

Independent simultaneous discoveries visualized through network analysis: the case of linear canonical transforms

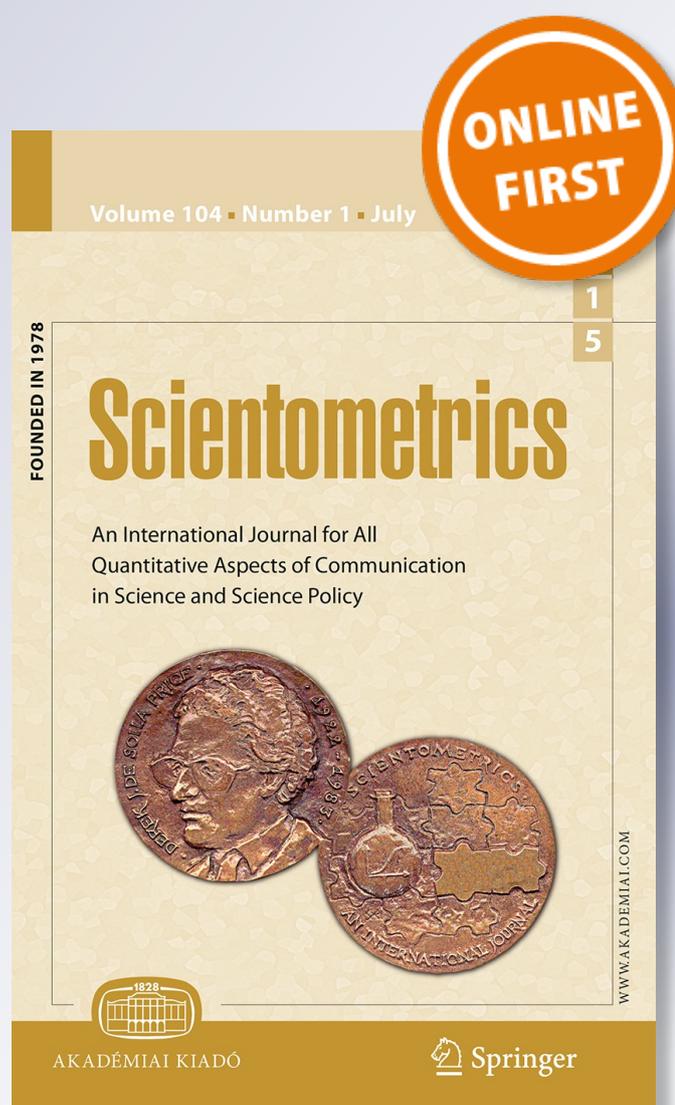
Sofia Liberman & Kurt Bernardo Wolf

Scientometrics

An International Journal for all
Quantitative Aspects of the Science of
Science, Communication in Science and
Science Policy

ISSN 0138-9130

Scientometrics
DOI 10.1007/s11192-015-1602-x



Your article is protected by copyright and all rights are held exclusively by Akadémiai Kiadó, Budapest, Hungary. This e-offprint is for personal use only and shall not be self-archived in electronic repositories. If you wish to self-archive your article, please use the accepted manuscript version for posting on your own website. You may further deposit the accepted manuscript version in any repository, provided it is only made publicly available 12 months after official publication or later and provided acknowledgement is given to the original source of publication and a link is inserted to the published article on Springer's website. The link must be accompanied by the following text: "The final publication is available at link.springer.com".

Independent simultaneous discoveries visualized through network analysis: the case of linear canonical transforms

Sofia Liberman¹  · Kurt Bernardo Wolf²

Received: 20 November 2014
© Akadémiai Kiadó, Budapest, Hungary 2015

Abstract We describe the structural dynamics of two groups of scientists in relation to the independent simultaneous discovery (i.e., definition and application) of linear canonical transforms. This mathematical construct was built as the transfer kernel of paraxial optical systems by Prof. Stuart A. Collins, working in the ElectroScience Laboratory in Ohio State University. At roughly the same time, it was established as the integral kernel that represents the preservation of uncertainty in quantum mechanics by Prof. Marcos Moshinsky and his postdoctoral associate, Dr. Christiane Quesne, at the Instituto de Física of the Universidad Nacional Autónoma de México. We are interested in the birth and parallel development of the two follower groups that have formed around the two seminal articles, which for more than two decades did not know and acknowledge each other. Each group had different motivations, purposes and applications, and worked in distinct professional environments. As we will show, Moshinsky–Quesne had been highly cited by his associates and students in Mexico and Europe when the importance of his work started to permeate various other mostly theoretical fields; Collins' paper took more time to be referenced, but later originated a vast following notably among Chinese applied optical scientists. Through social network analysis we visualize the structure and development of these two major coauthoring groups, whose community dynamics shows two distinct patterns of communication that illustrate the disparity in the diffusion of theoretical and technological research.

Keywords Psychology of science · Simultaneous discoveries · Network analysis · Scientific communication · Linear canonical transforms

Mathematical Subject Classification 91D30 · 01A65 · 65R10

✉ Sofia Liberman
liberman@unam.mx

¹ Facultad de Psicología, Universidad Nacional Autónoma de México, Mexico, Mexico

² Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Mexico, Mexico

Introduction

Independent multiple discoveries in science can appear simultaneously, or almost simultaneously, within a lapse of time that might be confused as being true separate discoveries, re-discoveries, co-discoveries, or plagiarism. For these to be real simultaneous discoveries, it is important that they be chronologically occurring within a short time from each other. Studies on simultaneous and independent multiple discoveries in science have concentrated in detecting and collecting evidence of their occurrence, often in the context of their discoverer's disputes. In the case we describe here, that of the linear canonical transformation integral kernel, discovery was almost simultaneous, but their simultaneity was not recognized during a couple of decades, and did not involve a priority conflict, because one of the follower groups works in theoretical and mathematical physics, and the other in separate applied fields of electromagnetic optics.

Reasons for the appearance of simultaneous discoveries have been approached from various points of view. Kroeber (1917) established that if one of the inventors had not made the discovery, it would have occurred in any case not much later, because of a supposed social determinism. Similarly, Ogburn and Thomas (1922) attributed the occurrence of multiple simultaneous discoveries to the scientific background existing at the time, which recognized some missing link, insight or experiment to complete a theory. These authors have listed 148 independent discoveries made between 1420 and 1901, observing that the rate of such simultaneous discoveries has increased over time, and concluding that the inventions were inevitable due to the cultural heritage of accumulated knowledge. Merton (1973) drew attention to the fact that in the context of sociological tradition, discoveries of some relevance occur in multiples; he argued that they are not random, but arise from the existing stock of knowledge. Merton's contribution generated a tradition for studying multiples based on the same idea that Galton (1874) proposed in his book "English men of science": basically, that discoveries come about when it is their time and where a genius scientist is working very hard on the problem to be solved.

Social interaction between scientists is a key for recognition in science and priority debates can generate passions that go far and become sordid and painful. Merton (1961) poses several lines of evidence for discoveries to be time multiples, which underlines the fact that singletons are very rare. Statistical analysis of historical data related to discoveries contribute to the explanation of the role of social context in their simultaneous occurrence. However, some confusion can be created by poor accessibility to the originally produced knowledge, publication times and/or the existence of unpublished material, which can come out much later, confirming the appearance of multiples that were thought to be singletons.

