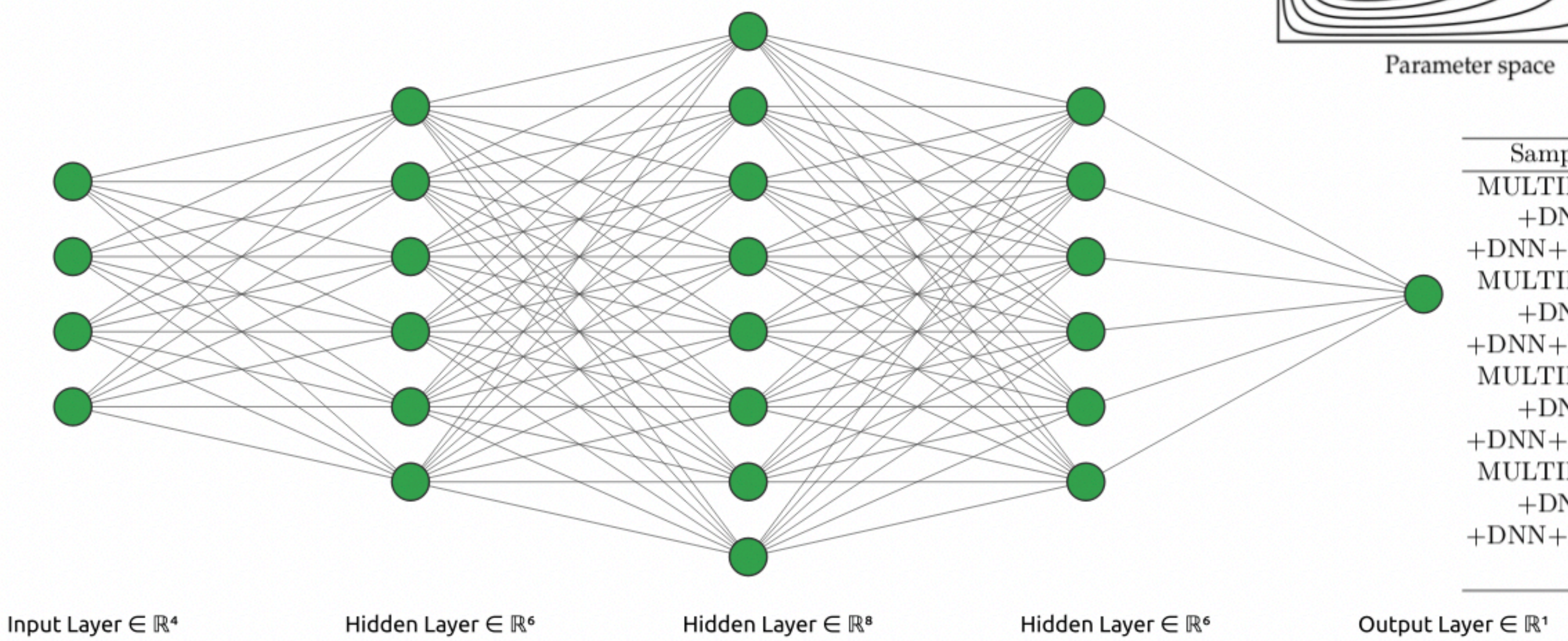












Sampler	steps	log(Evidence)	time (minutes)
MULTINEST	5 000	235.887 ± 0.035	1.12
+DNN	5 000	235.798 ± 0.035	1.93
+DNN+BN+D	5 000	235.809 ± 0.035	1.89
MULTINEST	10 000	235.745 ± 0.025	4.48
+DNN	10 000	235.865 ± 0.025	6.84
+DNN+BN+D	10 000	235.826 ± 0.025	6.77
MULTINEST	50 000	235.835 ± 0.011	132.70
+DNN	50 000	235.855 ± 0.011	172.907
+DNN+BN+D	50 000	235.835 ± 0.011	166.28
MULTINEST	100 000	235.840 ± 0.008	788.62
+DNN	100 000	235.854 ± 0.008	620.91
+DNN+BN+D	100 000	235.859 ± 0.0078	618.029

Artificial Neural Networks as optimizers of Bayesian evidence calculation

Isidro Gómez-Vargas[1a], Ricardo Medel Esquivel[1] Ricardo García Salcedo[1], and J. Alberto Vázquez[2]

The Bayesian evidence

A. MCEVIDENCE

B. NESTLE

NESTLE was developed by Kyle Barbary [11] and includes the single nested sampling [4] and the multimodal nested sampling [7].

C. CPNEST

CPNEST is an algorithm of Parallel Nested Sampling developed by John Veitch [12].

D. Dynesty

Dynamic Nested Sampling [13, 14].

E. DELFI

Density-estimation likelihood-free inference [15] is based on the previous works of the Refs [16, 17].

The Bayesian evidence

Primordial power spectrum

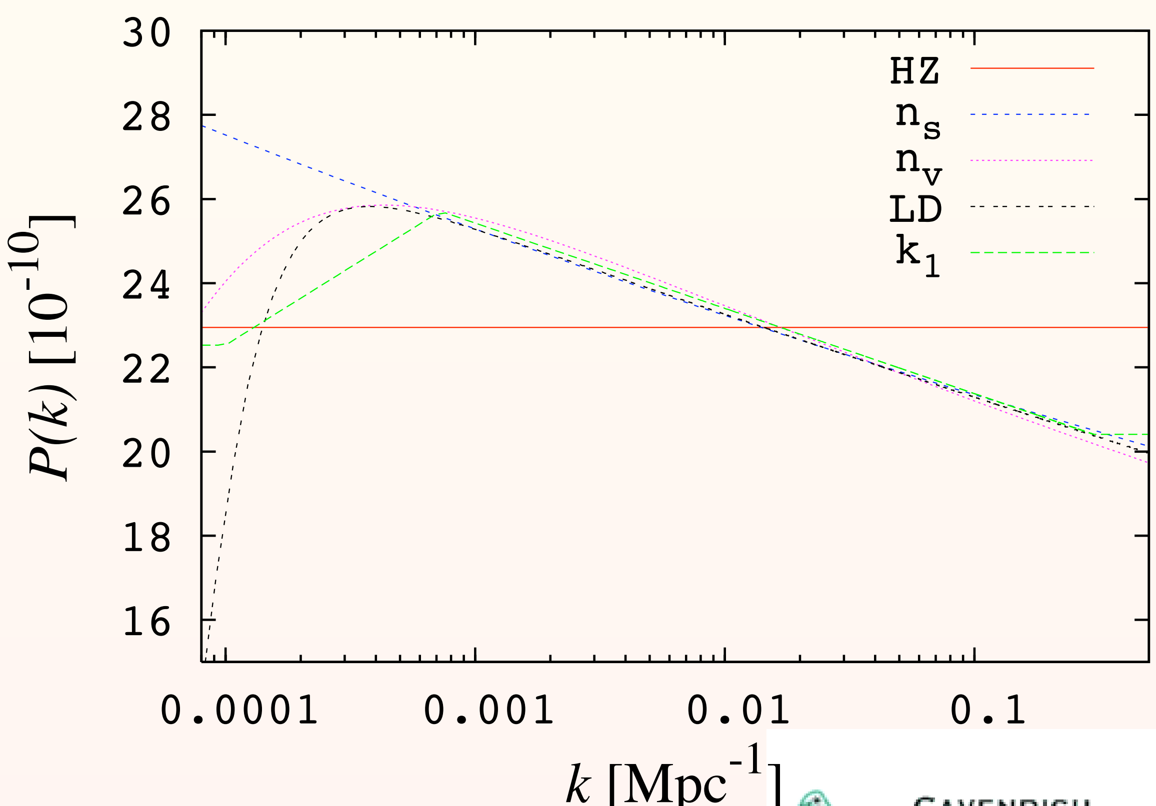
PR(k) is the **power spectrum of the initial curvature perturbations**

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\, dn_s/d\ln k \, \ln(k/k_*) + \frac{1}{6}\frac{d^2 n_s}{d\ln k^2}(\ln(k/k_*))^2 + \ldots},$$

The spectrum is a featureless power law with scalar spectral index n_S.

scale-invariant?

Model	N_{par}	χ^2_{min}	Bayes factor
~~HZ~~	~~8~~	~~0.0~~	~~$+0.0 \pm 0.3$~~
n_s	9	-8.6	$+3.3 \pm 0.3$
LD	10	-9.4	$+4.9 \pm 0.3$
k_1	11	-9.1	$+4.3 \pm 0.3$



CAVENDISH ASTROPHYSICS

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Model comparison

Competing model	$\Delta N_{\rm par}$	$\ln B$	Ref	Data	Outcome
Initial conditions					
Isocurvature modes					
CDM isocurvature	+1	-7.6	[58]	WMAP3+, LSS	Strong evidence for adiabaticity
+ arbitrary correlations	+4	-1.0	[46]	WMAP1+, LSS, SN Ia	Undecided
Neutrino entropy	+1	$[-2.5, -6.5]^p$	[60]	WMAP3+, LSS	Moderate to strong evidence for adiabaticity
+ arbitrary correlations	+4	-1.0	[46]	WMAP1+, LSS, SN Ia	Undecided
Neutrino velocity	+1	$[-2.5, -6.5]^p$	[60]	WMAP3+, LSS	Moderate to strong evidence for adiabaticity
+ arbitrary correlations	+4	-1.0	[46]	WMAP1+, LSS, SN Ia	Undecided
Primordial power spectrum					
No tilt ($n_s = 1$)	-1	$+0.4$	[47]	WMAP1+, LSS	Undecided
		$[-1.1, -0.6]^p$	[51]	WMAP1+, LSS	Undecided
		-0.7	[58]	WMAP1+, LSS	Undecided
		-0.9	[70]	WMAP1+	Undecided
		$[-0.7, -1.7]^{p,d}$	[185]	WMAP3+	$n_s = 1$ weakly disfavoured
		-2.0	[184]	WMAP3+, LSS	$n_s = 1$ weakly disfavoured
		-2.6	[70]	WMAP3+	$n_s = 1$ moderately disfavoured
		-2.9	[58]	WMAP3+, LSS	$n_s = 1$ moderately disfavoured
		$< -3.9^c$	[65]	WMAP3+, LSS	Moderate evidence at best against $n_s \neq 1$
Running	+1	$[-0.6, 1.0]^{p,d}$	[185]	WMAP3+, LSS	No evidence for running
		$< 0.2^c$	[165]	WMAP3+, LSS	Running not required
Running of running	+2	$< 0.4^c$	[165]	WMAP3+, LSS	Not required
Large scales cut–off	+2	$[1.3, 2.2]^{p,d}$	[185]	WMAP3+, LSS	Weak support for a cut–off
Matter–energy content					
Non–flat Universe	+1	-3.8	[70]	WMAP3+, HST	Flat Universe moderately favoured
		-3.4	[58]	WMAP3+, LSS, HST	Flat Universe moderately favoured
Coupled neutrinos	+1	-0.7	[192]	WMAP3+, LSS	No evidence for non–SM neutrinos
Dark energy sector					
$w(z) = w_{\rm eff} \neq -1$	+1	$[-1.3, -2.7]^p$	[186]	SN Ia	Weak to moderate support for Λ
		-3.0	[50]	SN Ia	Moderate support for Λ
		-1.1	[51]	WMAP1+, LSS, SN Ia	Weak support for Λ
		$[-0.2, -1]^p$	[187]	SN Ia, BAO, WMAP3	Undecided
		$[-1.6, -2.3]^d$	[188]	SN Ia, GRB	Weak support for Λ
$w(z) = w_0 + w_1 z$	+2	$[-1.5, -3.4]^p$	[186]	SN Ia	Weak to moderate support for Λ
		-6.0	[50]	SN Ia	Strong support for Λ
		-1.8	[187]	SN Ia, BAO, WMAP3	Weak support for Λ
$w(z) = w_0 + w_a(1-a)$	+2	-1.1	[187]	SN Ia, BAO, WMAP3	Weak support for Λ
		$[-1.2, -2.6]^d$	[188]	SN Ia, GRB	Weak to moderate support for Λ
Reionization history					
No reionization ($\tau = 0$)	-1	-2.6	[70]	WMAP3+, HST	$\tau \neq 0$ moderately favoured
No reionization and no tilt	-2	-10.3	[70]	WMAP3+, HST	Strongly disfavoured

