Notation

General Conventions

a (lowercase boldface)	vector
A (uppercase boldface)	matrix
$\mathscr{A}$ (script)	set
$\mathbb{A}$ (double)	operator
a^* (asterisk)	complex conjugate
$\mathbf{A}^\dagger$ (dagger)	adjoint

Re	real part
Im	imaginary part
Res	residue
$\mathscr{C}$	set of complex numbers
$\mathscr{R}$	set of real numbers
$\mathscr{Z}$	set of integer numbers
$\mathscr{Z}^+$	set of natural numbers
$\sum$	sum
$\prod$	product
(a, b)	open interval
$[a, b]$	closed interval
$\lvert a \rvert$	absolute value of a
$(\mathbf{a}, \mathbf{b})$	inner product of **a** and **b**
$\lVert \mathbf{a} \rVert$	norm of **a**
(a_n)	vector with components a_n
$\lVert A_{mn} \rVert$	matrix of elements A_{mn}
$\in$	is an element of
$:=$	is defined as
$=:$	defines
$\Rightarrow$	implies, only if
$\Leftarrow$	is implied by, if

Symbol List

$\mathrm{Ai}(z)$	Airy function of the first kind [Eqs. (B.24a), (B.24b)]
$\mathrm{Bi}(z)$	Airy function of the second kind [Eqs. (B.24c), (B.24d)]
$\mathbf{B}$	Bargmann transform matrix [Eqs. (9.55)]
$\mathscr{B}_M$	canonical transform of $\mathscr{L}^2(\mathscr{R})$-space [p. 394]
$C_M(q', q)$	canonical transform integral kernel [Eqs. (9.8)]
$\mathbb{C}_M$	canonical transform operator [p. 382]

$\mathscr{C}_{\downarrow}^{\infty}$	space of infinitely differentiable functions of fast decrease [p. 263]
$\mathbb{D}_a$	dilatation operator [Eq. (7.34)]
$D_k(x)$	Dirichlet kernel [Eq. (4.19)]
$\mathbb{E}_x$	multiplication by exponential operator [Eq. (7.29)]
$\{f_n\}_{n=1}^N$	coordinates of vector $\mathbf{f} \in \mathscr{V}^N$ [Eq. (1.2)]
$\{\tilde{f}_n\}_{n=1}^N$	Fourier transforms of $\{f_n\}_{n=1}^N$ [Eq. (1.51)]
$\{f_n\}_{n\in z}$	Fourier series coefficients of $f(x)$ [Eq. (4.17b)]
$\tilde{f}(p)$	Fourier transform of $f(q)$, $q \in \mathscr{R}$ [Eqs. (7.1)]
$f^B(g')$	Bargmann transform of $f(q)$ [p. 399]
$f^{BL}(s)$	bilateral Laplace transform of $f(q)$ [Eqs. (8.1)]
$f_\sigma^{BM}(\lambda)$	bilateral Mellin transform of $f(q)$ [Eqs. (8.26)]
$f^L(s)$	Laplace transform of $f(q)$ [Eqs. (8.9)]
$f^M(\lambda)$	Mellin transform of $f(q)$ [Eqs. (8.29)]
$f^M(q')$	M-canonical transform of $f(q)$ [Eq. (9.5)]
$\hat{f}_c(p)$	Cauchy representation of $f(q)$ [Eq. (7.136)]
$f_{a\infty}(q)$	function with support on $[a, \infty)$ [Eq. (7.125)]
$\mathbf{f}\,(\boldsymbol{\varepsilon})\,\mathbf{g}$	product relative to $\boldsymbol{\varepsilon}$-basis [p. 102]
$\mathbf{F}$	Fourier canonical transform matrix [Eq. (9.32)]
$\mathbf{F} = \|F_{mn}\|$	Fourier (finite) transform matrix [Eq. (1.48)]
$\mathbb{F}, \mathbb{F}^{-1}$	Fourier integral transform operator and its inverse [Eqs. (7.1)]
$\mathbb{F}_{(N)}, \mathbb{F}_{(N)}^{-1}$	N-dimensional Fourier transform and its inverse [Eqs. (8.38)]
$G_{\ldots}^{\ldots}(q, t)$	Green's function for a system
$G_\omega(q)$	Gaussian bell function of width ω [Eq. (7.20)]
$\mathbb{G}(t)$	time-evolution operator [finite lattice, p. 54; heat equation, p. 199; wave equation, p. 210]
$\mathbb{G}_{\text{II}}(t)$	phase-space evolution operator [Eq. (2.113)]
$\mathbb{G}_\omega$	Gaussian operator [Eq. (7.74)]
$H_n(q)$	hermite polynomials [Eq. (7.192)]
$\mathbb{H}_{\text{II}}$	phase-space evolution generator [Eq. (2.107)]
$\mathbb{H}_\mu, \mathbb{H}_\mu^{-1}$	Hankel transform operator and its inverse [Eqs. (8.83)]
$\mathbb{H}_\mu^B, \mathbb{H}_\mu^{B-1}$	Hankel–Bochner transform operator and its inverse [Eqs. (8. 66)]
$\mathbb{H}^f, \mathbb{H}^l, \mathbb{H}^r, \mathbb{H}^h$	Schrödinger Hamiltonian operators for the free particle, linear potential, repulsive and attractive oscillator [Eqs. (9.34), (9.76)]
I	group of inhomogeneous linear canonical transformations [p. 420]
$I_k(x)$	modified Bessel function [Eq. (B.18)]
$\mathbf{I}_0\,(\mathbb{I}_0)$	inversion matrix (operator) [Eqs. (1.54), (1.82), and (4.41)]
$\mathbb{I}_{\text{FP}(t)}$	time-evolution operator of the Fokker–Planck equation [Eq. (10.50)]