Hagstrom (1974, p. 15) established that competition in science leads to the recognition of missing links in a field, and the anticipation of the solution in independent simultaneous discoveries. Theoretical models have established diverse hypothesis about the causes of the appearance of multiples, either by implying the incidence of scientific genius, *zeitgeist*, or mere chance, as demonstrated through their Poisson distribution (Simonton 1979). From the observed occurrence of n -tuple simultaneous discoveries, Simonton (1978, 1979)

empirically fitted a Poissonian distribution $P(n)$ for n from 2 to 5, obtaining the least variance among two other models and concluding that the process is mostly stochastic (Simonton 1986). A recent review article (Simonton 2010) presents the stochastic and *zeitgeist* structures in various combinatorial models of creativity.

In this paper we describe the development of a two-fold simultaneous discovery in two relatively distant fields, optics and quantum mechanics, from 1970 to the present. We report rather briefly on the *zeitgeist* and stochasticity present at that time in both fields; while the optical applications were in some sense demanding a compact formalism to simplify calculations for the then-new Gaussian coherent states, in quantum mechanics the mathematical formalism of linear canonical transforms was available under various particular guises but was seriously broached by one researcher apparently by (informed) chance. There is hardly an objective way to reconstruct the previous process through interviews or data obtained from the scientists in a direct manner, even when the inventors are still alive. In any case, our main interest lies in the two contrasting developments that took place independently in the optics and theoretical physics communities until their common acknowledgement.

Brannigan and Wanner (1983) challenge Simonton's model of Poisson distribution of multiples, arguing that communication plays an important role in multiple discoveries. Here we examine the particular case of a twofold discovery in which the authors were clearly unaware of each other's motivation and contemplated applications. In each field there was a certain *zeitgeist* in which we could place their papers that we shall briefly mention below; however, we are interested in visualizing and analyzing their *later* impact, within their fields and their communities, up to the point when they started communicating with each other. We believe that placing the seminal articles in the scientific literature through coauthorship analysis, we can build a structural picture of how the knowledge has evolved with its utilization in the solution of problems in optics and nuclear physics. It is thus of special interest to follow what happened *after* the discovery was published. This could be a concrete case study that would contribute to refine the extant theories on multiples.

Informal communication between scientists and the channels they use to find information have changed rapidly as electronic access to databases have become increasingly efficient. The evolution of knowledge networks can be measured in many ways, from counting of the number of published articles and books that cite a discovery, to registering the increase of scientists dedicated to the subject or field opened by the discovery. What simultaneous discoveries may in fact reveal is that they there are different lines of thought distributed horizontally through the communication networks of science, based on the same abstract result, but focused to the specific systems studied by disjoint research communities.

Linear canonical transforms: some biographical data

The expression for the Linear canonical transform integral kernel was published in 1970 by Stuart A. Collins Jr. in the *Journal of the Optical Society of America* (Collins 1970), and by Marcos Moshinsky and Christiane Quesne in two adjoining articles in the *Journal of Mathematical Physics* (Moshinsky and Quesne 1971). The latter was presented at the 15th Solvay Conference in Physics of 1970, whose Proceedings were delayed 4 years (Moshinsky and Quesne 1974).

We will describe the context in which these scientists worked in order to understand the social bases of their recognition by the communities of optical engineers and theoretical physicists, which did not initially realize the coincidence and differences between the two seminal articles.

Stuart Collins was born in the United States of America in 1932. He obtained a Ph.D from the Massachusetts Institute of Technology, and has been working at the Department of Electrical Engineering of the Ohio State University since 1964, where he is an Emeritus Professor. He has 26 published articles and owns 10 patents in optical resonators and lens design, holography, fiber optics and photonics of phased microwave arrays. His seminal article (1970) accounts 893 citations (10/08/14), but Prof. Collins did not visibly pursue further the lines of research that followed. As Fig. 1 attests, this article was cited sporadically by US and Israeli optical researchers, until around 1990 when it was referenced by Lu Baida in China, in a paper that became highly cited among his colleagues together with the original formula referred to Prof. Collins.

Marcos Moshinsky was born in Kiev, Ukraine in 1921. He emigrated to México as a child refugee and started his studies at the Universidad Nacional Autónoma de México in 1942 where he obtained his B. Sc., and in 1949 his Ph.D. from Princeton University working under the advice of Nobel prizewinner Eugene Wigner. He worked at the Instituto de Física of the Universidad Nacional Autónoma de México, wrote over 200 scientific articles, received the Wigner Medal in 1988, was a highly respected scientific figure, and died in México in 2009 at the age of 87. He had been working in theoretical nuclear physics, in particular in alpha decay that modeled this particle as a Gaussian packet within the nucleus, when he received Dr. Christiane Quesne as a postdoctoral associate from the Université Libre de Bruxelles. Although Dr. Quesne does not recall the exact reason why Moshinsky thought the problem of canonical transformations in quantum mechanics was interesting, their resulting articles became the basis for several collaborations and independent work by his Mexican and European colleagues; his seminal paper has received 406 citations (10/08/14). A book on integral transforms by Wolf (1979) containing two chapters on the linear canonical transforms, their unitary complexification and applications to diffusive systems, condensed and expanded the existing knowledge, and served as reference for further citations to the source.

The context of geometric optical research at the beginning of the 70s had standardized the use of matrices to describe and concatenate optical elements in a setup, as formalized in the book by Brouwer (1964) and shortly thereafter by Gerrard and Burch (1975) for optical resonators. The corresponding wave model was known to proceed through the Fresnel

integral transform for free flights and a Gaussian phase for thin lenses. One may indeed wonder why the product of these elements was not compounded to find the integral kernel of generic systems, instead of the longer route taken by Collins, as we detail below. [Within] the community there does not seem to have dawned the realization that *any* of those matrices can be corresponded with an optical setup. This fundamental mathematical property of optical transforms was not recognized, as we witness in the book of Orestes Stavroudis (1972) the surprising statement that “If we exclude these [elements of negative thickness], we are left with a collection of constructible lenses, which fails to be a *group* by the failure of the fourth postulate [existence of inverses]” (see p. 294).

The context in the mathematical community was more conducive to consider the representations of the abstract structures called *groups* through matrices whose rows and columns are not discrete, but continuous, unitary and infinite. Thus we have the early works of Infeld and Plebański (1955) and Itzykson (1969) where the application to the Lorentz group of special three-dimensional relativity yields the same integral kernel later found by Moshinsky and Quesne; the same group had been studied by Valentin Bargmann (1970) from the point of view of functional analysis. Although Plebański, Itzykson and Moshinsky knew each other personally, they apparently did not communicate or hint these endeavors to Moshinsky, whose interests at the time lay relatively far in nuclear physics, and who did not quote their work in the two articles written with Christiane Quesne.

We should now explain briefly what linear canonical transforms are, and highlight the two paths taken to its discovery. In geometric optics, a system composed of thin lenses and free spaces, transforms incoming rays near to the optical center and axis into outgoing rays of the same type, called *paraxial*, through a matrix which can be 2×2 or 4×4 for two- or three-dimensional systems; to conserve rays, these must have the property of being *symplectic*, and whose action can be seen clearest in phase space. The coordinates of phase space are the positions and angles of the light rays (times the refractive index of the medium)—see Wolf (2004). Collins’ approach was to consider the three-dimensional wave-optical model and compute the optical distance following Fermat’s principle of least action and the *eikonal* equation to solve for it; after reducing a six-fold integral in three pages, he obtains the correct amplitude and one of the phases of the point-spread function of the system. This is the canonical transform kernel, which integrated with the incoming wave field yields the outgoing wave field. The applications offered are an optical Fourier transformer and the generic behavior of Gaussian wave packets in paraxial systems.