Since DE is an unknown component,
one is ‘forced’ to either take into account **additional features**

$$w(z) = w_0 + w_1 z/(1+z)$$

$$w(z) = w_0 + w_1 z/(1+z)[1+1/(1+z)]$$

$$w(z) = w_0 + w_1 [1/(1+z)]^\alpha \ln(1/(1+z))^\alpha$$

$$w(z) = w_0 + w_1 \ln(1+z)/z^\alpha$$

$$w(z) = w_0 + w_1 z/(1+z)^\alpha$$

$$w(z) = w_0 + w_1 \ln(1+z)/z^{5/6}$$

$$w(z) = \frac{w_0}{[1+w_a \ln(1+z)]^2}.$$

$$w(z) = w_0 + w_1 z \ln[(1+z)/z]$$

$$w(z) = w_0 + w_1 (1 - \ln(1+z)/z)$$

$$w(z) = w_0 + w_1 [z/(1+z)]^\alpha \ln(z/(1+z))^\alpha$$

$$w(z) = w_0 + w_1 z^\alpha \ln[(1+z)/z]$$

$$w(z) = w_0 + w_1 z^{4/9} \ln[(1+z)/z]$$

$$w(z) = w_0 + w_1 \left(\frac{\sin(1+z)}{1+z} - \sin(1)\right).$$

... amongst many others

or **assume a parameterisation w(z)**

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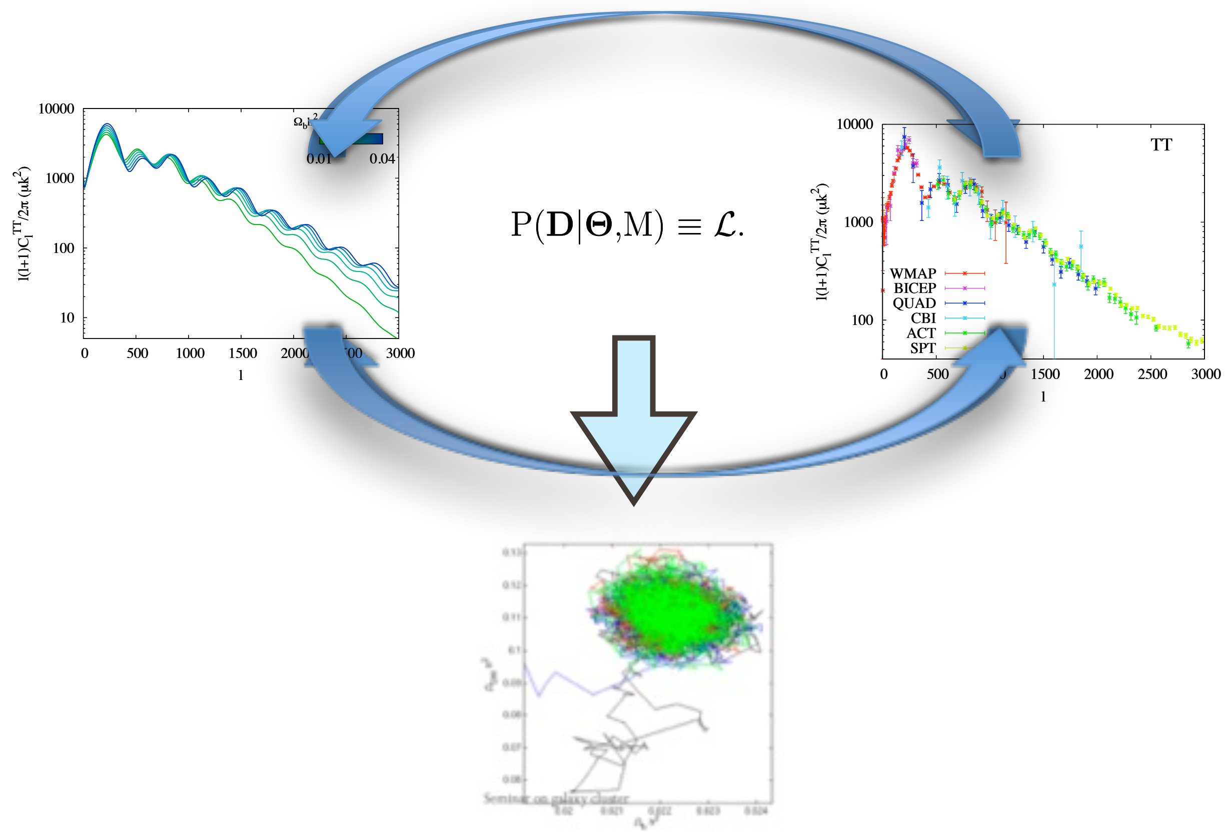
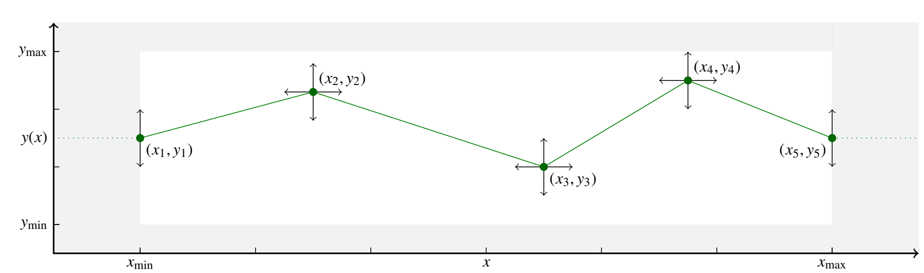
or **assume a parameterisation w(z)**

• Reconstructions



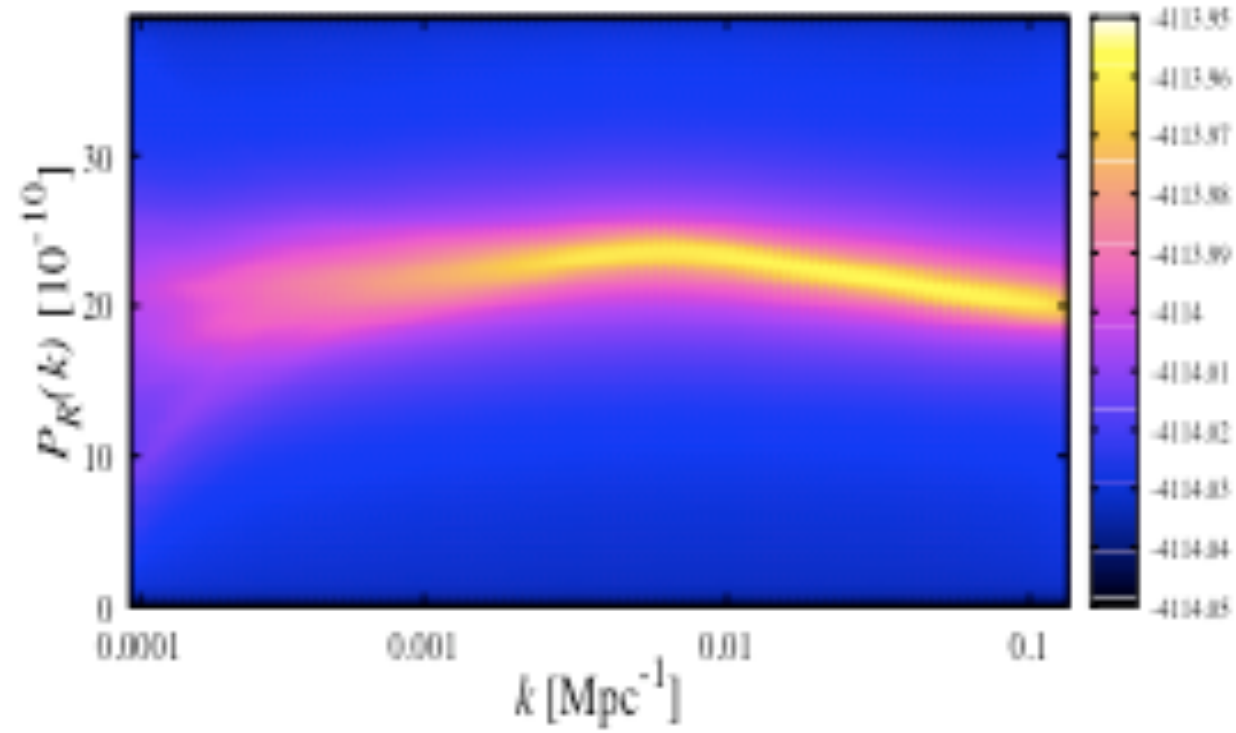
The **freedom** of the position of the internal z-nodes allows us to **localise the best position** for a turn-over (if any) and the amplitudes are able to **describe the global structure** of w(z).

f(x)



Model selection applied to reconstruction of the Primordial Power Spectrum

$(2\mathrm{k_i})\ \mathcal{B}_{2\mathrm{k_i},n_\mathrm{s}} = +2.3 \pm 0.3$

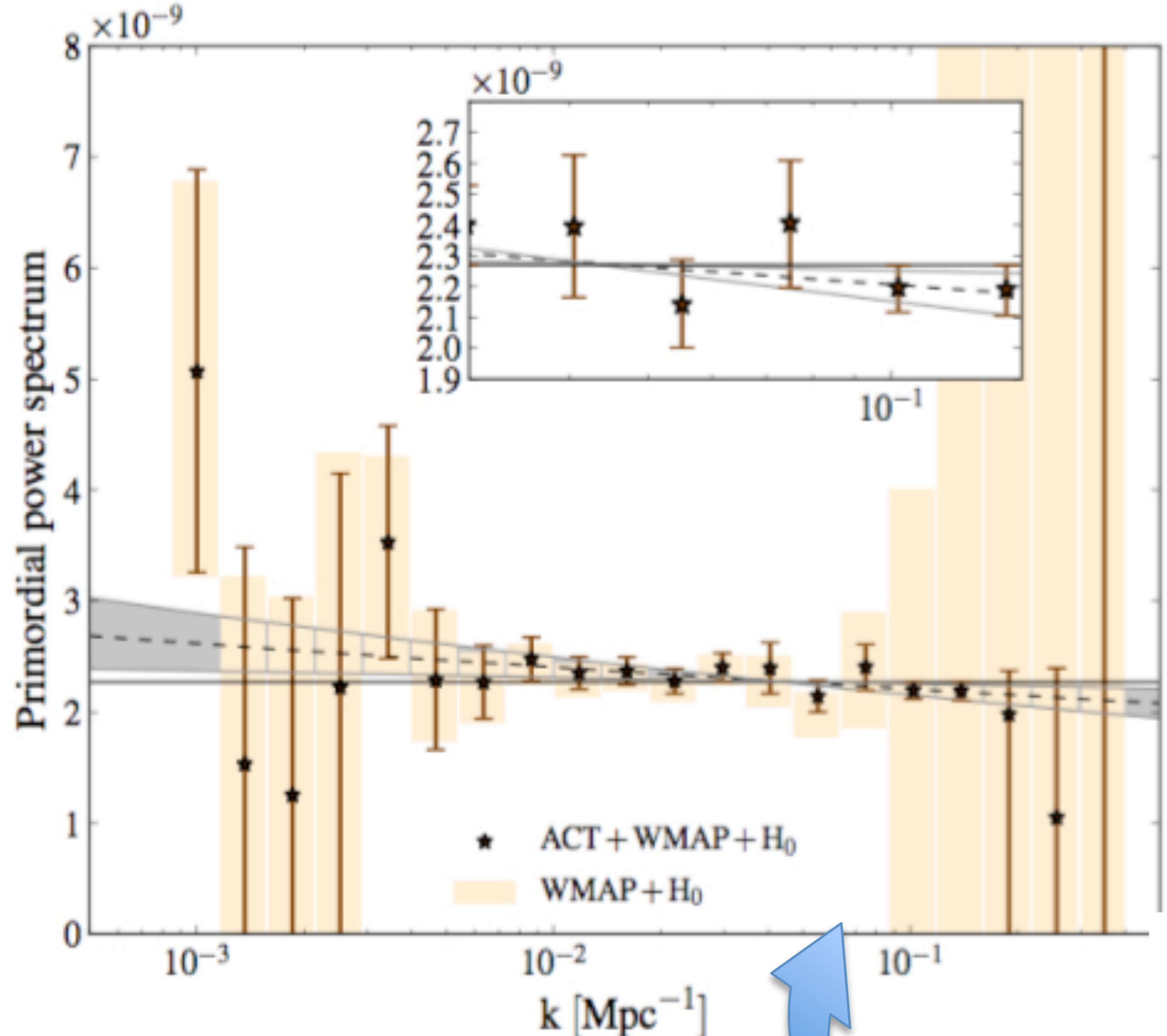


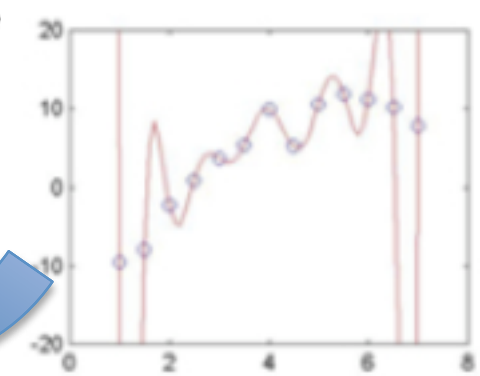
Model	$\mathrm{N_{par}}$	$-2\Delta \ln \mathcal{L}_{\max}$	Bayes factor
HZ	8	0.0	$\mathcal{B}_{1,1} = +0.0 \pm 0.3$
n_s	9	-8.6	$\mathcal{B}_{n_\mathrm{s},1} = +3.3 \pm 0.3$
n_v	10	-9.4	$\mathcal{B}_{n_\mathrm{v},1} = +4.7 \pm 0.3$
LD	10	-9.4	$\mathcal{B}_{\mathrm{LD},1} = +4.9 \pm 0.3$
k_1	11	-9.1	$\mathcal{B}_{k_1,1} = +4.3 \pm 0.3$