$\mathbb{I}_{H(t)}$ time-evolution operator of the diffusion equation [p. 442]
$\mathbb{I}\{\mathbf{M}, \boldsymbol{\xi}, z\}$ inhomogeneous linear canonical transform operator [Eqs. (10.7)]
$\mathbb{I}_{\omega(t)}$ time-evolution operator generated by $\mathbb{H}^{\omega}$ [Eq. (10.46)]
j_{mn} nth zero of $J_m(x)$ [Table B.1]
$J_m(x)$ Bessel function [Appendix B]
$\mathbb{J}_0, \mathbb{J}_1, \mathbb{J}_2, \mathbb{J}_{\pm}$ set of second-order differential operators [Eqs. (7.174), (9.34), (9.76), (10.37c)]
$K_B(q', q)$ Bargmann reproducing kernel [Eq. (9.63)]
$K_n(x)$ Macdonald function [Eq. (B.20)]
$\mathbb{K}, \mathbb{C}, \mathbb{M}$ lattice interaction, damping, and mass operators [p. 51]
$\mathbf{L}$ Laplace transform matrix [Eq. (9.68)]
$\mathbb{L}, \mathbb{L}^{-1}$ Laplace transform operator and its inverse [Eqs. (8.9)]
$\mathbb{L}_B, \mathbb{L}_B^{-1}$ bilateral Laplace transform operator and its inverse [Eqs. (8.1)]
$\mathscr{L}^2(\mathscr{I})$ space of (Lebesgue) square-integrable functions over the interval $\mathscr{I}$ [p. 142 and 264]
$\mathscr{L}_0{}^2(R)$ space of (Lebesgue) square-integrable functions which vanish at the boundary of a region R [p. 222]
$\mathscr{L}_{\omega}{}^2(\mathscr{I})$ space of (Lebesgue) square-integrable functions on $\mathscr{I} \subseteq \mathscr{R}$ with weight function $\omega(x)$ [p. 297]
$\mathbb{M}, \mathbb{M}^{-1}$ Mellin transform operator and its inverse [Eqs. (8.29)]
$\mathbb{M}_B, \mathbb{M}_B^{-1}$ bilateral Mellin transform operator and its inverse [Eqs. (8.26)]
$N_m(x)$ Neumann function [Eq. (B.14)]
$p_{\pm}, q_{\pm}$ "cut" power functions [Eq.(7.202)]
$\mathbf{P}(^{m}_{\pi(m)})$ permutation matrix [Eq. (1.45)]
$\mathbb{P}$ $-i$ times the differentiation operator (the quantum-mechanical momentum operator) [Eq. (7.56)]
$\mathscr{P}\int$ principle value of an integral [Eq. (7.138)]
$\mathbb{Q}$ multiplication by argument operator [Eq. (7.55)]
$R^{(\varepsilon,\eta)}(x)$ rectangle function of width ε and height η [Eqs. (4.24), (7.4)]
$\mathbf{R}$ ($\mathbb{R}$) rotation matrix (operator) [Eq. (1.81)]
$R_{\square}$ (hyper-) rectangular region [p. 223]
$R_{\bigcirc}$ circular region [p. 230]
$R_{\diamond}$ sectorial region [p. 240]
$R_{\odot}$ annular region [p. 243]
S_{N-1} sphere in N dimensions [p. 363]
$S^{(P,\eta)}(x)$ square wave of P pulses and height η [Eqs. (4.39)]
$SL(2, \mathscr{R})$ group of unimodular 2×2 real matrices [p. 390]
$\mathscr{S}_1, \mathscr{S}_2$ set of first- and second-order differential operators [p. 434]
$T^h(x)$ triangle function of height h [Eq. (4.28)]
$\mathbb{T}_a$ translation operator [Eqs. (4.36a), (7.27)]
$\mathscr{U}$ set of unitary matrixes [p. 14]

