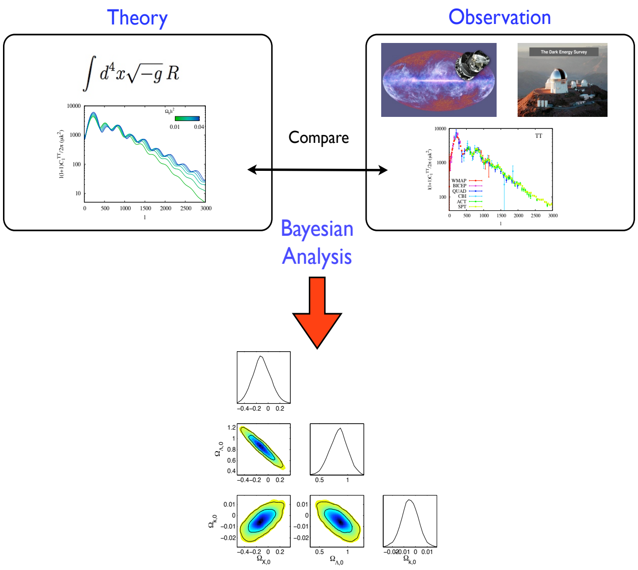
Updated Cosmology

with Python



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In progress

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0.1 Distances and Horizons

Now we have all the components of the universe and its dynamics, let's see how they may affect the distances in the universe.

The **particle horizon** is the distance light could have travelled since the origin of the universe. Regions further apart could never have been causally connected. In a time dt light travels a comoving distance $d\chi = cdt/R$, thus the total comoving distance travelled since the big-bang corresponds to,

$$\chi_{\rm p} \equiv c\int_0^t \frac{dt}{R(t)}. \tag{1}$$

considering

$$dz = d(1+z) = d\left(\frac{R_0}{R}\right) = -\frac{R_0}{R^2}dR = -\frac{R_0}{R^2}\dot{R}dt = -(1+z)H(z)dt, \tag{2}$$

therefore,

$$\chi_{\rm p} = \frac{c}{R_0}\int_0^R \frac{dR}{R^2H(R)} = \frac{c}{R_0}\int_z^\infty \frac{dz}{H(z)}. \tag{3}$$

We must know how $H(z)$ varies with z, which requieres knowledge of the evolution of the scale factor. No information could have propagated further than $\chi_{\rm p}$ on the comoving grid since the beginning of time [2].

Moreover, by changing the order of integration of (3), we can also define the *comoving distance* $d_{\rm c}$, or **event horizon**, as the distance light could have travelled between a source at scale factor R and an observer today [2], as

$$\chi_{\rm e} = c\int_t^{t_0} \frac{dt}{R(t)} = \frac{c}{R_0}\int_0^z \frac{dz}{H(z)}. \tag{4}$$

Considering the FRW metric in terms of the conformal time (**??**), the distance multiplying the solid angle provides the *metric distance*

$$d_{\rm m} = S_k(\chi). \tag{5}$$

In a flat universe ($k = 0$) the metric distance is equal to the comoving distance χ. We emphasize that the comoving distance $d_{\rm c}$ and the metric distance $d_{\rm m}$ are not observables.

A related concept is the *proper distance* $d_{\rm p}$ corresponding to the particle horizon:

$$d_{\rm p}(t) \equiv cR(t)\int_0^t \frac{dt}{R(t)} = R(t)\chi_{\rm p}(t). \tag{6}$$

Regions separated by distances greater that the proper distance $d_{\rm p}$ are not causally connected. Furthermore, the *Hubble radius* or *Hubble distance* is defined by

$$d_{\rm H}(t) = cH^{-1}(t). \tag{7}$$

The Hubble distance $d_{\rm H}(t)$, often described simply as the *'horizon'* and corresponds to the typical length-scale over which physical processes in the universe operate coherently. It is also the length-scale at which general-relativistic effects become important; on scales much less than $d_{\rm H}(t)$ (within the horizon), Newtonian theory is often sufficient to describe the effects of gravitation [4].