We allow the **data** to **decide the level of complexity of the model**

via the Bayesian Evidence

THE ATACAMA COSMOLOGY TELESCOPE: A MEASUREMENT OF THE PRIMORDIAL POWER SPECTRUM





Bridges et al. 2009; Peiris & Verde 2010; Vazquez et al. 2011). The standard models of inflation predict a power

Planck 2013 results. XXII. Constraints on inflation

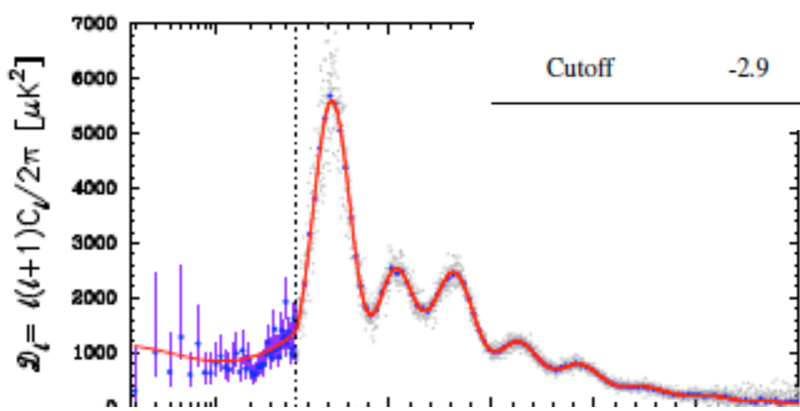
7. Primordial power spectrum reconstruction

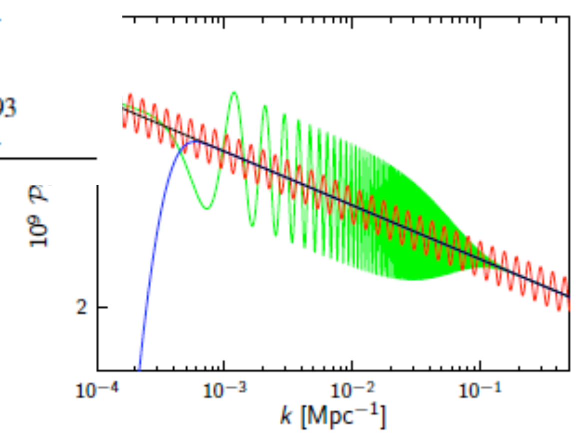
et al., 2003), and Bayesian model selection (Bridges et al., 2009; Vázquez et al., 2012). The approach pursued here follows

3.4. Model selection

$$\mathcal{E}_i = \int \mathrm{d}x^X P(\hat{x}|\mathcal{M}_i)\mathcal{L}(\mathrm{data}|\hat{x})\,.$$

Model	$-2\Delta \ln \mathcal{L}_{\max}$	$\ln B_{0X}$	Parameter	Best fit value
Wiggles	-9.0	1.5	α_w	0.0294
			ω	28.90
			φ	0.075 π
Step-inflation	-11.7	0.3	$\mathcal{A}_f$	0.102
			$\ln(\eta_f/\mathrm{Mpc})$	8.214
			$\ln x_d$	4.47
Cutoff	-2.9	0.3	$\ln(k_c/\mathrm{Mpc}^{-1})$	−8.493
			λ_c	0.474

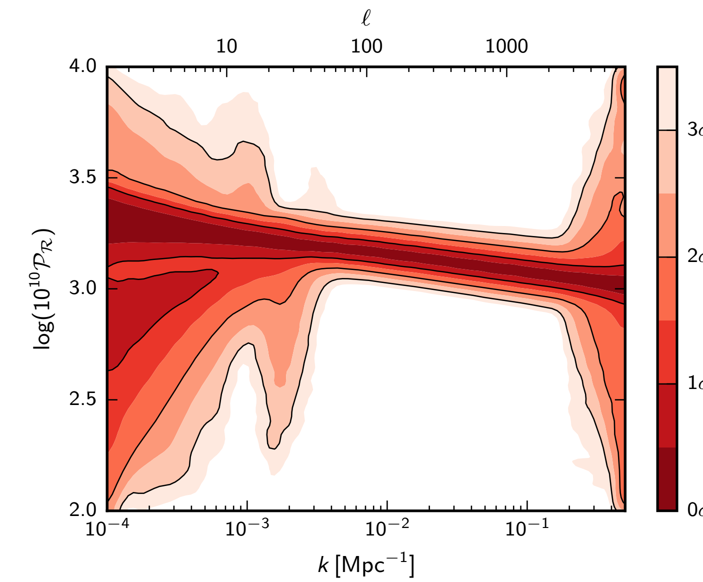




Planck 2015 results. XX. Constraints on inflation

8. $\mathcal{P}(k)$ reconstruction

8.2. Method II: Bayesian model comparison

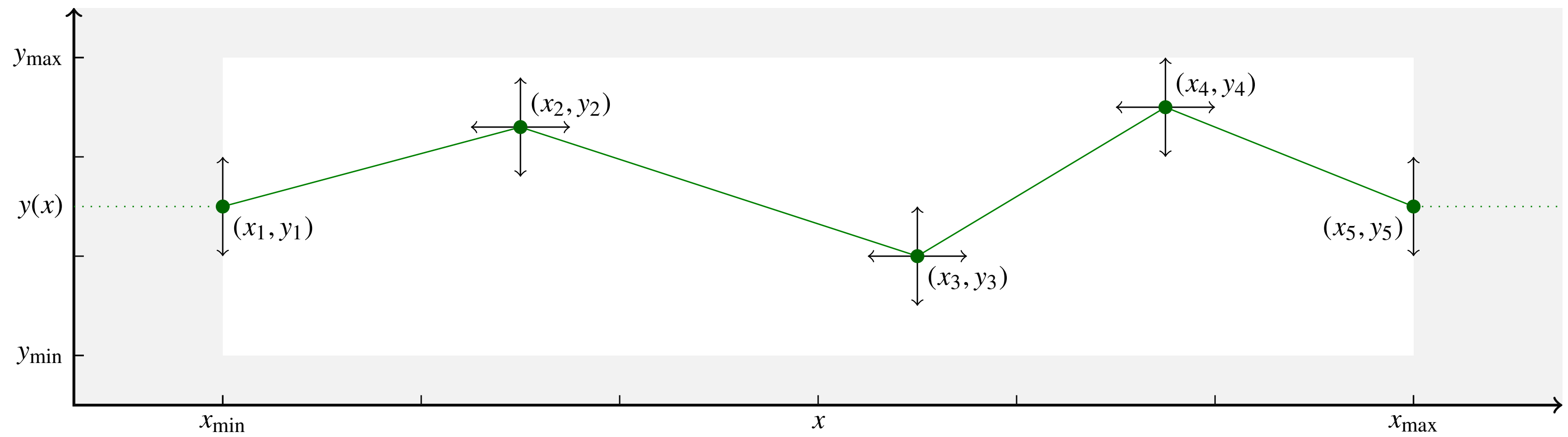


Planck 2018 results. VI. Cosmological parameters

reconstructing the primordial power spectrum from *Planck* data (Vázquez et al. 2012; Planck Collaboration XX 2016).

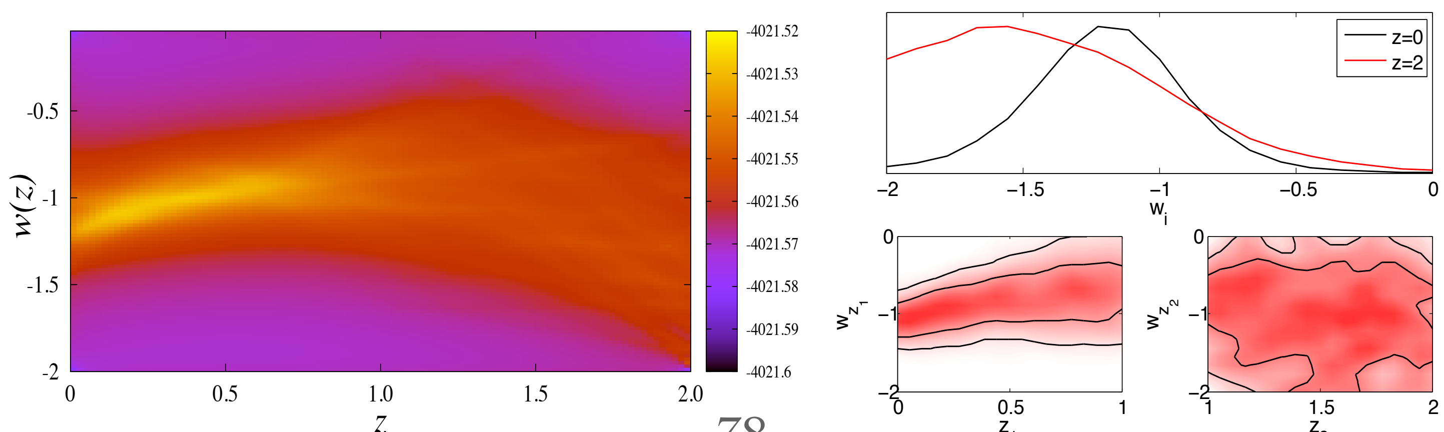
Reconstruction of the dark energy equation of state

J. Alberto Vázquez,[a,b] M. Bridges,[a,b] M.P. Hobson[b] and A.N. Lasenby[a,b]



The **freedom** of the position of the internal z-nodes allows us to **localise the best position** for a turn-over (if any) and the amplitudes are able to **describe the global structure** of w(z).

$(z_2)\ \mathcal{B}_{z_2,\Lambda} = -0.81 \pm 0.35$



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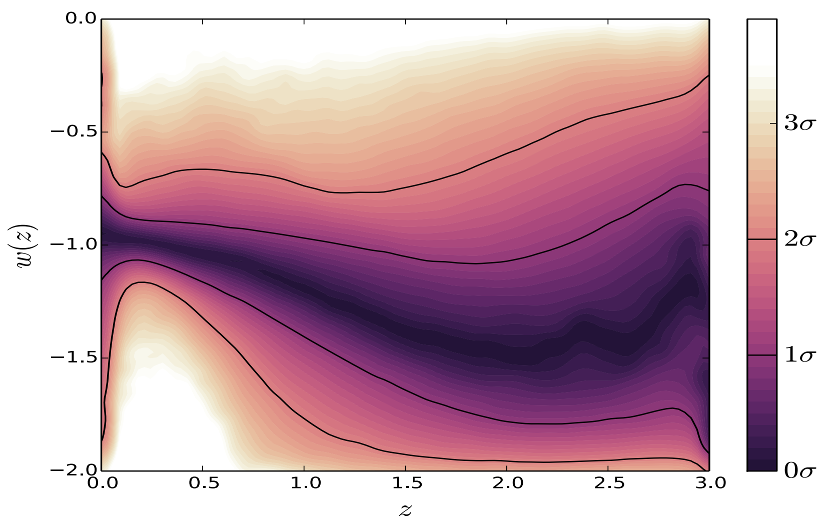
Constraining the dark energy equation of state using Bayes theorem and the Kullback–Leibler divergence