As we mentioned above, the work of Moshinsky and Quesne was apparently not motivated by any concrete scientific problem, but addressed the properties of quantum-mechanical phase space, whose canonical coordinates are the positions and momenta of particles, and the linear transformations that preserve the Heisenberg commutation relations that embody their uncertainty principle. They used a pair of coupled differential equations to elegantly find the integral kernel in one page. Their viewpoint was to regard linear canonical transforms as integral representations of the group of $N \times N$ real symplectic matrices. The concatenation of two symplectic transformations was the subject of much of their concern due to the problem of adjusting the product phase, which they did not yet associate with the peculiar structure of the symplectic groups: their *multiple cover*.

The intrinsic mathematical interest of the construct presented by Moshinsky and Quesne led to further developments applied to other quantum systems, to diffusive heat systems, to

the matrix and integral representations of the group of relativity in two space and one time dimensions, to additions in special function theory and, rather belatedly, to scalar wave optics. It was in this last realm where the two follower communities of Collins and of Moshinsky finally met and sporadically started to acknowledge each other's source discovery. For readers interested in the mathematical formulations used by those authors and further developments that have taken place since, we direct them to the historical introduction in a book dedicated to linear canonical transform that gathers contributions from many of the workers in this field (Wolf 2015).

Technological application and theoretical physics had an impact on the growth of the two canonical transform research networks over time, but they kept separate for two decades. There still is an optics community and a theoretical physics community that do not often cite each other despite their common use of these transforms, because their journals, institutes, countries, subjects and applications continue to be different. The theoretical trend has shifted towards the analysis of discrete and finite data sets such as computers can handle; this also evinces a bifurcation between the search for efficient algorithms to use in encryption, metrology, holography and optical implementations, while for mathematical physicists the quest for subtler constructions based on symmetry is to catch the eye and please the mind.

Coauthorship analysis

Coauthorship and citation analyses are useful to understand how a seminal paper impacts on a research field over time, and both may be motivated by many reasons. Still, we believe that two or more scientists agree to coauthor a paper because they share interest in the problem to be solved, because they find each other's bags of knowledge complementary, and because the interaction is to the advantage of all. Similarly, most citations in the research literature we assume to be expressions of recognition of previous discoveries. Based on the topological structure of the coauthor and citations network of the seminal articles of Collins and Moshinsky–Quesne we can give a specific instance and topic to illustrate the spread of ideas in the scientific community over time.

Interaction at the informal level appears to be an important route to initiate formal collaboration and coauthorship in scientific papers. In the social psychological tradition, Homans established that the idea of *participating together* accounts for social interaction, and that a group is defined by the interaction of its members "...just by counting interactions we can map out a group quantitatively distinct from others" (1950, p. 70). This does not mean that a person cannot belong to more than one group, or that the group under description is not a subgroup in a much larger group; subgroups stand for different levels of analysis and have boundaries. Arranging data in rows and columns in a matrix describing social ties can be used to reveal their underlying structure: Moreno (1953) studied the patterns of interaction between children in order to describe and map the sociodynamic structure of choices and rejections; in this way it is possible to develop mathematical models of group behavior. Bavelas (1948, 1968) developed a mathematical model of group interaction under the hypothesis that structural centrality is related to influence in a group, and his model has contributed to the ideation of many other models to measure interaction (Freeman 1978). Last but not least, Milgram's famous study on the 'small world

experiment' showed how people are connected everywhere to a certain degree (Travers and Milgram 1969). These discoveries have influenced many studies in social network analysis.

The links and the structure of a network are symbolic in the sense that a link is not an actual face-to-face communication. Henry Small (1978, p. 337) claims that "A theory of citation practice, if such a theory is possible, must take account of the symbolic act of authors associating particular ideas with particular documents", showing how information is shared and accepted by a community of scientists. We also assume that, following scientific practices, scientists usually attend specialized meetings because these are opportunities to engage in face-to-face encounters and dialogue with their peers.

Scientific communication starts with an *inner cycle* of personal contacts between scientists in informal situations, either electronic contacts or face-to-face meetings at the workplace. In a second *outer cycle*, scientific communication becomes formal and is validated through publications that will appear in the public archive of knowledge contained in large databases that can be accessed through the Web. The collaboration between coauthors is the basis for the formation and structure of groups in science (Lieberman and Wolf 2013), and we presume that there is some awareness of this structure among the scientists involved. From the point of view of group psychology, we believe that a theory of group identity in a field or subfield of science can be based on the extant data on coauthorship. These observations follow the approach of group dynamics in social psychology according to the tradition of Bavelas' work and further developments of social network analysis (Freeman 2004). Alex Bavelas was a social psychologist located at MIT and is considered a very relevant researcher on group communication, information diffusion, network models and mathematical models of group communication. In his laboratory studies, communication between subjects is viewed as a behavioral event; since we study scientists, coauthorship is a behavioral event where researchers are forced to interact and build an interaction structure called a 'group' which can be studied from the point of view of social network analysis.

For a scientist that belongs to such a *reference group*, it entails the recognition and validation of his or her work that builds consensus and group identity. There are surely several other reasons for citations that we cannot tackle with our analysis; our intention is to understand the structural development of group formation assuming that citations acknowledge scientific priority. Here we study the topological progression in time, from a small cabal to a more complex network in the two communities, which addressed linear canonical transforms and their applications, as determined by their accumulating coauthor structure.

Method and data

We built a database from a search in the ISI Web of Knowledge in the Science Citation Index Expanded (SCI-EXPANDED) from the end of 1971 to June 2013 for the two seminal articles, knowing that authors and citations will keep increasing in the future. Our study is intended to portray how that community of scientists evolved during that period. We also downloaded results of the citation analysis provided by the Web of Knowledge, which helped us to describe the dynamics of both groups of scientists, where we also

manually normalized the names that were misspelled or duplicated, and excluded homonyms in other fields.

The search was based on:

Paper Citation Report

Title: LINEAR CANONICAL TRANSFORMATIONS AND THEIR UNITARY REPRESENTATIONS

Author(s): MOSHINSKY, M; QUESNE, C

Source: JOURNAL OF MATHEMATICAL PHYSICS Volume: 12, Issue: 8, Pages: 1772-& DOI: [10.1063/1.1665805](https://doi.org/10.1063/1.1665805) Published: 1971

Timespan = 1970–2013. Databases = SCI-EXPANDED, A&HCI, SSCI, CPCI-SSH, CPCI-S.

This report reflects citations to source items indexed within Web of Science.

Paper Citation Report

Title: LENS-SYSTEM DIFFRACTION INTEGRAL WRITTEN IN TERMS OF MATRIX OPTICS

Author(s): COLLINS, SA

Source: JOURNAL OF THE OPTICAL SOCIETY OF AMERICA Volume: 60, Issue: 9, Pages: 1168-& Published: 1970

Timespan = 1970–2013. Databases = SCI-EXPANDED, A&HCI, SSCI, CPCI-SSH, CPCI-S.

This report reflects citations to source items indexed within Web of Science.

The seminal articles have a considerable number of citations that constitute a recognizable body of knowledge initiated by those authors on the subject. Both articles, by Collins (1970) and Moshinsky and Quesne (1971), have been highly cited, so identifying their clusters can help us understand the development of their discovery in time.

With this objective we sliced the data into 8-year periods and created the coauthor networks for each period with Sci2 Tool (Sci2 2009) and (Weingart et al. 2010), and we visualized them with Gephi (Bastian et al. 2009). The visualization of these networks is based on community detection through a modularity layout (Blondel et al. 2008) implemented in Gephi; some statistics from these networks were computed simultaneously with both tools.