We also introduce the *comoving Hubble distance* as:

$$\chi_{\rm H} = \frac{d_{\rm H}(t)}{R(t)} = \frac{c}{H(t)R(t)} = \frac{c}{\dot{R}(t)}, \tag{8}$$

which gives the χ-coordinate corresponding to the Hubble distance.

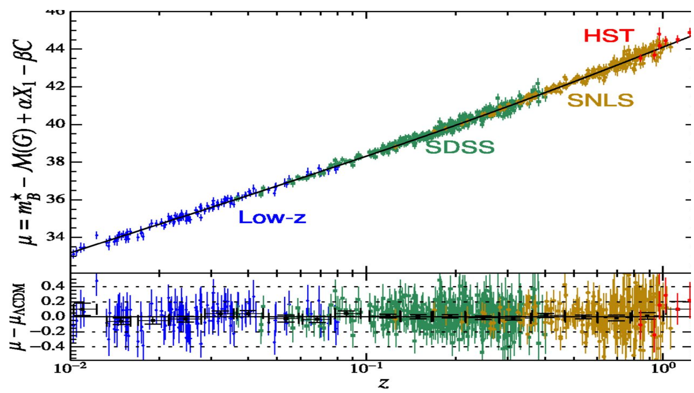

Figure 1: Supernovae

A classical way of measuring distances in astronomy is to measure the flux from an object of known luminosity, for example from Supernovae Type Ia (SNe Ia). Let us consider the observed flux F_o at a distance d_L from an emitting source of known luminosity L_e (J s^{-1}):

$$F_o = \frac{L_e}{4\pi d^2}. \tag{9}$$

The quantity

$$d_L = \left(\frac{L_e}{4\pi F_o}\right)^{1/2} \tag{10}$$

is called the **luminosity distance** of the source.
In a FRW Universe, the proper area of this sphere is

$$A = 4\pi R^2(t_0)S_k^2(\chi). \tag{11}$$

The photon frequency received by an observer is redshifted by a factor

$$\frac{\nu_0}{\nu_e} = \frac{R(t_e)}{R(t_0)} = \frac{1}{1+z},$$

and also the rate of the photons that fall into the detector is also reduced by the same factor. Therefore, the observed flux will be

$$F(t_0) = \frac{L(t_e)}{4\pi[R_0 S_k(\chi)]^2}\frac{1}{(1+z)^2}.$$

Then, the *luminosity distance* d_L in terms of measurable quantities is

$$d_L(z) \equiv (1+z)R_0 S_k(\chi). \tag{12}$$

The distance-redshift relation is, in fact, one of the most important cosmological tests. This is because given the observables H_0, $\Omega_{i,0}$ and the expression (12) we can compute the luminosity distance to an object at any redshift z. Conversely, for a population of standard candles with absolute magnitude M, and apparent magnitude m, we can measure the object's distance modulus μ at a given redshift z, defined by

$$\mu \equiv m - M = 5\log_{10}\left(\frac{d_L(z)}{1\text{Mpc}}\right) + 25. \tag{13}$$

Then, the relationship of μ with redshift allows us to estimate the luminosity distance and thereby constrain the cosmological parameters, as we will see in Chapter **??**.

Another classical distance measurement in astronomy is to measure the angle $\delta\theta$ subtended by an object of known physical size l. The *angular distance* is then defined as

$$d_A = l/\delta\theta.$$

From the angular part of the FRW metric, we have

$$l = R(t_e)S(\chi)\delta\theta$$

so that

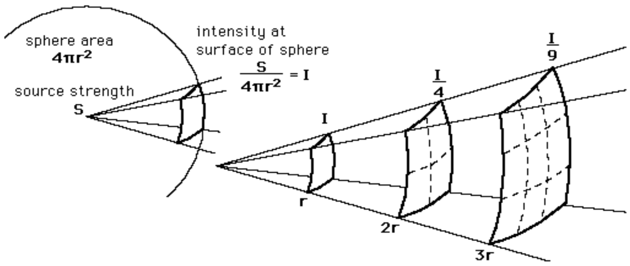

Figure 2: (jav: caption)

$$d_A = R(t_e)S(\chi) = R(t_0)\frac{R(t_e)}{R(t_0)}S(\chi) = \frac{R(t_0)S(\chi)}{1+z}.$$