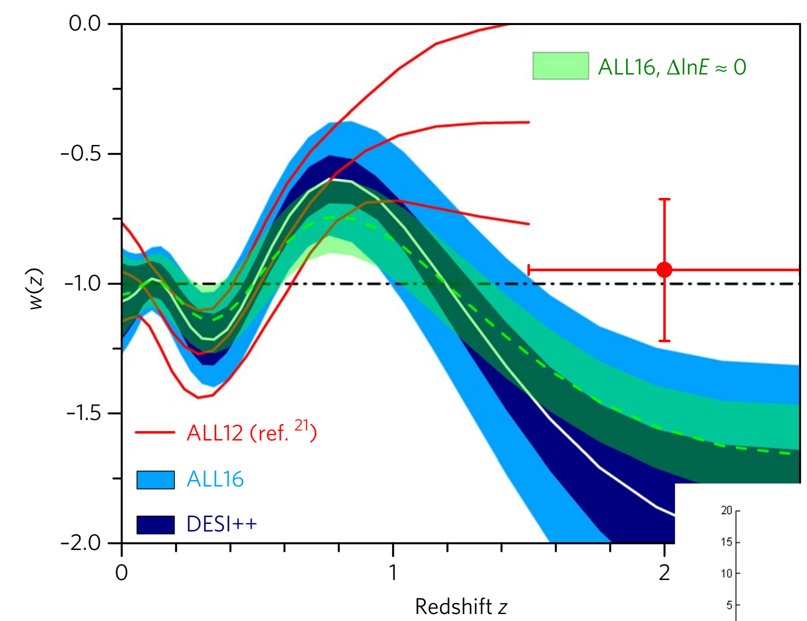
S. Hee,[1,2★] J.A. Vázquez,[3] W.J. Handley,[1,2] M.P. Hobson[1] and A.N. Lasenby[1,2]

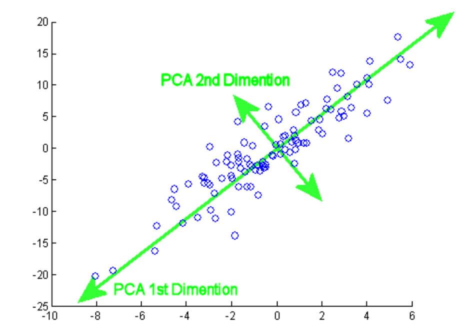
$\mathcal{B}_{\Lambda 3} = -3.36 \pm 0.29$



Dynamical dark energy in light of the latest observations

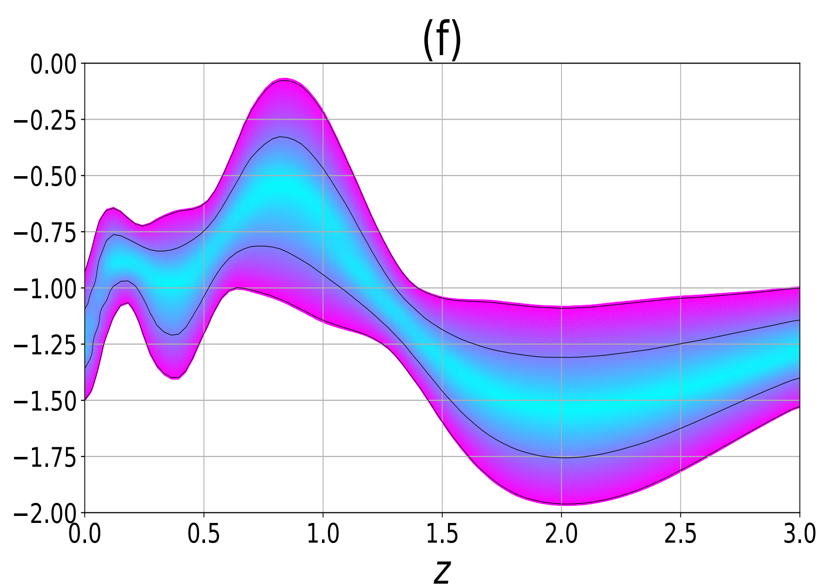
w/ BOSS

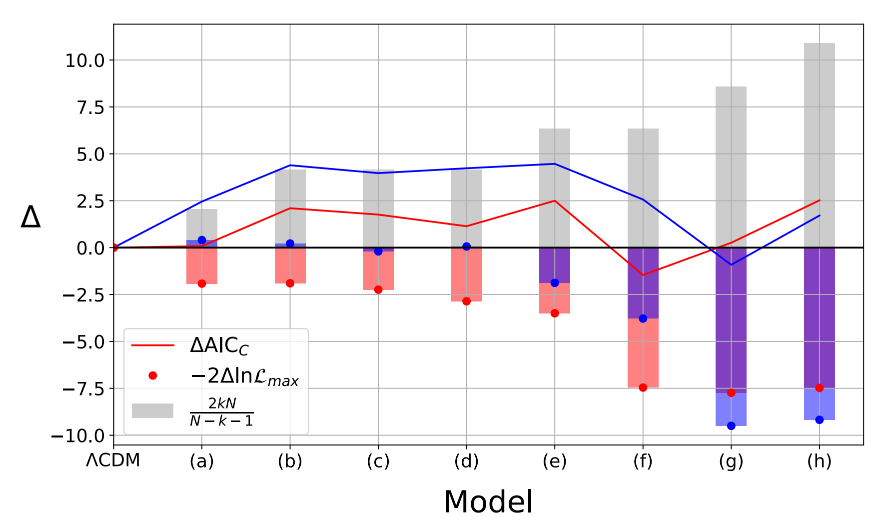




PCA

Fourier series expansion of the dark energy equation of state





David Tamayo,[1★] and J. Alberto Vázquez,[2†]

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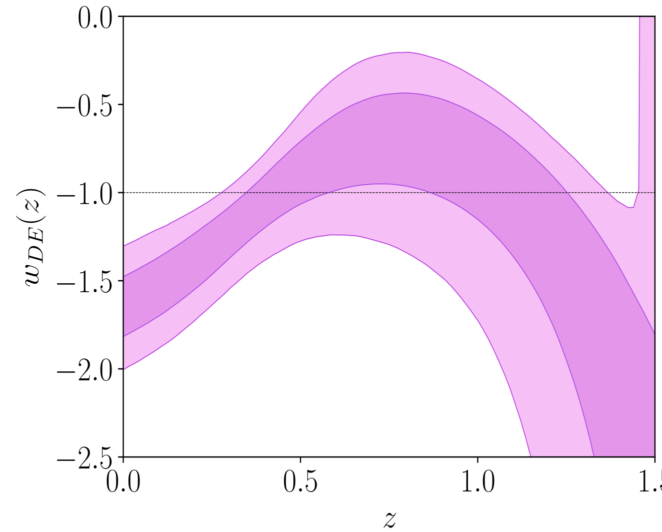
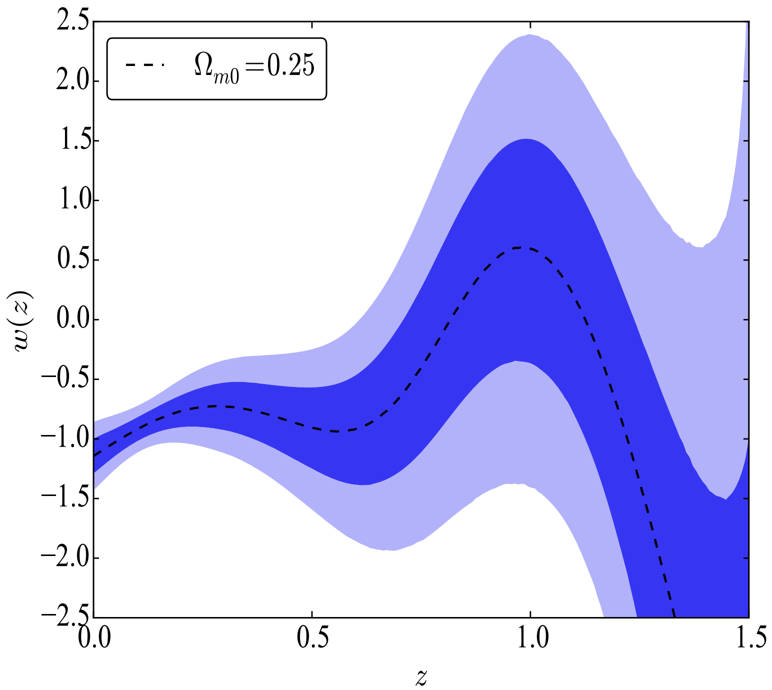
Model independent constraints on dark energy evolution from low-redshift observations

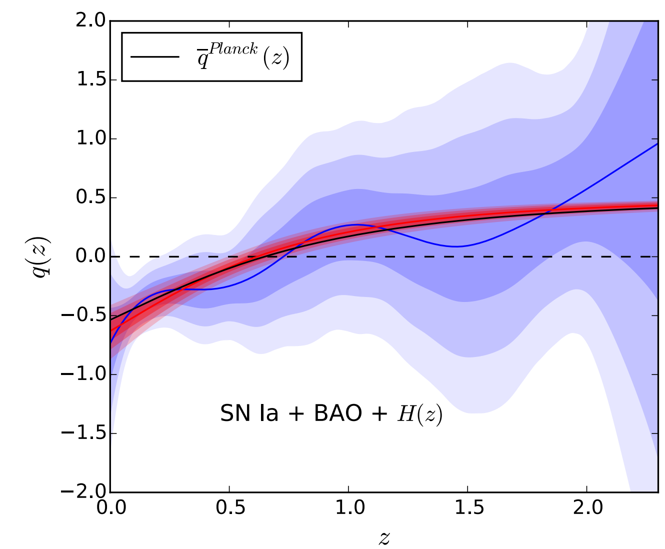
Salvatore Capozziello [1⋆], Ruchika [2†], Anjan A Sen [2 ‡]





A general reconstruction of the recent expansion history of the universe

S. D. P. Vitenti[a] M. Penna-Lima[b,c]



Gaussian processes reconstruction of dark energy from observational data

Ming-Jian Zhang, Hong Li[a]

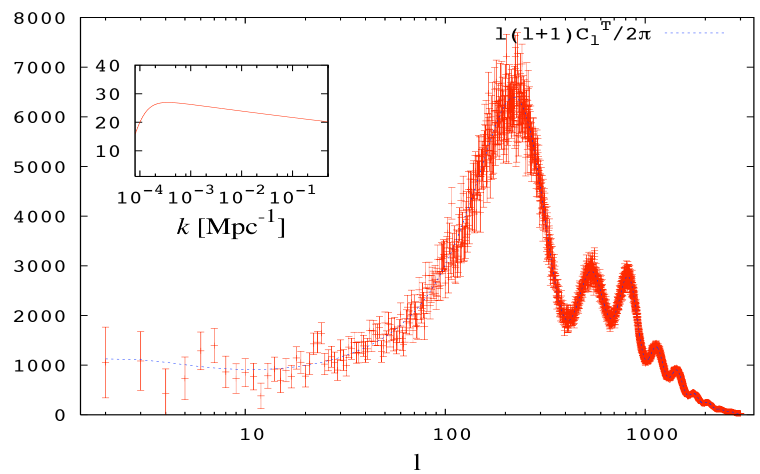
However, as pointed out by Montiel et al. [28], these methods find difficulties due to the limited amount of observational data or even due to some features of the methods themselves,

Nonparametric reconstruction of the cosmic expansion with local regression smoothing and simulation extrapolation

Ariadna Montiel[1], Ruth Lazkoz[2], Irene Sendra[2],

Forecast

Assume a “true” model of the Universe



Add your favourite experiment

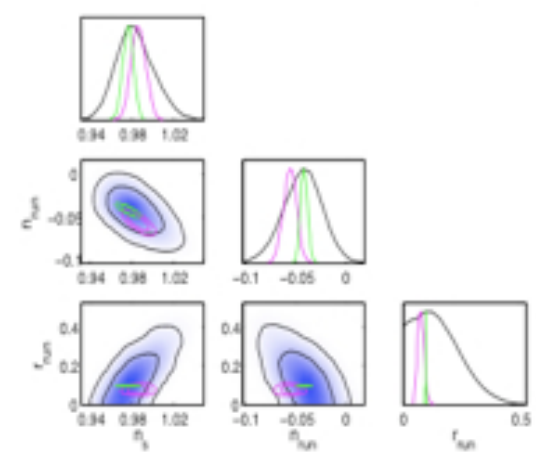
$$
\begin{aligned}
(\Delta \hat{C}_l^{XX})^2 &= \frac{2}{(2l+1) f_{sky}} \left(C_l^{XX} + N_l^{XX}\right)^2, \\
(\Delta \hat{C}_l^{TE})^2 &= \frac{2}{(2l+1) f_{sky}} \left[\left(C_l^{TE}\right)^2 + \left(C_l^{TT} + N_l^{TT}\right)\left(C_l^{EE} + N_l^{EE}\right)\right]
\end{aligned}
$$