$\mathscr{V}^N$ N-dimensional complex vector space [Sect. 1.1]
W Heisenberg–Weyl group [p. 418]
W_f equivalent width of $f(q)$ [Eqs. (4.69a), (7.222)]
$\mathbf{W}(t)$ Gauss–Weierstrass transform matrix [Eq. (9.67)]
$\mathbb{W}(x, y, z)$ (Heisenberg–Weyl) $\mathbb{W}$ transform operator [Eqs. (10.3)]
$\mathbb{Z}^\dagger, \mathbb{Z}$ harmonic oscillator raising and lowering operators [Eqs. (7.160)]

$\Gamma(x)$ gamma function [Appendix A]
$\mathbf{\Gamma}(g)$ 6×6 matrix representation of the inhomogeneous canonical transformation g [Eqs. (10.38)]
$\delta(q)$ Dirac δ [Eqs. (4.79), (7.85)]
$\delta^{(n)}(q)$ nth derivative of the Dirac δ [Eqs. (4.94), (7.87)]
$\boldsymbol{\delta}_y,\ y \in \mathscr{I}$ Dirac's generalized basis [pp. 133, 183]
$\delta_{m,n}$ Kronecker δ [Eq. (1.6)]
Δ_f dispersion of $f(q)$ [Eq. (7.217)]
$\mathbf{\Delta}$ matrix representative of $\mathbb{\Delta}$ in the ε-basis [Eq. (1.60)]
$\tilde{\mathbf{\Delta}}$ matrix representative of $\mathbb{\Delta}$ in the $\boldsymbol{\varphi}$-basis [Eq. (1.62)]
$\mathbb{\Delta}$ second-difference operator [p. 21]
$\{\boldsymbol{\varepsilon}_n\}_{n=1}^N$ basis for $\mathscr{V}^N$ [Sect. 1.1]
$\boldsymbol{\varepsilon}^m(t)$ lattice fundamental solutions [Eqs. (2.41), (2.42)]
$\theta(x, t)$ (Jacobi) theta function [Eq. (4.64)]
$\Theta(q)$ Heaviside theta function [Eq. (7.89)]
$\Theta_\varepsilon(q)$ causal exponentially damped function [Eq. (7.123)]
$\nu_M(q, q^*)$ weight function for the $\mathbb{C}_M$ transform space [Eq. (9.47)]
$\Upsilon_c(q)$ coherent states [Eqs. (7.188)]
$\{\boldsymbol{\varphi}_n\}_{n=1}^N$ finite Fourier transform basis [Eqs. (1.52)]
$\boldsymbol{\varphi}^m(t)$ lattice normal modes [Eqs. (2.46)]
$\varphi_n(x)$ imaginary exponential functions [Eq. (4.9)]
$\varphi_n(x, t)$ vibrating string normal modes [Eqs. (5.31)]
$\varphi_n{}^y(t)$ infinite lattice normal modes [Eqs. (5.52)]
$\varphi_{n_1,n_2}(x_1, x_2, t)$ rectangular membrane normal modes [Eq. (6.12)]
$\varphi_{mn}^{\bigcirc}(r, \phi, t)$ circular membrane normal modes [Eq. (6.27)]
$\varphi_{mn}^{\diamond}(r, \phi, t)$ sectorial membrane normal modes [Eq. (6.34)]
$\chi_\lambda{}^{\pm}(q)$ repulsive oscillator wave functions [Eqs. (7.203)]
$\Psi_n(q), \Psi_n{}^h(q)$ harmonic oscillator wave functions [Eq. (7.166)]
$\Psi_\lambda{}^{\cdot}, {}^H(q, t)$ time evolution of $\Psi_\lambda{}^{\cdot}(q)$ under the diffusion equation [Eqs. (10.27), (10.28), (10.32), (10.33)]
$\mathbf{\Omega}$ symplectic 2×2 metric matrix [Eqs. (10.8), (10.39) *et seq.*]
Ω^ω class of operators in the same orbit as $\mathbb{H}^\omega$ [Eq. (10.40)].
$\binom{m}{\pi(m)}$ permutation [p. 14]
$\mathbb{\nabla}$ differentiation operator
$\mathbb{\nabla}^2, \mathbb{\Delta}$ Laplacian operator