Thus the angular distance is given by

$$d_\mathrm{A} \equiv \frac{R_0 S_k(\chi)}{(1+z)}, \tag{14}$$

or the comoving angular distance

$$d_\mathrm{M} = R_0 S_k(\chi). \tag{15}$$

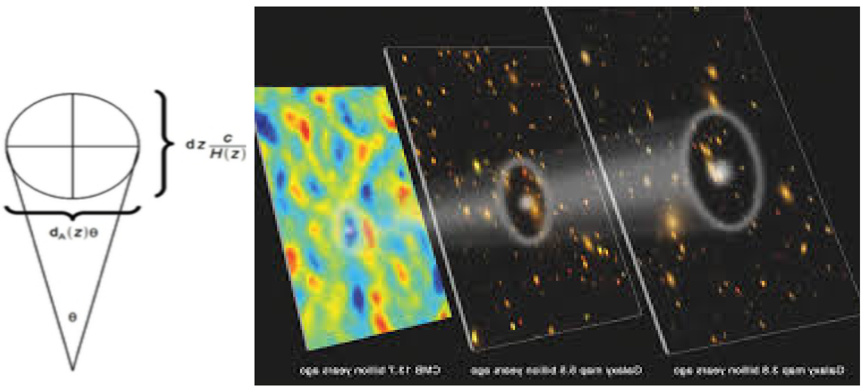

Figure 3: (jav: caption)

Curvature affects $D_M(z)$ both through its influence on $H(z)$ and through the geometrical factor. The luminosity distance (relevant to supernovae) is related to the angular distance by

$d_L = d_M(1+z)$.

Hubble tension: https://arxiv.org/pdf/2012.13932.pdf

Graduated paper, Fig 4: https://arxiv.org/pdf/2108.09239.pdf

If redshift-space distortions are weak, which is a good approximation for luminous galaxy surveys after reconstruction, but not for the LyaF, then the constrained quantity is the volume averaged distance

$$d_V(z) = [z d_H(z) d_M^2(z)]^{1/3}. \tag{16}$$

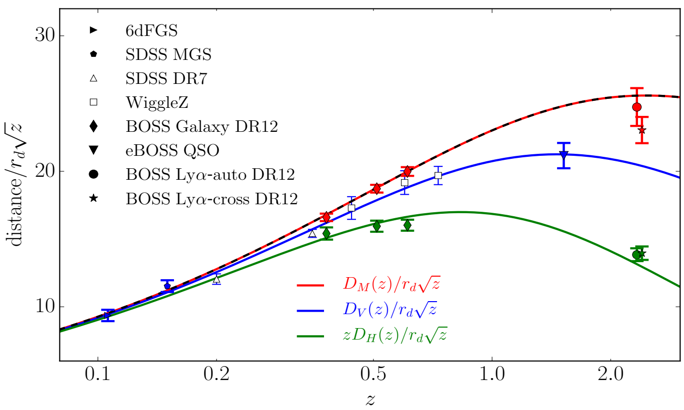

Figure 4: (jav: see: https://arxiv.org/pdf/1411.1074.pdf)

Figure 5 sketches the distances d_c, d_L and d_A in terms of redshift. It is worthwhile noticing that for small scales, all these distance measures coincide

$$d \simeq \frac{z}{H_0}, \tag{17}$$

where the linear evolution of distance with redshift is referred as the *Hubble law* [5].

0.1.1 Look-back time

A general expression for the look-back time

$$t_0 - t = \int_t^{t_0} dt = \int_0^z \frac{dz}{(1+z)H(z)} = \tag{18}$$

t emitted, and t_0 received.

