Preface

Integral transforms are among the main mathematical methods for the solution of equations describing physical systems, because, quite generally, the coupling between the elements which constitute such a system—these can be the mass points in a finite spring lattice or the continuum of a diffusive or elastic medium—prevents a straightforward "single-particle" solution. By describing the same system in an appropriate reference frame, one can often bring about a mathematical *uncoupling* of the equations in such a way that the solution becomes that of noninteracting constituents. The "tilt" in the reference frame is a finite or integral transform, according to whether the system has a finite or infinite number of elements. The types of coupling which yield to the integral transform method include diffusive and elastic interactions in "classical" systems as well as the more common quantum-mechanical potentials.

The purpose of this volume is to present an orderly exposition of the theory and some of the applications of the finite and integral transforms associated with the names of Fourier, Bessel, Laplace, Hankel, Gauss, Bargmann, and several others in the same vein.

The volume is divided into four parts dealing, respectively, with finite, series, integral, and canonical transforms. They are intended to serve as independent units. The reader is assumed to have greater mathematical sophistication in the later parts, though.

Part I, which deals with finite transforms, covers the field of complex vector analysis with emphasis on particular linear operators, their eigenvectors, and their eigenvalues. Finite transforms apply naturally to lattice structures such as (finite) crystals, electric networks, and finite signal sets.

Fourier and Bessel series are treated in Part II. The basic theorems are proven here in the customary classical analysis framework, but when introducing the Dirac δ, we do not hesitate in translating the vector space concepts from their finite-dimensional counterparts, aiming for the rigor of most mathematical physics developments. The appropriate warning signs are placed where one is bound, nevertheless, to be led astray by finite-dimensional analogues. Applications include diffusive and elastic media of finite extent and infinite lattices.

Fourier transforms occupy the major portion of Part III. After their introduction and the study of their main properties, we turn to the treatment of certain special functions which have close connection with Fourier transforms and which are, moreover, of considerable physical interest: the attractive and repulsive quantum oscillator wave functions and coherent states. Other integral transforms (Laplace, Mellin, Hankel, etc.) related to the Fourier transform and applications occupy the rest of this part.

"Canonical transforms" is the name of a parametrized continuum of transforms which include, as particular cases, most of the integral transforms of Part III. They also include Bargmann transforms, a rather modern tool used for the description of shell-model nuclear physics and second-quantized boson field theories. In the presentation given in Part IV, we are adapting recent research material such as canonical transformations in quantum mechanics, hyperdifferential operator realizations for the transforms, and similarity groups for a class of differential equations. We do not explicitly use Lie group theory, although the applications we present in the study of the diffusion and related Schrödinger equations should cater to the taste of the connoisseur.

On the whole, the pace and tone of the text have been set by the balance of intuition and rigor as practiced in applied mathematics with the aim that the contents should be useful for senior undergraduate and graduate students in the scientific and technical fields. Each part contains a flux diagram showing the logical concatenation of the sections so as to facilitate their use in a variety of courses. The graduate student or research worker may be interested in some particular sections such as the fast Fourier transform computer algorithm, the Gibbs phenomenon, causality, or oscillator wave functions. These are subjects which have not been commonly included under the same cover. Part IV, moreover, may spur his or her interest in new directions. We have tried to give an adequate bibliography whenever our account of an area had to stop for reasons of specialization or space. References are cited by author's name and year of publication, and they are listed alphabetically at the end of the book. A generous number of figures and some tables should enable the reader to browse easily. New literals are defined by "$:=$"; thus $f := A$ means f is defined as the expression A. Vectors and unargumented functions are denoted by lowercase boldface type and matrices by uppercase boldface. Operators appear in "double" type, e.g., $\mathbb{Q}$, $\mathbb{P}$, and sets in script, e.g., $\mathscr{R}, \mathscr{C}$. A symbol list is included at the end.

Exercises are used mainly to suggest alternative proofs, extensions to the text material, or cross references, usually providing the answers as well as further comments. They are meant to be read at least cursorily as part of the text. Equations are numbered by chapter.

I would like to express my gratitude to Professor Tomás Garza for his encouragement and support of this project; to my colleagues Drs. Charles P. Boyer, Jorge Ize, and Antonmaría Minzoni among many others at IIMAS, Instituto de Física, and Facultad de Ciencias, for their critical comments on the manuscript; and to my students for bearing with the first versions of the material in this volume. The graphics were programmed by the author on the facilities of the Centro de Servicios y Cómputo and plotted at the Instituto de Ingeniería, UNAM. Special acknowledgment is due to Miss Alicia Vázquez for her fine secretarial work despite many difficulties.

Ciudad Universitaria, México D.F. Kurt Bernardo Wolf