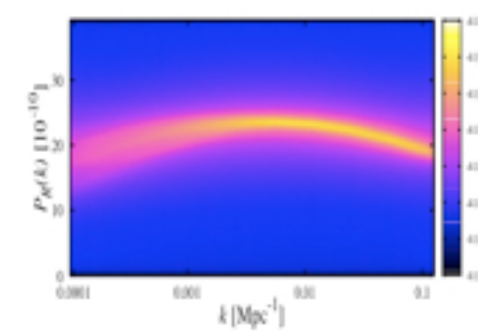
differentiate models through future surveys

?

$$
\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{1}{2}\ln\left(\frac{k}{k_0}\right) n_{\rm run}}.
$$

$(n_{\rm run})\ \mathcal{B}_{n_{\rm run}, n_s} = +2.0 \pm 0.3$





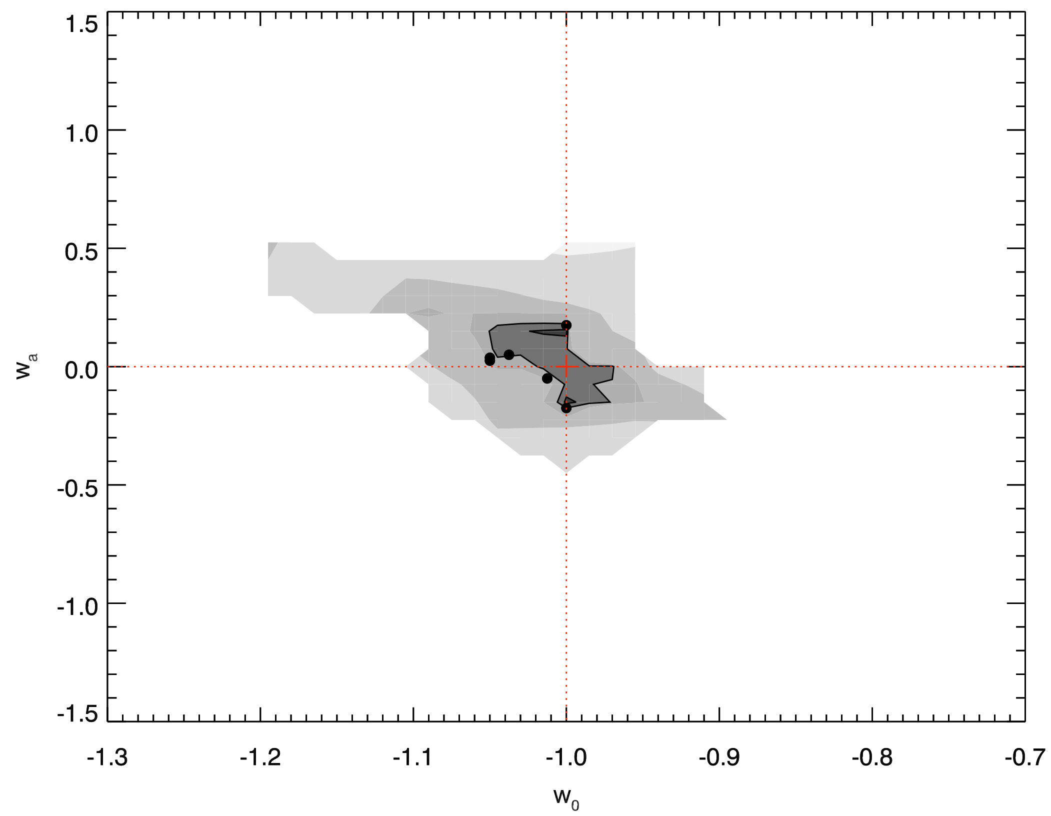
	Planck	CMBPol
LD	0.0 ± 0.3	0.0 ± 0.3
n_s	-6.3 ± 0.3	-13.0 ± 0.3
$n_{\rm run}$	-6.5 ± 0.3	-15.5 ± 0.3
$2k_i$	-3.1 ± 0.3	-10.2 ± 0.3

Constraints on the Tensor-to-Scalar ratio for non-power-law models

J. Alberto Vázquez[a,b] M. Bridges[a,b] Yin-Zhe Ma[c,d] M.P. Hobson[b]

... same for dark energy

Euclid



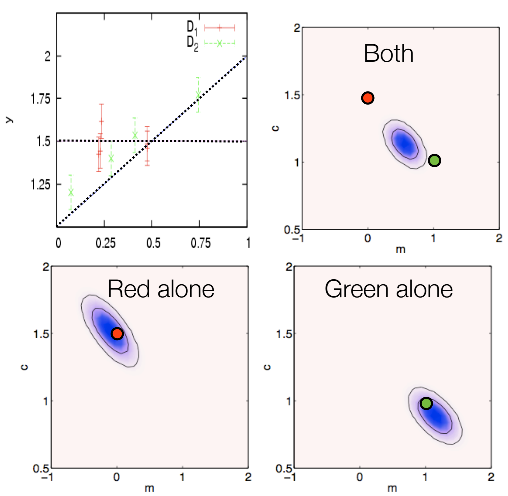
Ivan Debono

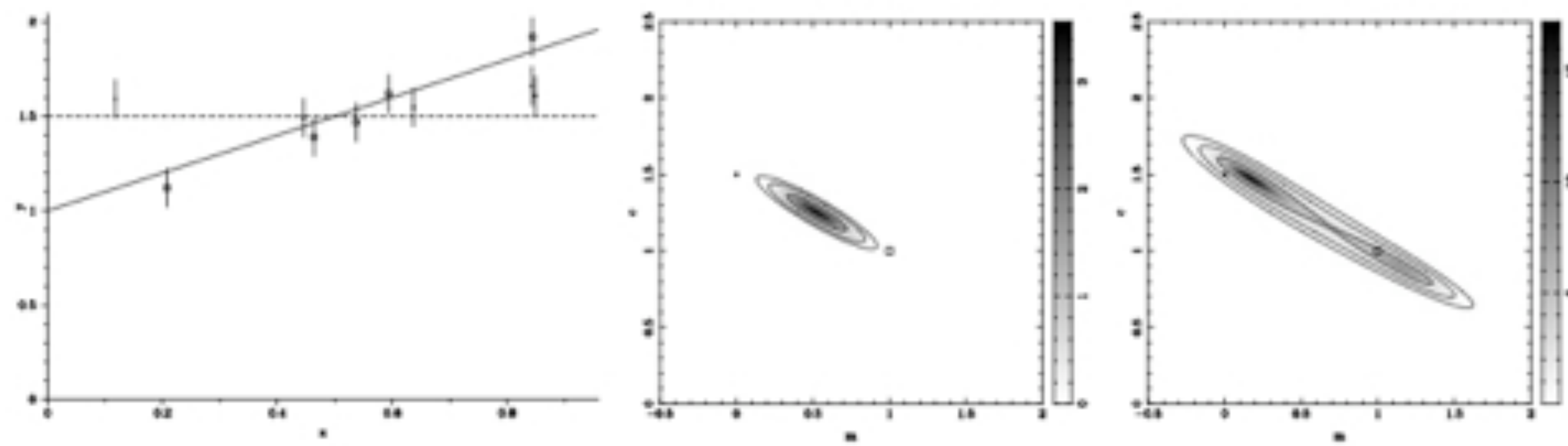
Bayesian model selection for dark energy using weak lensing forecasts

Dataset consistency

$$R = \frac{\Pr(D|H)}{\prod_{i=1}^{n} \Pr(D_i|H)},$$

$$\frac{\Pr(\boldsymbol{D}|H_1)}{\Pr(\boldsymbol{D}|H_0)} = 11.6,$$

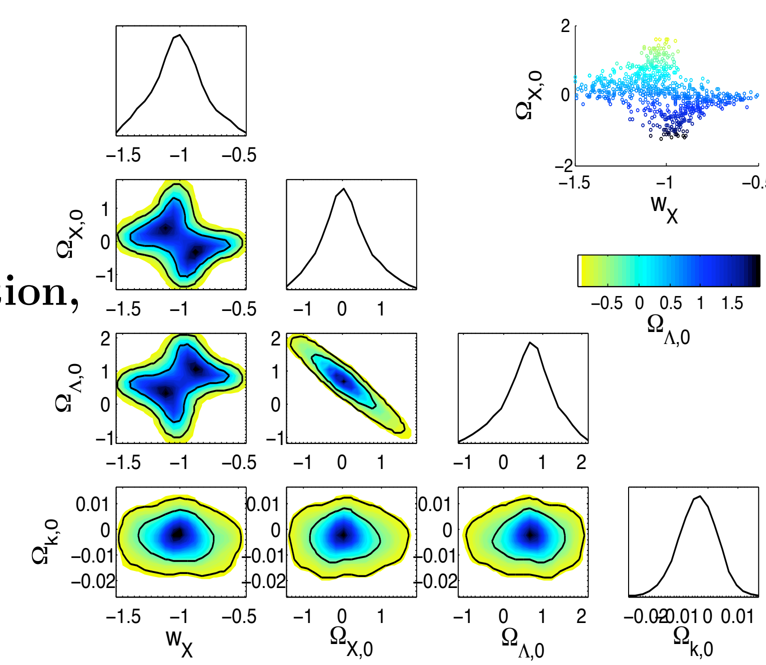




Combining cosmological datasets: hyperparameters and Bayesian evidence

M.P. Hobson[1], S.L. Bridle[2] and O. Lahav[2]

$$R = 2.15$$

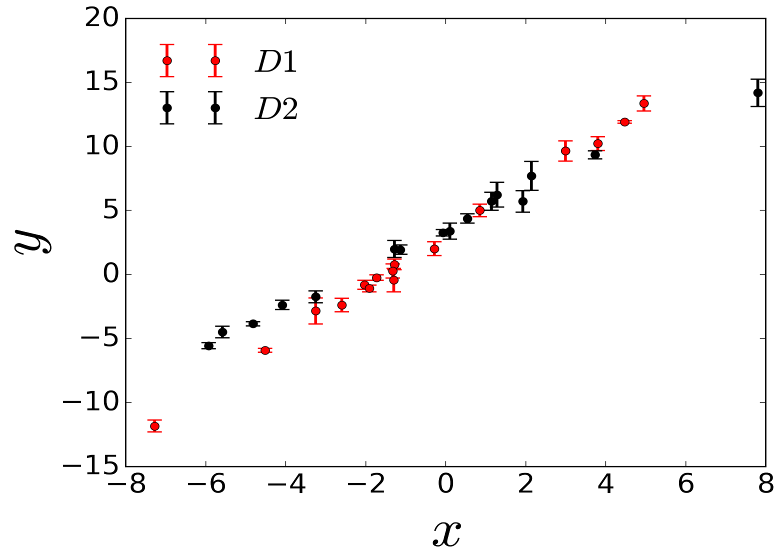


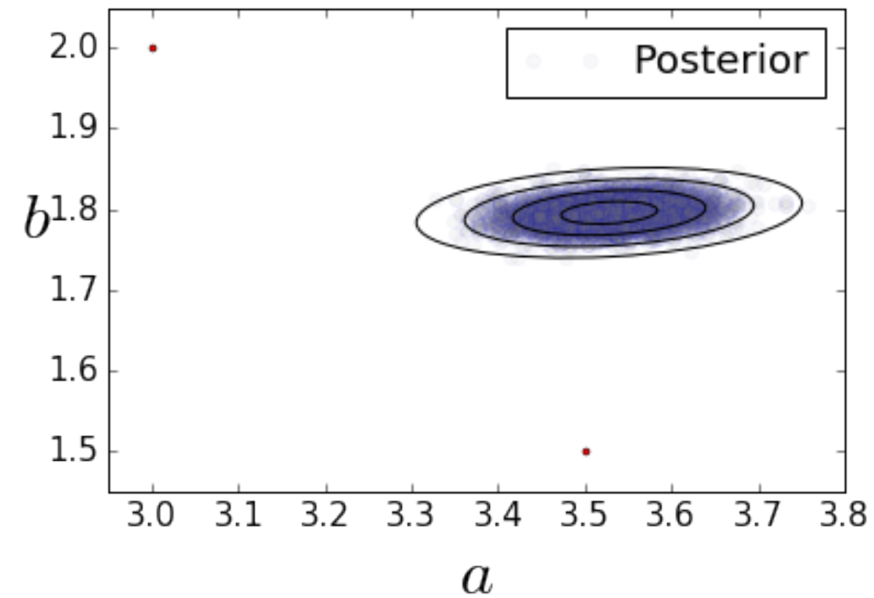
Reciprocity invariance of the Friedmann equation, Missing Matter and double Dark Energy

