UNIVERSIDAD DE GUANAJUATO
DEPARTAMENTO DE FISICA

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Efectos del Minisuperespacio Deformado en Cosmología

Seminario del Departamento
de Física de la DCeI

August 6, 2014

Cuernavaca.

PLAN OF THE TALK

INTRODUCTION, MOTIVATION

NC-GRAVITY

NC-COSMOLOGY

NC AND LAMBDA

FINAL REMARKS

INTRODUCTION

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Unfortunately on the gravity side of things, the story has been more complex.

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From these equations we can write the noncommutative lagrangian and the noncommutative theory.
This results where reproduced by J. Wess independent of string theory (Wess , EPJC 2005).

NC- GRAVITY

$$I = \int d^4x e_a^\mu e_b^\nu R_{ab}^{\mu\nu}$$

GR

$$\widehat{I}_{\theta^2} = \frac{1}{2^4}\theta^{\gamma\delta}\theta^{\tau\xi}\int dx^4\Big\{4e\Big[4R_\delta{}^\rho(R_{\rho\tau}{}^{ab}R_{\gamma\xi ab} - \omega_\tau{}^{ab}\partial_\xi R_{\rho\gamma ab}) + \omega_\gamma{}^{\rho\sigma}\partial_\tau\omega_\delta{}^{ab}\partial_\xi R_{\rho\sigma ab}$$

$$+R\partial_\delta(\omega_\tau{}^{ab}(\partial_\xi\omega_{\gamma ab} + R_{\gamma\xi ab})) + 2\omega_\gamma{}^{\rho\sigma}\partial_\delta(R_{\rho\tau}{}^{ab}R_{\sigma\xi ab} - \omega_\tau{}^{ab}\partial_\xi R_{\rho\sigma ab})\Big]$$

$$+\epsilon^{\mu\nu\rho\sigma}\Big\{4e\Big[\epsilon_{\gamma\delta\alpha\beta}R_{\rho\sigma}{}^{\alpha\beta}(R_{\mu\tau}{}^{ab}R_{\nu\xi ab} - \omega_\tau{}^{ab}\partial_\xi R_{\mu\nu ab}) + 2\epsilon_{\tau\xi\alpha\beta}R_{\mu\nu}{}^{ab}R_{\rho\sigma ab}R_{\gamma\delta}{}^{\alpha\beta}\Big]$$

$$+\epsilon_{abcd}\Big[4R_{\rho\sigma\gamma\delta}(R_{\mu\tau}{}^{ab}R_{\nu\xi}{}^{cd} - \omega_\tau{}^{ab}\partial_\xi R_{\mu\nu}{}^{cd}) + 4R_{\mu\nu}{}^{ab}R_{\rho\sigma}{}^{cd}(2R_{\gamma\delta\tau\xi} - \omega_{\tau ef}\partial_\xi(e_\gamma{}^e e_\delta{}^f))$$

$$-2\omega_{\gamma\mu\nu}\partial_\delta(R_{\rho\tau}{}^{ab}R_{\sigma\xi}{}^{cd} - \omega_\tau{}^{ab}\partial_\xi R_{\rho\sigma}{}^{cd}) - 2\omega_\gamma{}^{ef}R_{\rho\sigma ef}\partial_\delta(2R_{\mu\nu}{}^{ab}e_\tau{}^c e_\xi{}^d - \omega_\tau{}^{ab}\partial_\xi(e_\mu{}^c e_\nu{}^d))$$

$$-2\omega_\gamma{}^{ab}R_{\rho\sigma}{}^{cd}\partial_\delta(2R_{\mu\nu\tau\xi} - \omega_{\tau ef}\partial_\xi(e_\mu{}^e e_\nu{}^f)) - \omega_{\gamma\mu\nu}\partial_\tau\omega_\delta{}^{ab}\partial_\xi R_{\rho\sigma}{}^{cd} - \omega_\gamma{}^{ef}R_{\rho\sigma ef}\partial_\tau\omega_\delta{}^{ab}\partial_\xi(e_\mu{}^c e_\nu{}^d)$$

(O. Obregon, H.Compean, C. Ramirez and **M.S.**, PRD 68 (2003) .

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Noncommutative Quantum Cosmology

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What happens if space time does not commute?