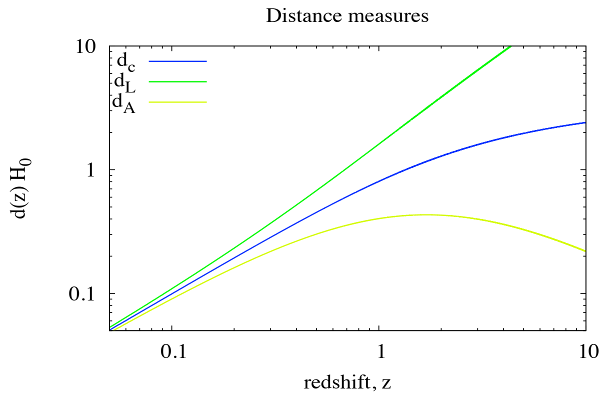

Figure 5: Comoving distance d_c, luminosity distance d_L, and angular distance d_A for a universe filled with the same constituents as in Figure **??**. (jav: Add a dash line with different components. Use python)

$\Omega_{m,0}$	$\Omega_{\Lambda,0}$	H_0=50	70	90
1.0	0.0	13.1	9.3	7.2
0.3	0.0	15.8	11.3	8.8
0.3	0.7	18.9	13.5	10.5

Table 1: Age of the Universe (Gyr). Fijar parametros, usar w0=-1.5, -1, -0.5, wa=-0.5, 0, 0.5

$$t_0 - t = \int_0^z \frac{d\bar{z}}{(1+\bar{z})H(\bar{z})} \tag{19}$$

$$= \frac{1}{H_0}\int_{(1+z)^{-1}}^{1} \frac{x dx}{\sqrt{\Omega_{m,0}x + \Omega_{r,0} + \Omega_{\Lambda,0}x^4 + \Omega_{k,0}x^2}} \tag{20}$$

The oldest star in globular clusters $t_{\rm star} \approx 11.5 \pm 1.3$ Gys, hence $t_0 > t_{\rm star}$.

0.1.1.1 Alternatives to the ΛCDM model

The ΛCDM model has had great success in modeling a wide range of astronomical observations. However, it is in apparent conflict with some observations on small-scales within galaxies (e.g. cuspy halo density profiles, overproduction of satellite dwarfs within the Local Group, amongst many others, see for example [**? ?**]). In addition, all attempts to detect WIMPs either directly in the laboratory, or indirectly by astronomical signals of distant objects have failed so far. Also, a large range of the particle parameters – predicted to be detectable – have thereby been ruled out. For some of these reasons, it seems necessary to explore alternatives to the standard ΛCDM model. With this in mind, several alternatives have been suggested. For instance the Scalar Field Dark Matter (SFDM) model proposes the dark matter is a spin 0 bosson particle [**? ? ? ? ?**]; or the Self Interacting Dark Matter, as its name states, it relies on the cold dark matter to be made of self interacting particles [**?**]. On the other hand, in order to explain the accelerated expansion of the universe there exist different modifications to the theory of General Relativity, i.e. $f(R)$ theories [3**?**], braneworld models [**? ?**]. There are also several candidates to be the dark energy of the universe – alternatives to the cosmological constant –, i.e. scalar fields (quintessence, K-essence, phantom, quintom, non-minimally coupled scalar fields [**? ? ?**] ; or many more alternatives i.e. anisotropic universes [**? ? ?**]. Finally, if the dark energy is assumed to be a perfect fluid, then one of the most popular time-evolving parameterization for its equation of state consists of expanding ω in a Taylor series, for example the Chevallier-Polarski-Linder (CPL) $\omega = \omega_0 + \omega_a\,(1 - a)$, with two free parameters ω_0, ω_a [1, 6]. It may also be expanded into Fourier series [**?**] or many more Bayesian approaches have been suggested to account for a dynamical dark energy [**?**].

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