The modular decomposition is presented as an undirected graph linking connected components or partitions of the same graph. A module or community is defined as a set of nodes that connect to each other within the group, but which connect much less with other modules in the network. The density of these graphs is computed to identify the hub of interaction between author clusters, and to show the structure of the network with the purpose of determining a natural division of vertices and groups or communities that do not overlap (Newman 2006). This measure can be useful to determine whether there exists any natural division between its vertices into non-overlapping subgroups that may be of any size. Following Blondel's algorithm (Blondel et al. 2008) implemented in Gephi, modularity is calculated through:

$$\text{Modularity} = \frac{\# \text{ edges in communities} - \# \text{ expected edges in communities}}{\text{total number of edges}}$$

Although modularity cannot be compared for different graphs, our purpose is to show that the evolution of the two research communities—optical engineers and mathematical physicists—follows through time different patterns of communication. Modules may contain smaller sub-modules that can be identified through appropriate visualization. We present the full coauthor network for each seminal article and, as a second step we identify the largest component in the structure of these networks. With the purpose of illustrating their development, we calculated modularity slicing the data into 8-year periods, and to differentiate the two groups we present bipartite graphs showing the research fields of the authors, as well as two tables of authors by country as also reported in the Web of Science.

Results

The ISI Web of Science search resulted in 565 records for the seminal article by Collins and 350 for the seminal article by Moshinsky–Quesne from 1971 to 2013. Paper count and citations per year show a difference in the acceptance for both seminal articles (Fig. 1).

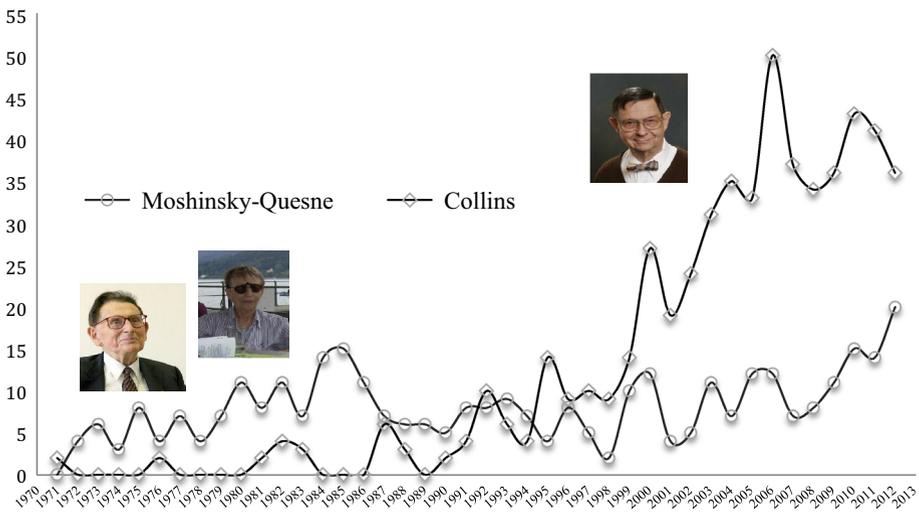


Fig. 1 Number of publications citing the seminal papers of Collins and Moshinsky–Quesne per year

Longitudinal study of co-authorship through network analysis

Our procedure was to visualize the full network first, identify the largest component and then slice the networks in periods of 8 years to show the structural dynamics of the two communities of scientists. For the visualization of networks we used Gephi and a ForceAtlas2 layout (Bastian et al. 2009, Jacomy et al. 2014) to render Figs. 2, 3, 5, 6, 7, 8, 9, 10, 11 and 12. Figures 15 and 16 were rendered with the Yifan Hu algorithm (2006).

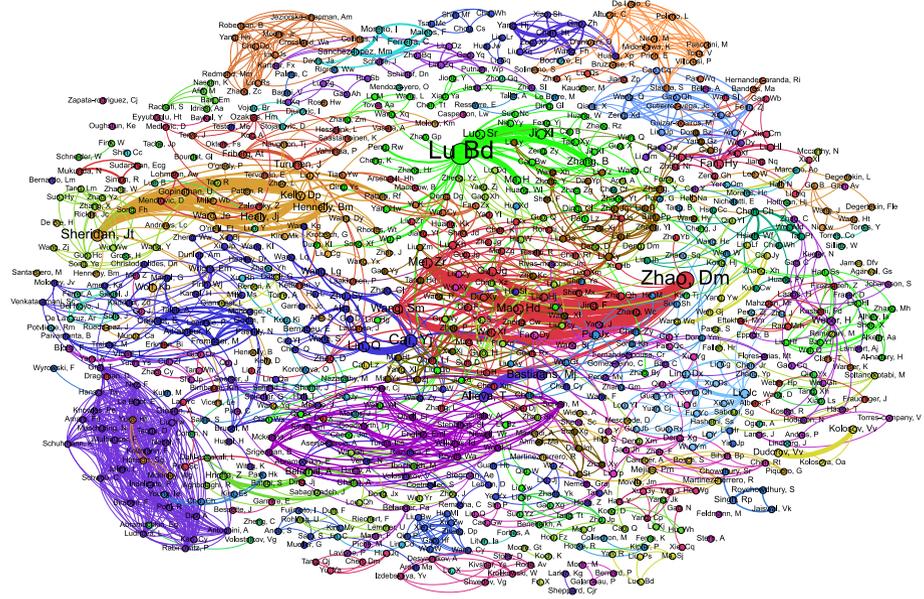


Fig. 2 Coauthorship network for Collins 1971–2013

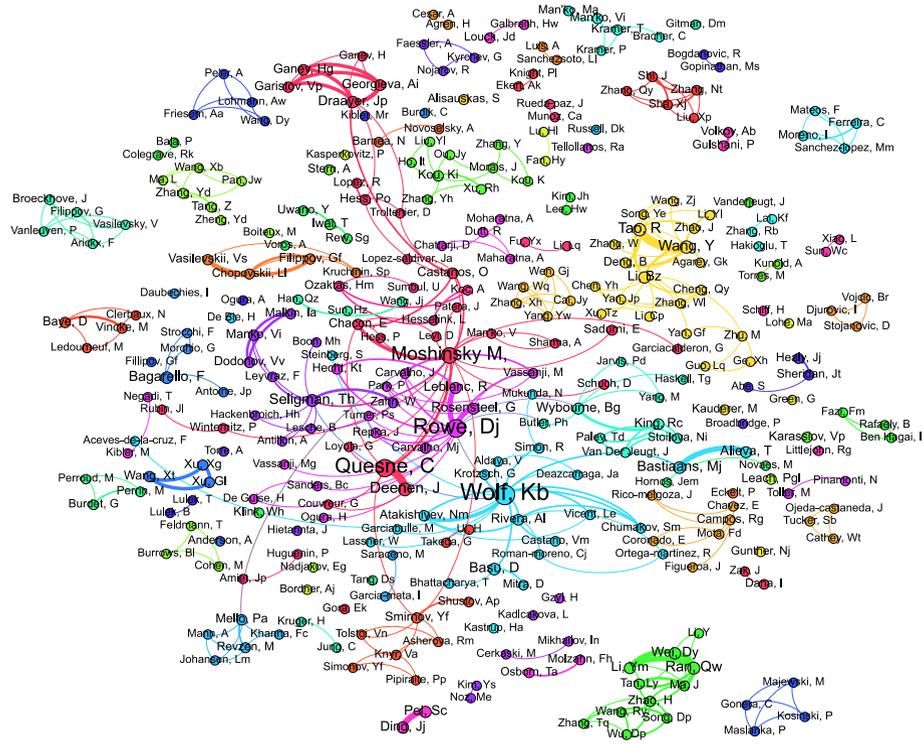


Fig. 3 Coauthorship network for Moshinsky–Quesne 1971–2013

Table 1 Statistical measures for the coauthorship networks of both seminal articles

	Nodes	Edges	Average degree	Density	Clustering coefficient	Modularity	Number of communities
Collins	782	1802	4.609	0.006	0.746	0.941	140
Moshinsky–Quesne	320	363	2.269	0.007	0.513	0.919	99

The size of the nodes is based on the number of authored works with undirected links connecting coauthors. The color of the nodes is assigned by modularity through Blondel’s algorithm in Gephi, detecting densely connected subsets of nodes. We show the full network of both seminal articles for a 42 year period in Figs. 2 and 3, and in Table 1.