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$$\left[-\nabla^2 + V(x,y)\right] * \Psi(x,y) = E\Psi(x,y)$$

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$$\left[-\nabla^2 + V\left(x + \frac{\theta}{2}P_y, y - \frac{\theta}{2}P_x\right)\right]\Psi(x,y) = E\Psi(x,y).$$

The original proposal for NCQC was the Kantowski-Sachs model

$$ds^2 = -N^2dt^2 + e^{\left(2\sqrt{3}\gamma\right)}dr^2 + e^{\left(-2\sqrt{3}\gamma\right)} \times e^{\left(-2\sqrt{3}\Omega\right)}\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

WDW equation for KS

$$\left[-\frac{\partial^2}{\partial\Omega^2} + \frac{\partial^2}{\partial\gamma^2} + 48e^{-2\sqrt{3}\Omega}\right]\psi(\Omega,\gamma) = 0,$$

$$\psi_\nu^\pm(\beta,\Omega) = e^{\pm i\nu\sqrt{3}\beta}K_{i\nu}(4e^{-\sqrt{3}\Omega}),$$

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$$\left[-\frac{\partial^2}{\partial\Omega^2} + \frac{\partial^2}{\partial\gamma^2} + 48e^{\left(-2\sqrt{3}\Omega + \sqrt{3}\theta P_\gamma\right)}\right]\psi(\Omega,\gamma) = 0$$

NC COSMOLOGY

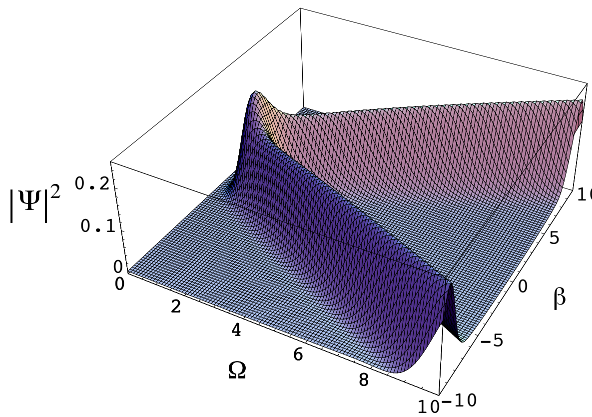

FIG. 1: Variation of $|\Psi|^2$ with respect to Ω and β, at the value $\theta = 0$. It shows only one possible universe around $\Omega = 4.812$ and $\beta = 0$.

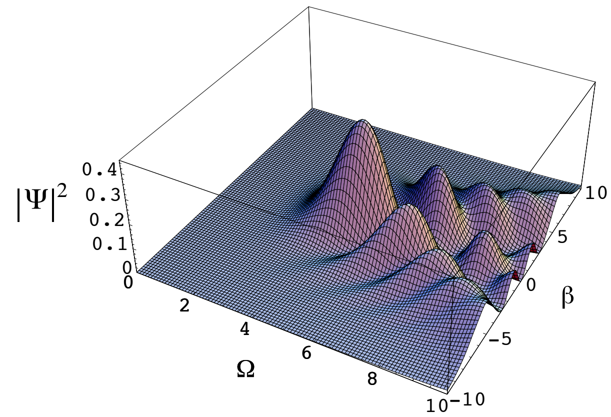

FIG. 2: Variation of $|\Psi|^2$ with respect to Ω and β, at the value $\theta = 4$. It shows many peaks corresponding to different possible universes.

NONCOMMUTATIVE COSMOLOGY

Seminario del Departamento
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This is an effective noncommutativity between the minisuperspace fields, and is similar to the modification used in noncommutative quantum mechanics. H. Compean, O. Obregón and C. Ramírez PRL 88 (2002), W. Guzmán, **M.S**, J. Socorro Phys. Rev D (2007),) One of a noncommutative mini-superspace point to the possibility of late time acceleration of the universe. W. Guzmán, M.S. and J.Socorro PRD (2007).

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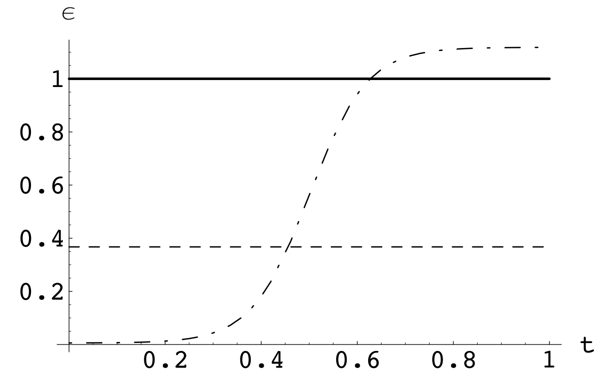

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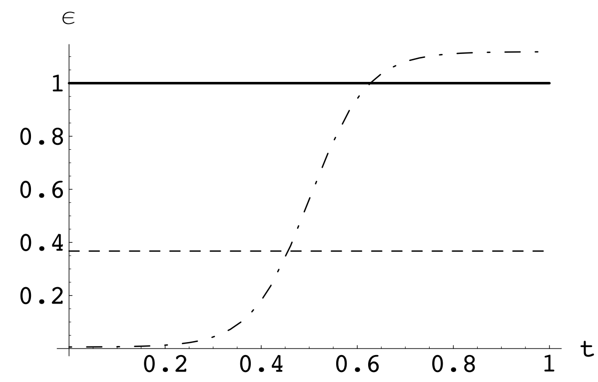

$$\epsilon = \frac{\lambda^2}{2}\left[1 - \theta\mu\left(1 - \frac{\sqrt{6}}{\lambda}\tanh\left[3\mu(\lambda^2-6)t\right]\right)\right],$$