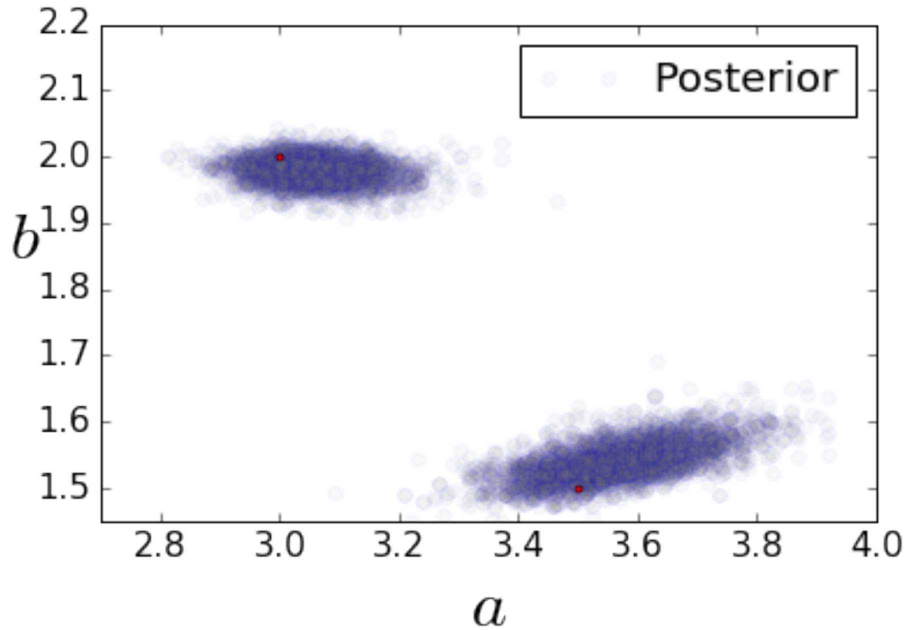
J. Alberto Vázquez[1,2],* M.P. Hobson[2],
A.N. Lasenby[1,2], M. Ibison[3], and M. Bridges[1,2]

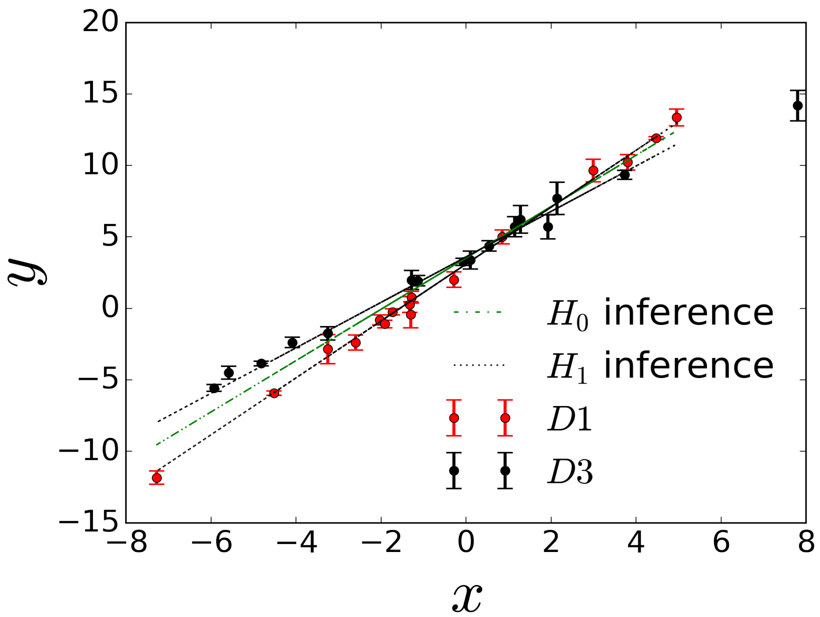
Dataset consistency

$$R = \frac{\Pr(D|H)}{\prod_{i=1}^{n} \Pr(D_i|H)},$$









Model Averaging

This model-averaged posterior encodes the uncertainty as to the correct model

$$P(\bar{\theta}|D) = \frac{\sum_k P(\bar{\theta}|D, \mathcal{M}_k) P(\mathcal{M}_k|D)}{\sum_k P(\mathcal{M}_k|D)} \, .$$

5

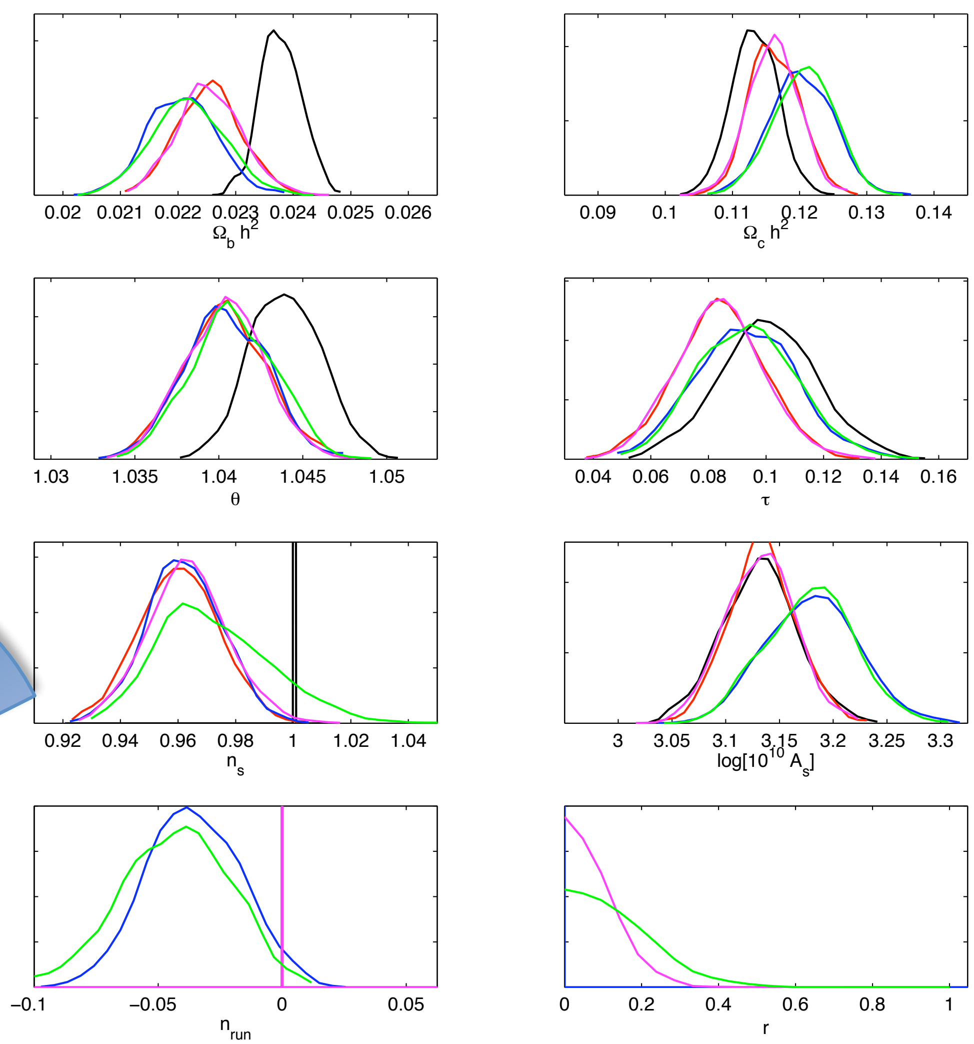
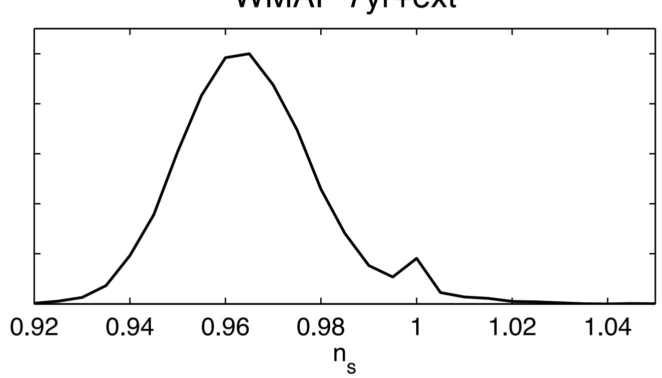


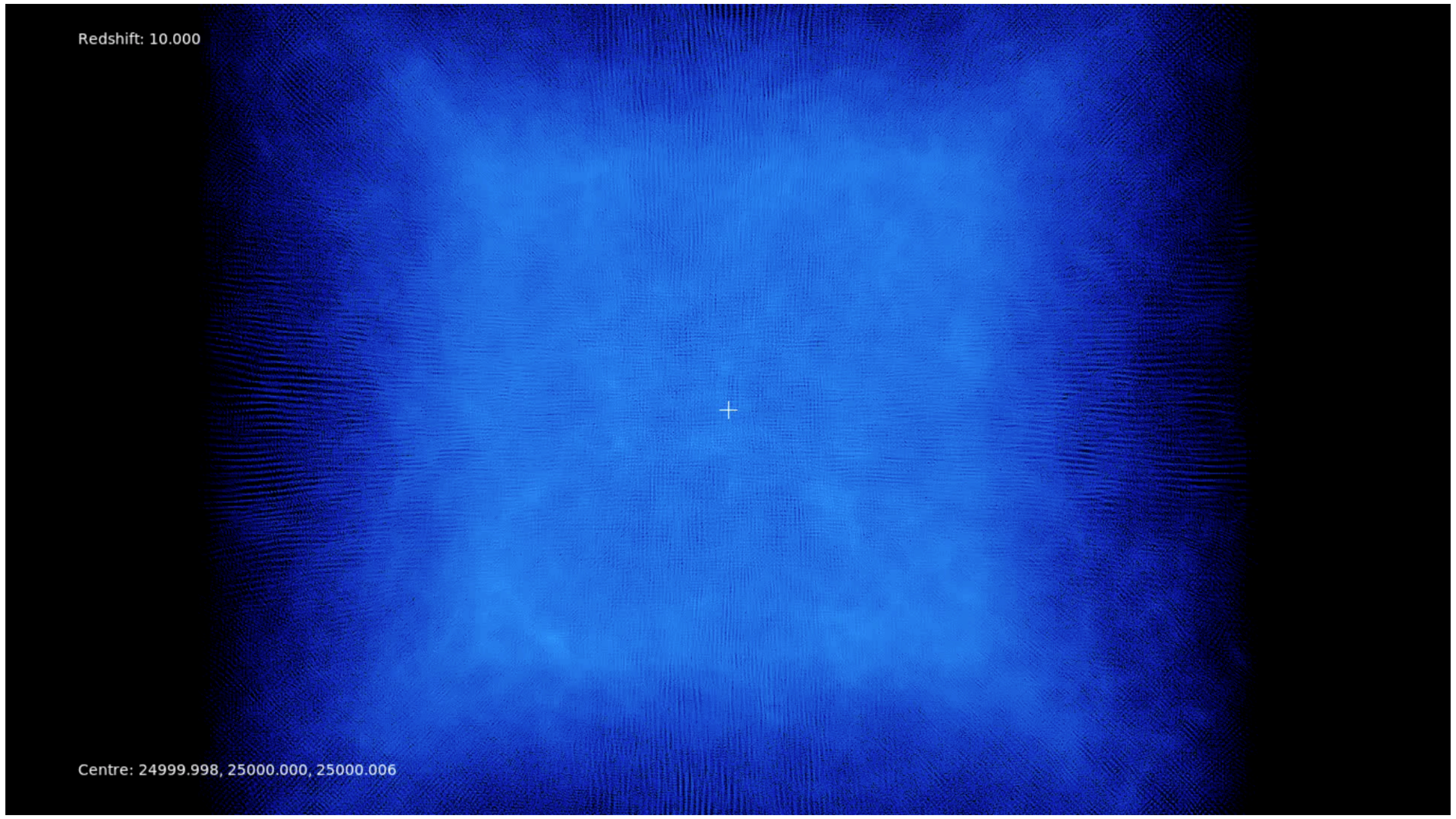
FIG. 3: The posterior probability distributions for the different models, using only the WMAP 7yr data plus other datasets. The models are HZ (black), tilted (red), running n_s+n_{run} (blue), 'inflation' n_s+r (magenta) and tensor+running (green).