In order to contrast the size of the interacting groups in each community, we obtained the frequency distribution of the number of coauthors per published article, shown in Fig. 4 and Table 2, finding that optical engineering and applied technology groups that cite Collins are clearly larger than the mathematical and theoretical physics groups that refer to the Moshinsky–Quesne articles.

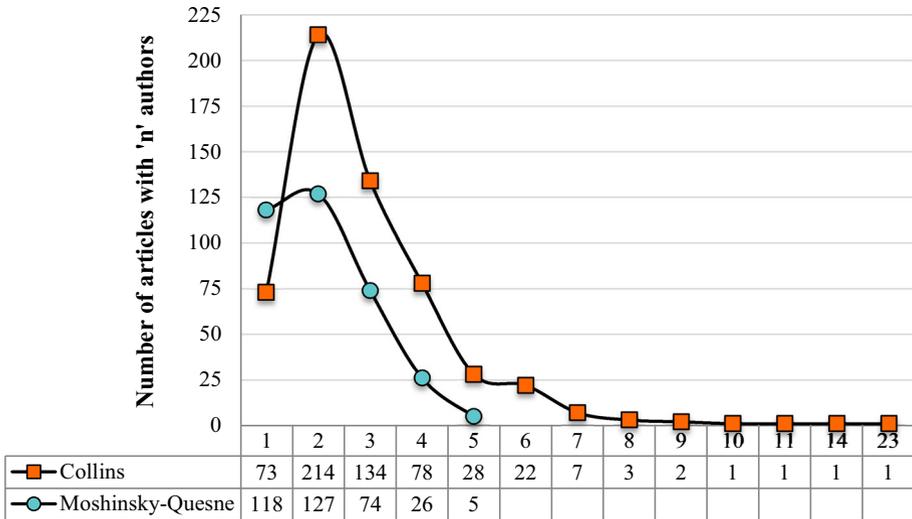


Fig. 4 Frequency distribution of authors per article

Table 2 Frequency distribution of number of authors per article

Number of authors/article	1	2	3	4	5	6	7	8	9	10	11	12	13
Collins	73	214	134	78	28	22	7	3	2	1	1	1	1
Moshinsky–Quesne	118	127	74	26	5								

To identify the core group following each of the two seminal articles we identified the largest component for the full network for each author. In Figs. 5, 6 and Table 3 we show the largest component for each network.

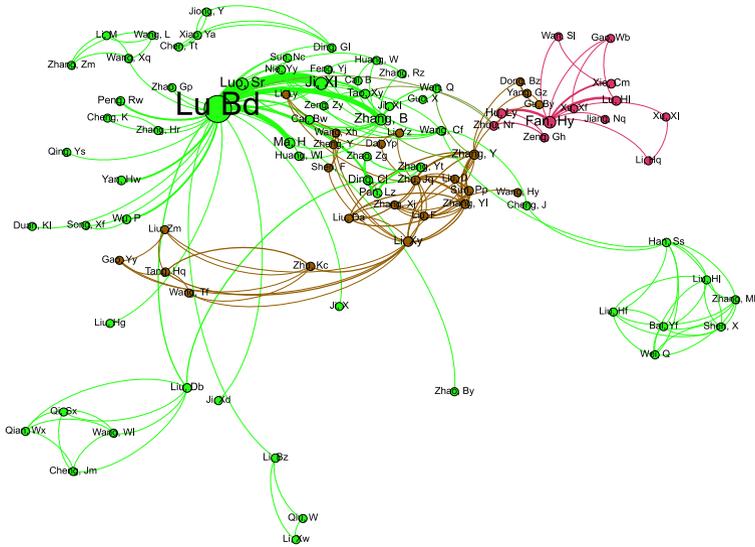


Fig. 5 Collins giant component

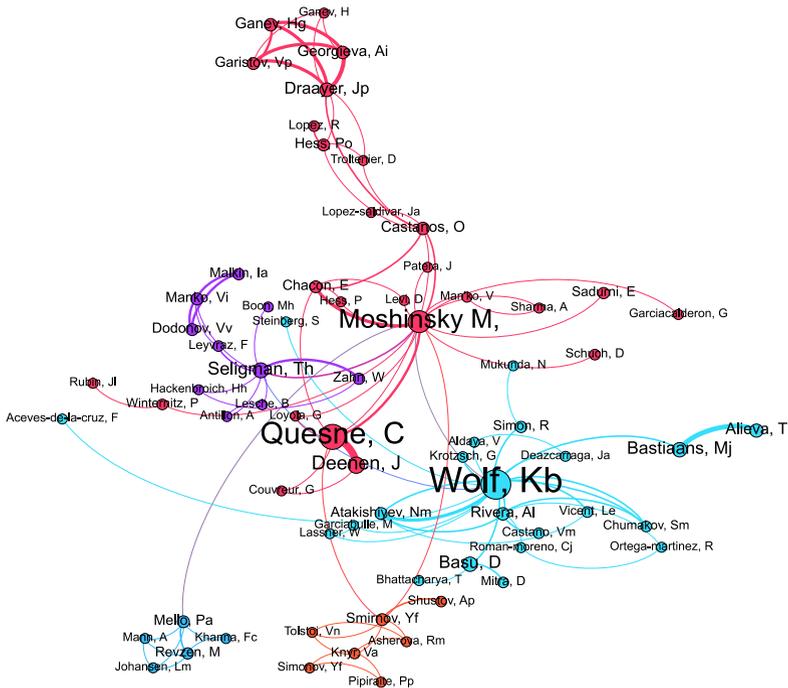


Fig. 6 Moshinsky and Quesne giant component

Table 3 Statistical measures of the largest component

Nodes	Edges	Average degree	Modularity	Clustering coefficient	Number of communities
96 (12.28 % visible)	198 (10.99 % visible)	4.125	0.693	0.752	10
70 (21.88 % visible)	106 (29.2 % visible)	3.029	0.732	0.555	7

The evolution of the network is visualized by slicing the 42 years in periods of 8 years as shown in Figs. 7, 8, 9, 10, 11 and 12. The values of the number of nodes and edges, average degree, modularity and number of communities for each period appear in Tables 4 and 5.

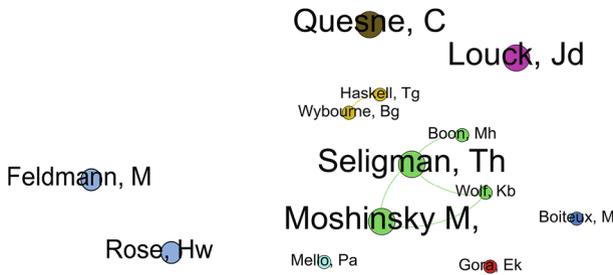


Fig. 7 Coauthorship networks from 1971 to 1973 for both seminal articles

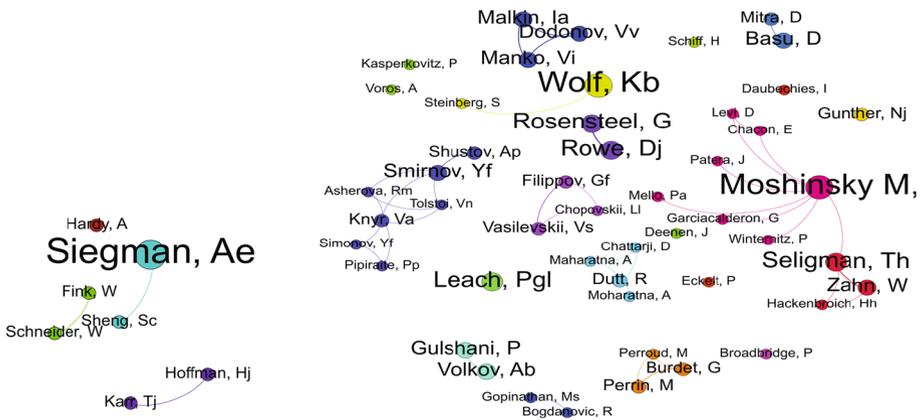


Fig. 8 Coauthorship networks from 1974 to 1981 for both seminal articles

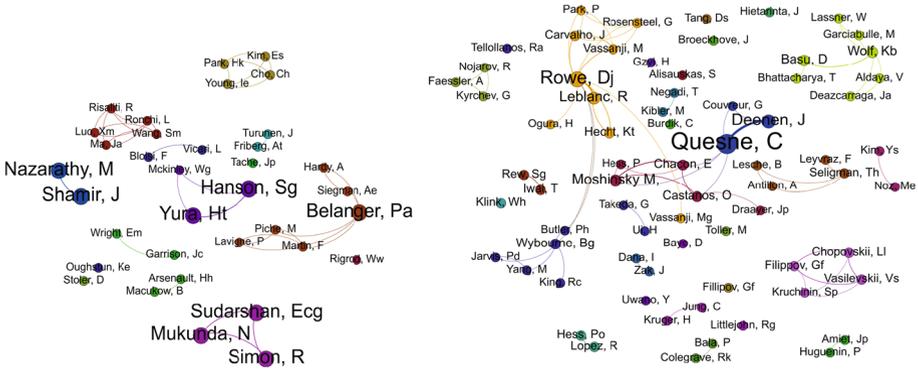


Fig. 9 Coauthorship networks from 1982 to 1989 for both seminal articles

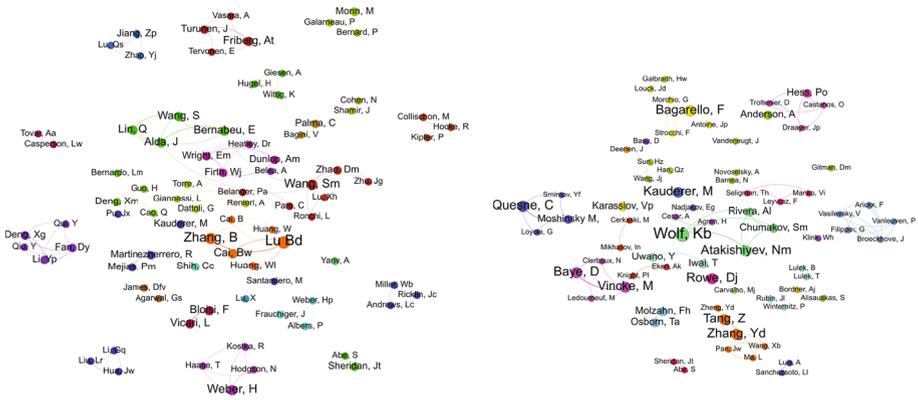


Fig. 10 Coauthorship networks from 1990 to 1997 for both seminal articles

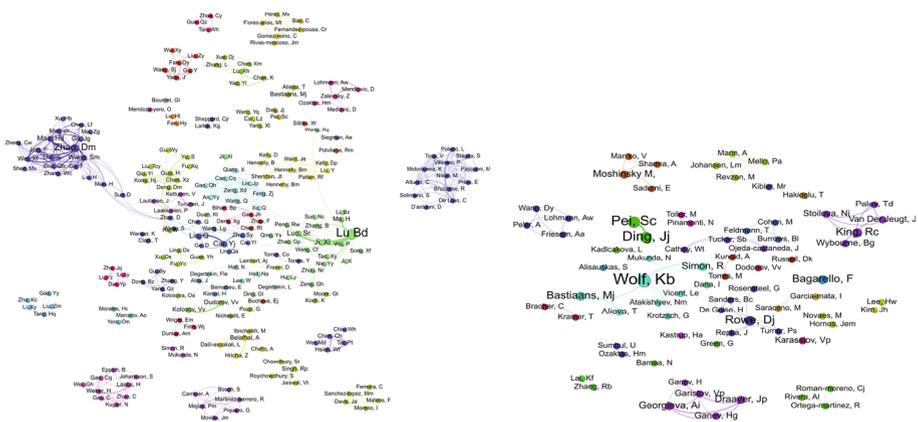


Fig. 11 Coauthorship networks from 1998 to 2005 for both seminal articles

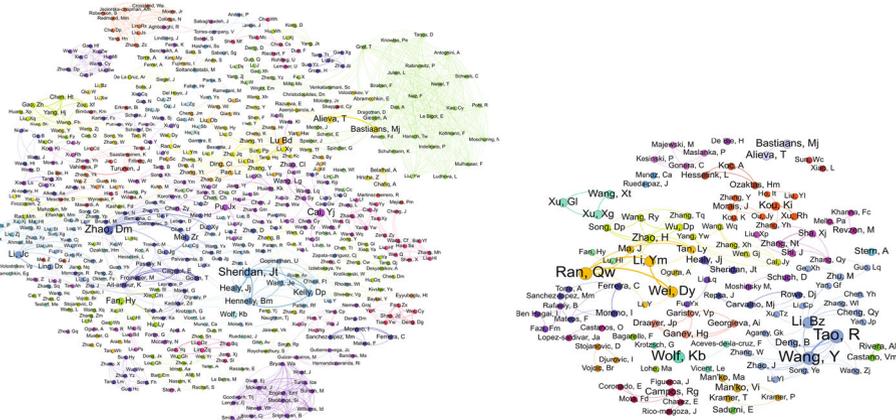


Fig. 12 Coauthorship networks from 2006 to 2013 for both seminal articles

Table 4 Statistical measures of nodes, edges and Average degree per periods of 8 years for both networks

	Collins nodes	Moshinsky–Quesne nodes	Collins edges	Moshinsky–Quesne edges	Collins Av. degree	Moshinsky–Quesne Av. degree
1971–1973	2	11	0	5	0	0.909
1974–1981	7	49	3	39	0.857	1.592
1982–1989	35	71	36	64	2.057	1.803
1990–1997	84	72	87	61	2.071	1.694
1998–2005	225	75	434	65	3.858	1.733
2006–2013	539	119	1304	152	4.839	2.555

Table 5 Statistical measures of modularity and number of communities per periods of 8 years for both networks

	Collins modularity	Moshinsky–Quesne modularity	Collins communities	Moshinsky–Quesne communities
1971–1973	0	0.32	2	7
1974–1981	0.667	0.865	4	21
1982–1989	0.835	0.804	14	30
1990–1997	0.902	0.905	32	33
1998–2005	0.872	0.899	46	31
2006–2013	0.932	0.865	100	34

The development in the number of nodes and edges for both coauthor networks is different. Tables 4 and 5 show how Collins' network increasingly involves larger groups of coauthors, while the Moshinsky and Quesne network remains active with the typically smaller theoretical physics group size.