WMAP 7yr+ext



Bayesian Model Averaging in Astrophysics: A Review

David Parkinson[1]*and Andrew R. Liddle[2]



Análisis Comparativo de dos Modelos de Evolución Cosmológica Utilizando Simulaciones Numéricas

Jazhiel Chacon

& Erick Almaraz

: parameter estimation

$$P(\mathbf{\Theta}|\mathbf{D}, M)$$

: model selection

$$\mathcal{B}_{i,j} = \ln \frac{\mathcal{Z}_i}{\mathcal{Z}_j}.$$

dataset consistency

$$R = \frac{\Pr(D|H)}{\prod_{i=1}^{n} \Pr(D_i|H)},$$

model averaging

$$P(\bar{\theta}|D) = \frac{\sum_k P(\bar{\theta}|D, \mathcal{M}_k) P(\mathcal{M}_k|D)}{\sum_k P(\mathcal{M}_k|D)}.$$

SuperBayeS

Supersymmetry Parameters Extraction Routines for Bayesian Statistics

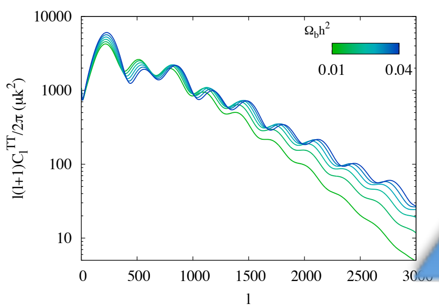
SuperBayeS is a package for fast and efficient **sampling of supersymmetric theories**. It uses Bayesian techniques to explore multidimensional SUSY parameter spaces and to compare SUSY predictions with observable quantities, including sparticle masses, collider observables, dark matter abundance, direct detection cross sections, indirect detection quantities etc. Scanning can be performed using Markov Chain Monte Carlo (MCMC) technology or even more efficiently by employing a new scanning technique called, **MultiNest**. which implements the nested sampling algorithm. Using MultiNest, a full 8-dimensional scan of the CMSSM takes about 12 hours on 10 2.4GHz CPU's. There is also an option for old-style fixed-grid scanning. More info about the package can be found throughout this site. If you discover any bug or if you have any questions, please login on the SuperBayeS discussion **forum**.

The package combines **SoftSusy**, **DarkSusy**, **FeynHiggs**, **Bdecay**, **MultiNest** and **MicrOMEGAs**. Some of the routines and the plotting tools are based on **CosmoMC**.

Conclusions

Theory

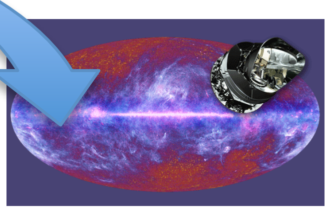
$$\int d^4x\sqrt{-g}\, R$$

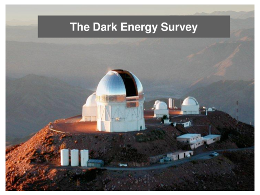


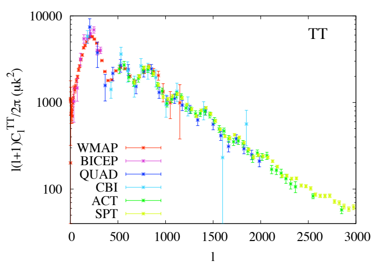
H

$$\mathrm{P}(\mathbf{D}|\boldsymbol{\Theta},\mathrm{M}) \equiv \mathcal{L}.$$

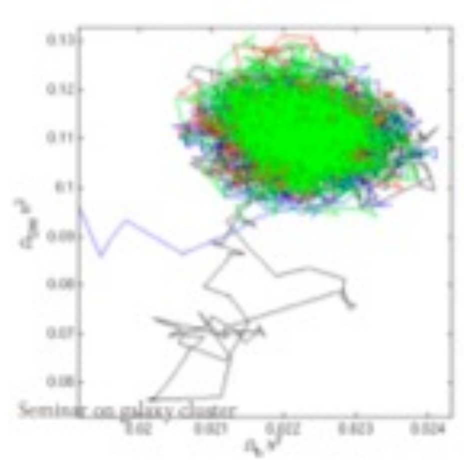
Observations

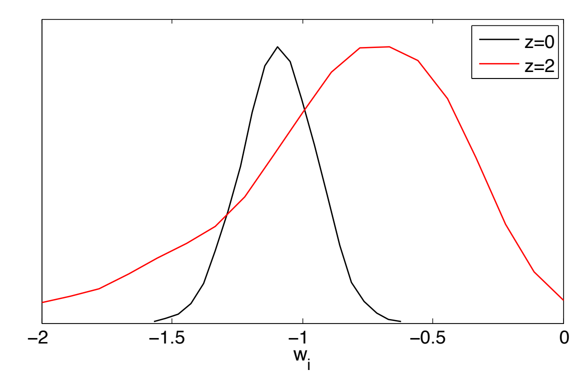






D





$$P(\boldsymbol{\Theta}|\mathbf{D},M) = \frac{P(\mathbf{D}|\boldsymbol{\Theta},M)\ P(\boldsymbol{\Theta}|M)}{P(\mathbf{D}|M)}.$$

Conclusions

.. let the observations decide

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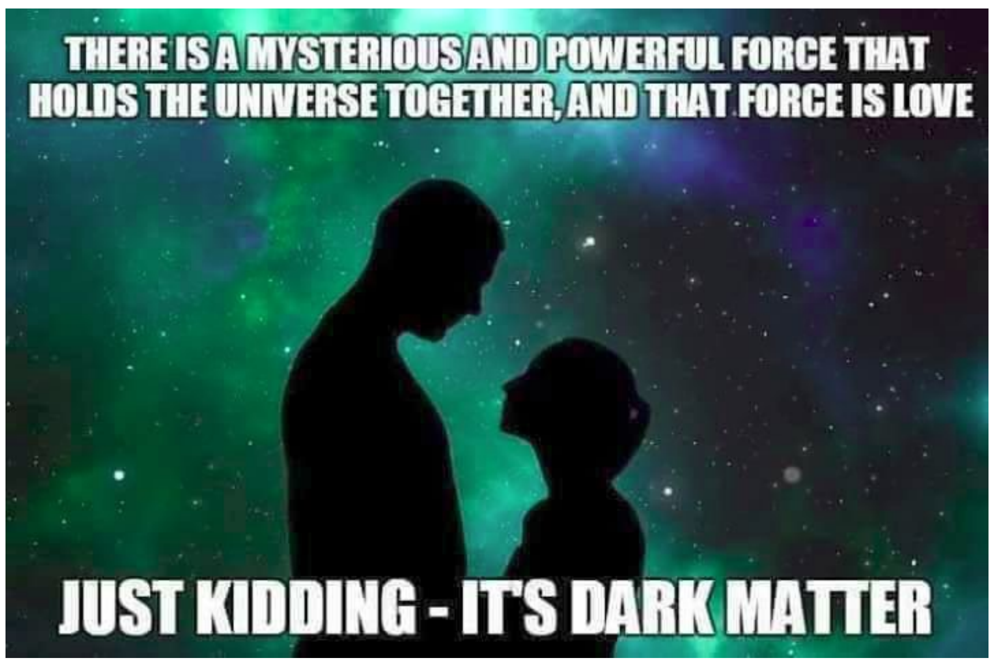
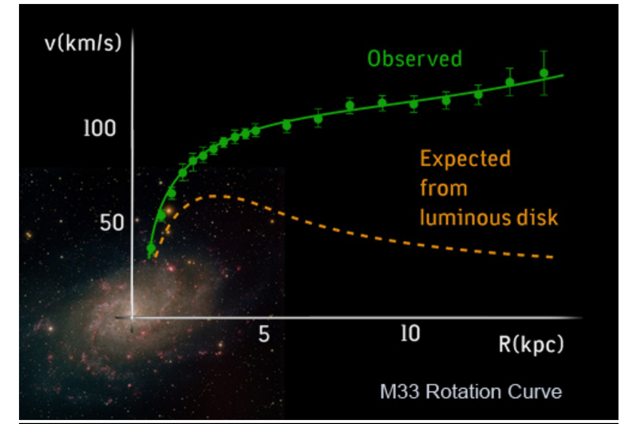
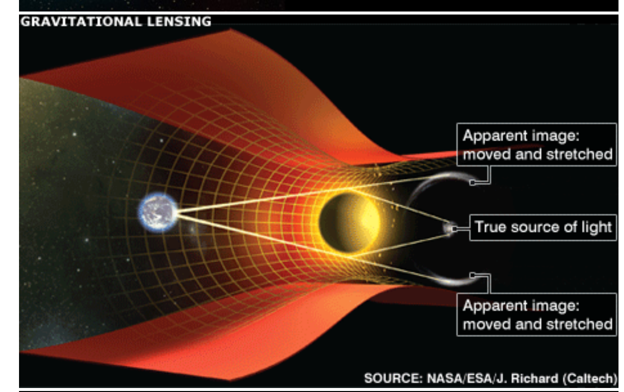
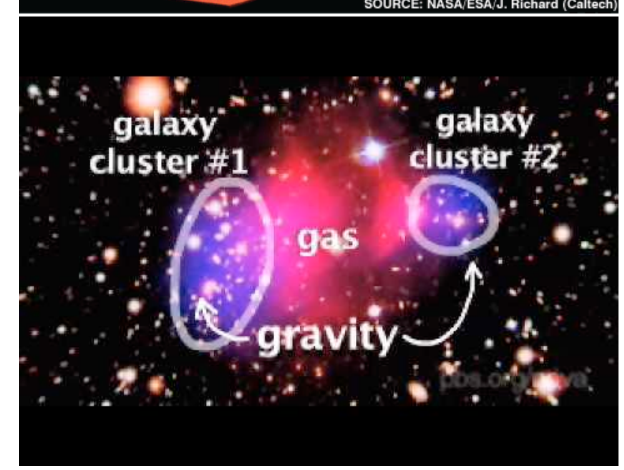
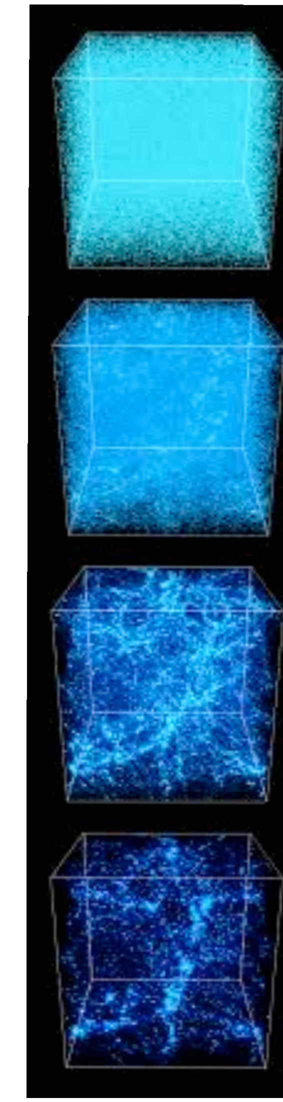
Materia Oscura







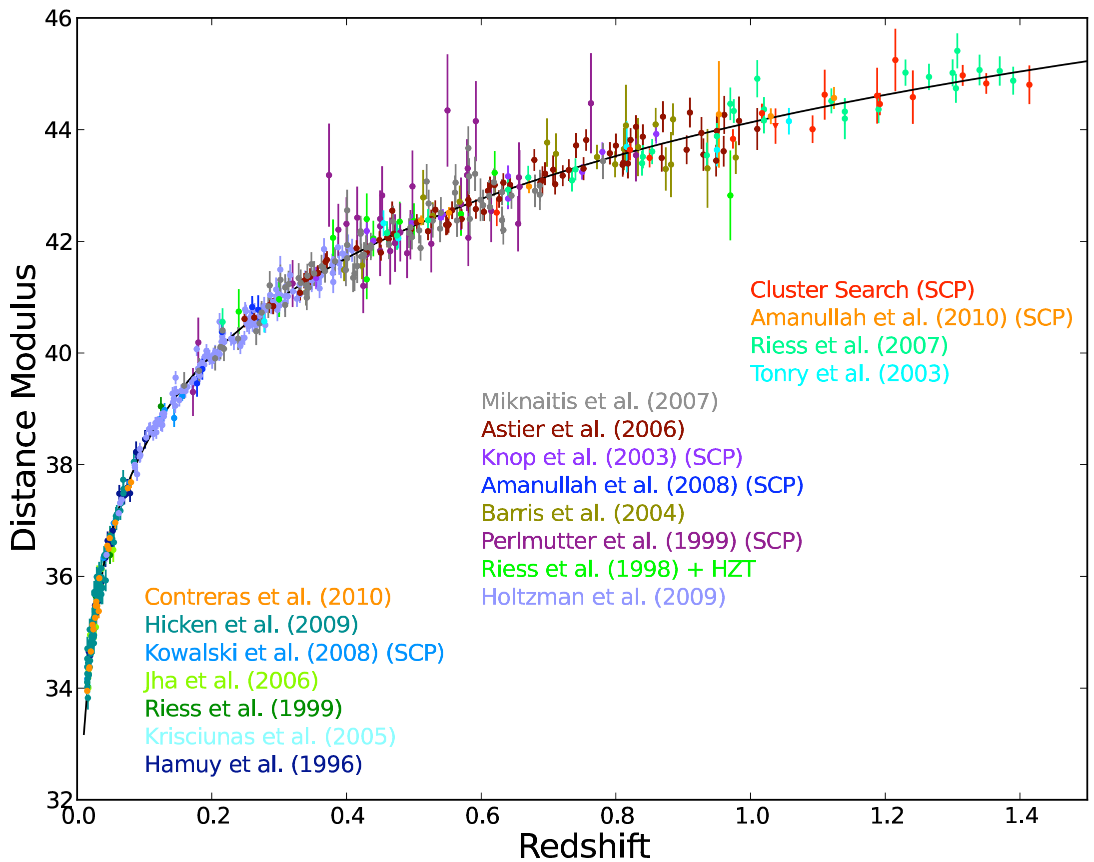




Uno de los **enigmas más fascinantes en la Física**, es el problema de la existencia de materia oscura en el Universo. **Seis veces** mas abundante que la materia ordinaria, una cuarta parte de la densidad total y el componente principal para la **formación de estructura** en el Universo.