The number of communities did change drastically between 1990 and 1997: their average degree between 1982 and 1989 is shown in Fig. 13a, b.

We computed modularity in order to see how clearly a network decomposes into modular communities or sub-networks; the high modularity score that indicates the complexity of network structure is shown in Fig. 14.

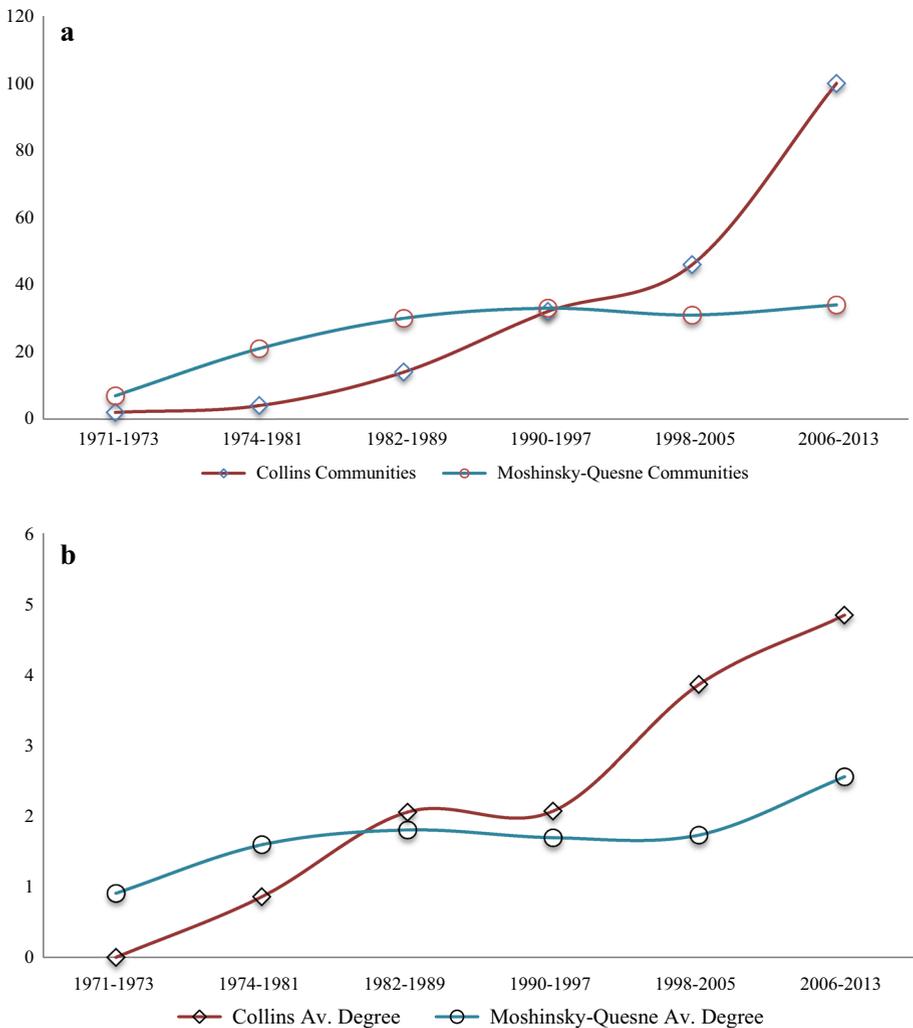


Fig. 13 a Number of communities in 8 year periods for both networks. b Average degree in 8 year periods for both networks

Table 6 Collins authors of papers identified with research fields

Web of science categories	Records	% of 565
Optics	473	83.717
Physics applied	71	12.566
Physics multidisciplinary	52	9.204
Engineering electrical electronic	49	8.673
Physics mathematical	12	2.124
Imaging science photographic technology	9	1.593
Physics atomic molecular chemical	9	1.593
Mathematics applied	6	1.062

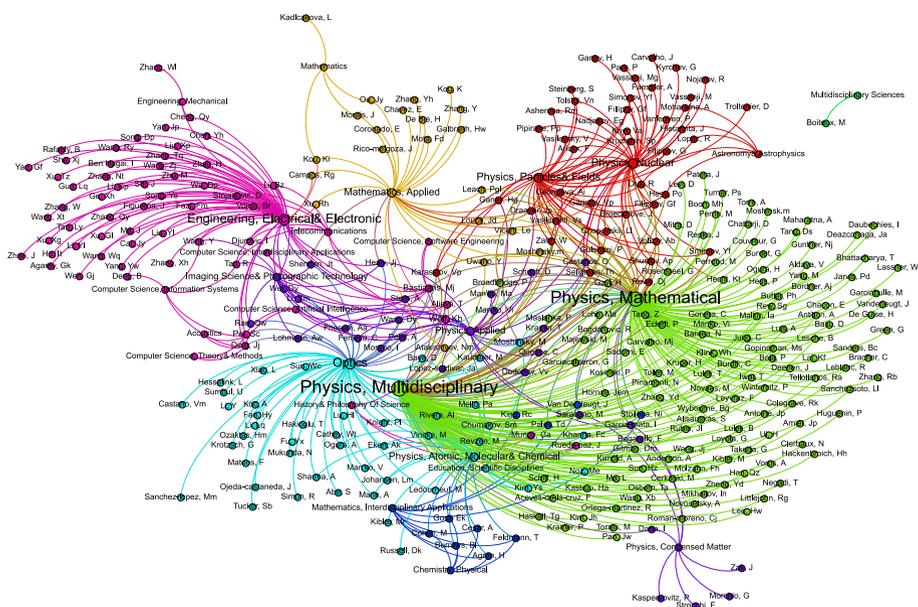


Fig. 16 Moshinsky–Quesne bipartite graph of author–research field

Table 7 Moshinsky–Quesne bi-partite graph of authors identified with research fields

Web of science categories	Records	% of 350
Physics mathematical	139	39.714
Physics multidisciplinary	125	35.714
Optics	44	12.571
Engineering electrical electronic	41	11.714
Physics nuclear	27	7.714
Physics particles fields	23	6.571
Mathematics applied	15	4.286
Physics applied	15	4.286
Physics atomic molecular chemical	12	3.429
Imaging science photographic technology	6	1.714
Astronomy astrophysics	4	1.143
Mathematics interdisciplinary applications	4	1.143
Physics condensed matter	4	1.143

The geographical prominence of research groups described by articles per country is shown in Table 8, with the list of largest records (down to 5).

Table 8 Number of published papers per country for both seminal articles

Countries	Collins records	Countries	Moshinsky–Quesne records
China	314	Mexico	69
United States	54	China	47
Spain	32	Canada	38
Germany	29	Belgium	37
Ireland	22	United states	32
Italy	17	Russia	16
Netherlands	14	Germany	15
United Kingdom	13	United Kingdom	13
Finland	12	Italy	13
France	12	India	12
Taiwan	12	Israel	11
Israel	10	Australia	9
Mexico	10	Japan	9
Morocco	10	Spain	9
Russia	10	France	8
Canada	9	Netherlands	8
India	9	Taiwan	7
Sweden	6	Bulgaria	6
Iran	5	New Zealand	5

Concluding remarks

Network analysis has served to visualize the growth of the scientific communities that work on and use the linear canonical transformations originated by their simultaneous independent discovery as published in the seminal papers by Collins and Moshinsky–Quesne in 1970–1971. In this case it is evident that there are two distinct groups (communities) of scientists dedicated to this subject, who were disjoint before the end of the nineties, and up to now very few authors belong to both networks. These two groups differ in the ways they use and apply their research results. Technological and theoretical modes of research have different patterns of knowledge creation, recognition and diffusion, as illustrated by their average degree difference. Technological applications of knowledge entail large research teams while theoretical and mathematical elaborations are made typically by smaller blackboard sessions between coauthoring colleagues. In the latter, the increase in number of authors, citations and number of papers published, is far more stable than in technological fields, where a “hot” new development or device quickly assembles the interest of many large research groups.

The structural dynamics of the formation of the two groups of scientists in their networks is different, as is their intellectual attitude directed toward use or understanding of the subject matter, and continues so today. The two networks remained disjoint for more than two decades and no dispute over priority has openly arisen since then. In the longitudinal study, modularity confirms that after 1990 the Collins group grew considerably and communicated more, while size and communication in the Moshinsky–Quesne group has been roughly stable. This demonstrates that in the social dynamics of science there are distinct patterns of development between different disciplinary traditions, in particular between technological and theoretical research.

Today the harvest provided by the discovery of linear canonical transforms includes applications to encryption technology, metrology, holography and optical implementations, while as a mathematical construct it has provided insight into many branches of mathematical physics that contain the foundations of quantum mechanics.

Acknowledgments The work of the first author has been supported by the project DGAPA-PAPIIT N° IN305013_2014, of the Universidad Nacional Autónoma de México; the work of the second author has been supported by DGAPA-PAPIIT N°101115. The authors also acknowledge the very useful comments of two anonymous reviewers.

References

- Bargmann, V. (1970). Group representations in Hilbert spaces of analytic functions. In P. Gilbert & R. G. Newton (Eds.), *Analytical methods in mathematical physics* (pp. 27–63). New York: Gordon and Breach.
- Bastian, M., Heymann, S. & Jacomy, M. (2009). Gephi: an open source software for exploring and manipulating networks. In *International AAAI conference on weblogs and social media*. Association for the Advancement of Artificial Intelligence.
- Bavelas, A. (1948). A mathematical model for group structures. *Human Organization*, 7, 16–30.
- Bavelas, A. (1968). Communication patterns in task-oriented groups. In D. Cartwright & A. F. Zander (Eds.), *Group dynamics: Research and theory*. New York: Harper & Row.
- Blondel, V. D., Guillaume J., Lambiotte R., & Lefebvre E. (2008). Fast unfolding of communities in large networks. *Journal of Statistical Mechanics: Theory and Experiment*, 10, 1–12.
- Brannigan, A., & Wanner, R. A. (1983). Multiple discoveries in science: A test of the communication theory. *Canadian Journal of Sociology*, 8, 135–151.
- Brouwer, W. (1964). *Matrix methods in optical instrument design*. New York: Benjamin.

- Collins, S. A. (1970). Lens-system diffraction integral written in terms of matrix optics. *Journal of Optical Society of America*, *60*, 1168–1177.
- Freeman, L. C. (1978). Centrality in social networks conceptual clarification. *Social Networks*, *1*(3), 215–239.
- Freeman, L. C. (2004). *The development of social network analysis: A study in the sociology of science*. Vancouver, BC: Empirical Press.
- Galton, F. (1874). *English men of science: Their nature and nurture*. London: Macmillan & Co.
- Gerrard, A., & Burch, B. (1975). *Introduction to matrix methods in optics*. New York: Wiley.
- Hagstrom, W. O. (1974). Competition in science. *American Sociological Review*, *39*(1), 1–18.
- Homans, G. C. (1950). *The human group*. New York: Harcourt, Brace & Co.
- Hu, Y. (2006). Efficient, high-quality force-directed graph drawing. *Mathematica Journal*, *10*(1), 37–71.
- Infeld, L., & Plebański, J. (1955). On a certain class of unitary transformations. *Acta Physica Polonica*, *14*, 41–75.
- Itzykson, C. (1969). Group representation in a continuous basis: An example. *Journal of Mathematical Physics*, *10*, 1109–1114.
- Jacomy, M., Venturini, T., Heymann, S., & Bastian, M. (2014). ForceAtlas2, a continuous graph layout algorithm for handy network visualization designed for the Gephi software. *PLoS ONE*, *9*, 6.
- Kroeber, A. L. (1917). The superorganic. *American Anthropologist*, *19*, 163–213.
- Liberman, S., & Wolf, K. B. (2013). Scientific communication in the process to coauthorship. In G. J. Feist & M. Gorman (Eds.), *Handbook of the psychology of science*. Berlin: Springer.
- Merton, R. K. (1961). Singletons and multiples in scientific discovery. *Proceedings of the American Philosophical Society*, *105*(5), 470–486.
- Merton, R. K. (1973). *The sociology of science: Theoretical and empirical investigations*. Chicago: University of Chicago Press.
- Moreno, J. L. (1953). *Who shall survive?*. New York: Beacon House.
- Moshinsky, M., & Quesne, C. (1971). Linear canonical transformations and their unitary representations. *Journal of Mathematical Physics*. (N. Y.), *12*(8), 1772–1780.
- Moshinsky, M. & Quesne, C. (1974) Oscillator systems. In *Proceedings of the 15th solvay conference in physics*. Gordon and Breach, New York.
- Newman, M. E. J. (2006). Modularity and community structure in networks. *Proceedings of the National Academy of Sciences*, *103*(23), 8577–8582. doi:10.1073/pnas.0601602103.
- Ogburn, W. F., & Thomas, D. (1922). Are inventions inevitable? A note on social evolution. *Political Science Quarterly*, *37*, 83–98.
- Quesne, C., & Moshinsky, M. (1971). Linear canonical transformations and matrix elements. *Journal of Mathematical Physics*. (N. Y.), *12*(8), 1780–1783.
- Sci2 Team. (2009). Science of Science (Sci2) Tool: Indiana University and SciTech Strategies, Inc. <http://sci2.cns.iu.edu>
- Simonton, D. K. (1978). Independent discovery in science and technology: A closer look at the Poisson distribution. *Social Studies of Science*, *8*, 521–532.
- Simonton, D. K. (1979). Multiple discovery and invention: Zeitgeist, genius, or chance? *Journal of Personality and Social Psychology*, *37*, 1603.
- Simonton, D. K. (1986). Multiple discovery: Some Monte Carlo simulations and Gedanken experiments. *Scientometrics*, *9*, 269–280.
- Simonton, D. K. (2010). Creative thought as blind-variation and selective-retention: Combinatorial models of exceptional creativity. *Physics of Life Reviews*, *7*(2), 190–194.
- Small, H. G. (1978). Cited documents as concept symbols. *Social Studies of Science*, *8*(3), 327–340.
- Stavroudis, O. (1972). *The optics of rays, wavefronts, and caustics*. New York: Academic Press.
- Travers, J., & Milgram, S. (1969). An experimental study of the small world problem. *Sociometry*, *32*(4), 425–443.
- Weingart, S., Hanning, G., Börner, K., Boyac, K. W., Linnemeier, M., Duhon, R. J., Phillips, P. A., et al. (2010) Science of Science (Sci2) Tool User Manual. http://sci.slis.indiana.edu/registration/docs/Sci2_Tutorial.pdf. Accessed 16 July 2013.
- Wolf, K. B. (1979). *Integral transforms in science and engineering*. New York: Plenum Publ. Corp. <http://www.fis.unam.mx/~bwolf/>
- Wolf, K. B. (2004). *Geometric optics on phase space*. Heidelberg: Springer.
- Wolf, K. B. (2015). Developments of linear canonical transforms—a historical sketch. In M. Alper Kutay, J. J. Healy, H. M. Ozaktas, & J. T. Sheridan (Eds.), *Linear canonical transforms: theory and applications*. Heidelberg: Springer.