Dark Energy

SN Ia → Accelerated expansion



acceleration — gravity — cosmological constant

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

slows down expansion — speeds up expansion

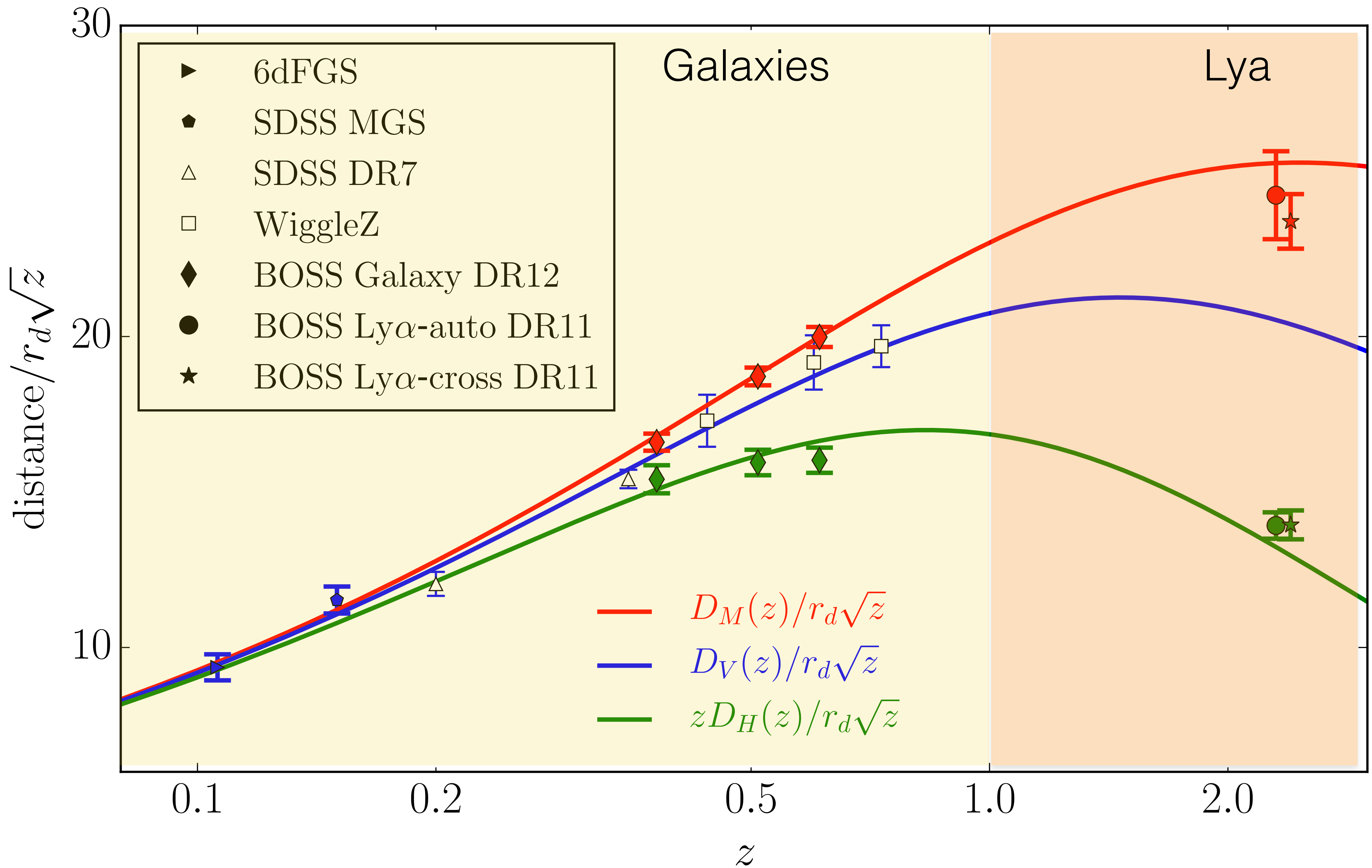
Dark energy !

$$\ddot{a} > 0 \iff p < -\rho/3$$

BAO data

new

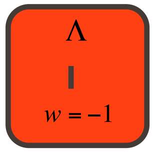
The BAO "Hubble diagram" from a world collection of detections.

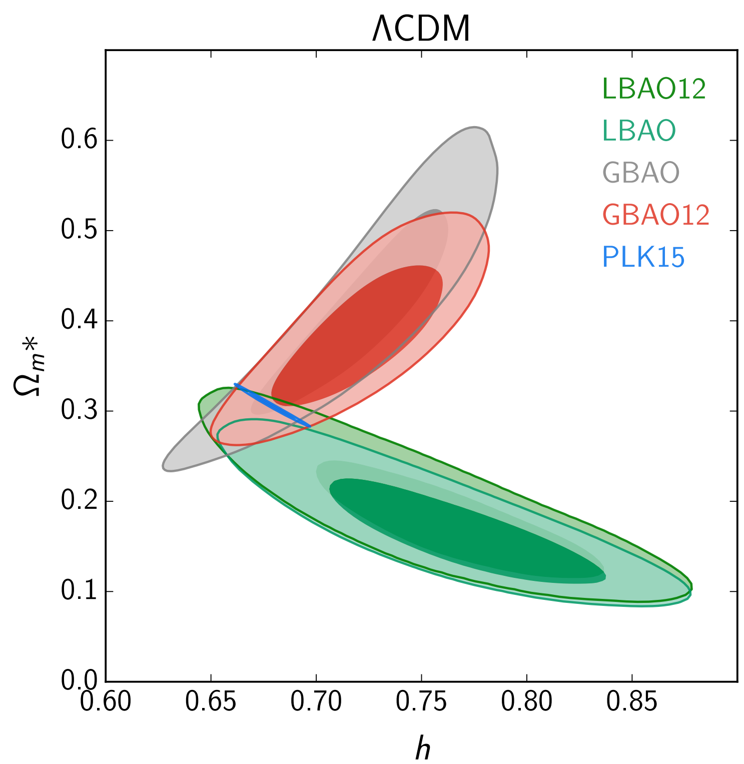


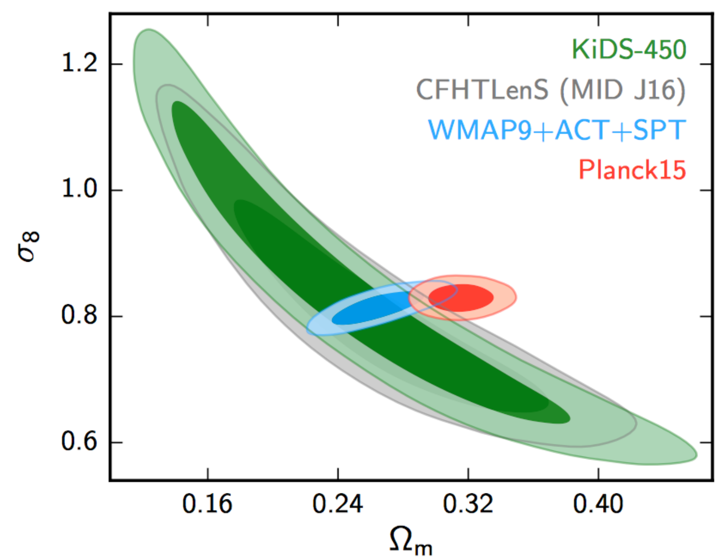
Cosmological implications of baryon acoustic oscillation (BAO) measurements

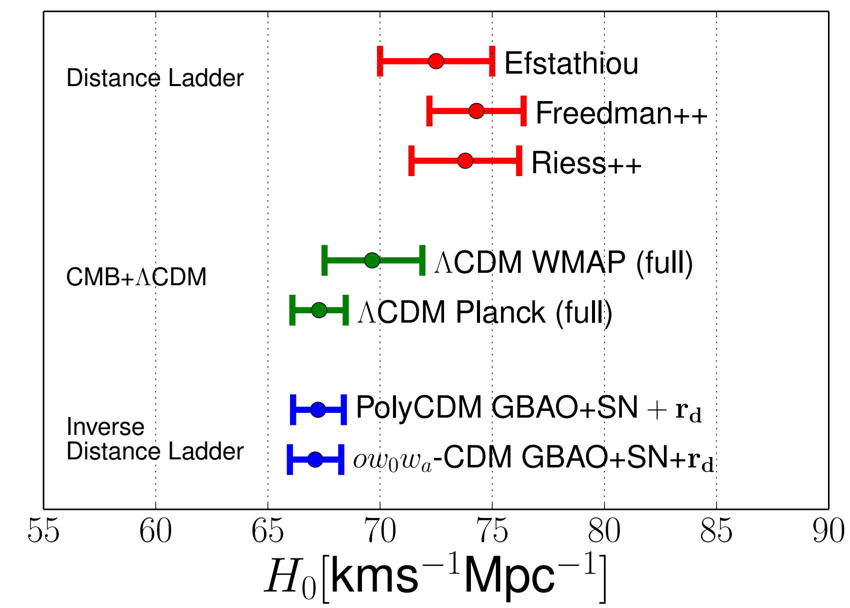
w/ BOSS Collaboration

but …









- The tension of the LyaF BAO with the Planck LCDM model **manifests** itself here as a best fit at relatively low matter density and high Hubble parameter.

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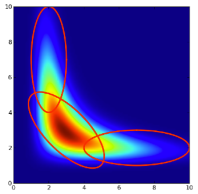
... require around 10^7 likelihood evaluations, for a five-parameter cosmological model.

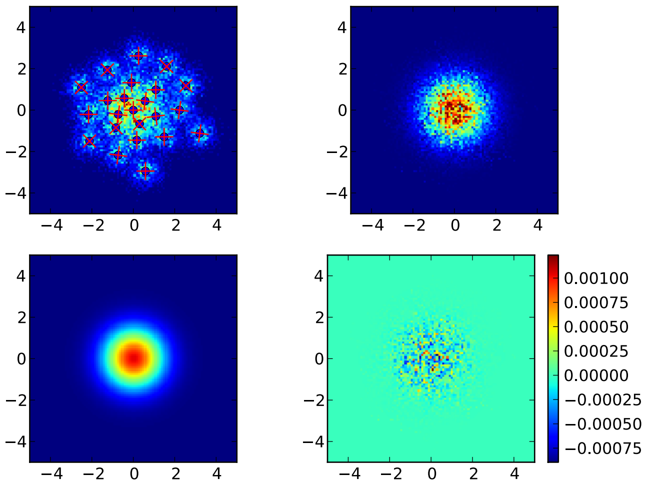
When your MCMC converges

To perform the analysis we built a simple and fast MCMC code: Simple MC

w/ Anze S

Gaussian Mixer Sampler





w/ Anze S

A review of samplers for cosmological model comparison

Isidro Gómez Vargas,[1,a] Ricardo Medel Esquivel,[1,b] J. Alberto Vázquez,[2] and Ricardo García Salcedo[1]

to be continued …

1301.8673

www.cimat.mx/~jac/twalk/