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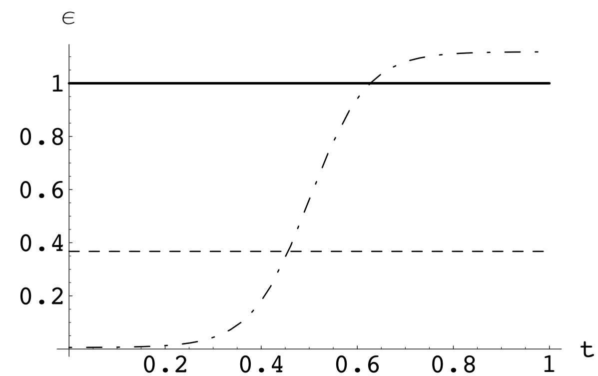

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One can ask if the noncommutative deformation can be related to the late time acceleration or to the cosmological constant.

There was also evidence that noncommutativity reduced the number of states for a Schwarzchild black Hole (J. Lopez,O. Obregon **M. S.,** Phys Rev D 74 084024, 2006)

By the same procedure as in the black hole, we use the appropriate metric for the universe

$$ds^2 = -N^2 dt^2 + e^{2\alpha(t)} \left[dr^2 + r^2(d\vartheta^2 + sin^2\vartheta d\varphi^2)\right]$$

where $a(t) = e^{\alpha(t)}$

And the NCWDW equation is

$$\alpha(t) = \frac{1}{6}\ln\left(\frac{P_{\phi_0}^2}{4\Lambda}\right) + \frac{1}{3}\ln\left(\mathrm{sech}\left[\frac{\sqrt{3}}{2}P_{\phi_0}(t-t_0)\right]\right).$$

$$\alpha_{nc}(t) = \frac{1}{6}\ln\left(\frac{P_{\phi_0}^2}{4\Lambda e^{-3\theta P_{\phi_0}}}\right) + \frac{1}{3}\ln\left(\mathrm{sech}\left[\frac{\sqrt{3}}{2}P_{\phi_0}(t-t_0)\right]\right),$$

By comparing these equations we arrive to the relationships

$$g_{\mu\nu}^{(nc)} = diag(e^{3\theta P_{\phi_0}} g_{00}, e^{\theta P_{\phi_0}} g_{ij}),$$

From which the vacuum density energy is

$$\langle \rho_{vac} \rangle_{nc} \approx e^{-\theta P_{\phi_0}} k_{max}^4$$

And the ratio of the observed to calculated densities is

$$\frac{\langle \rho_{obs} \rangle}{\langle \rho_{vac} \rangle_{nc}} = e^{\theta P_{\phi_0}} \frac{\langle \rho_{obs} \rangle}{k_{max}^4},$$

For $k_{max}=M_p$ we only need ΘP_ϕ of order 240!! (E. Mena, O. Obregon, MS, MPLA 2009)

PRELIMINARIES: NC COSMOLOGY

In order to find the NC metric, we needed that the noncommutative model had a classical limit.

Is there a classical interpretation and formulation of NC Quantum Cosmology?

We want

$$\{x_i, x_j\} = \theta_{ij}, \quad \{P_{x_i}, P_{x_j}\} = 0 \quad \{x_i, P_{x_j}\} = \delta_{ij}.$$

This can be constructed from Hamiltonian Manifolds (W. Guzman,M.S., J. Socorro PLB 2011)

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$$i_{X_H}\omega_c = -dH \Rightarrow \begin{array}{c} \frac{dq^i}{dt} = \frac{\partial H}{\partial p_i}, \\ \frac{dp_j}{dt} = -\frac{\partial H}{\partial p^j}. \end{array}$$

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$$\{q'^i, q'^j\} = \theta_{ij}, \quad \{q'^i, p'_j\} = \delta^i_j, \quad \{p'_i, p'_j\} = \gamma_{ij}$$

COSMOLOGIA NO CONMUTATIVA

Minisuperspace Models

+

Canonical Quantization

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Noncommutative Quantum Mechanics

NC Quantum Cosmology

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NC Classical Cosmology.
Deformed phase space cosmology.

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Deformed Phase Space Cosmology

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$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\theta a^3\left[\left(\frac{\dot{a}}{a}\right)\frac{dV}{d\phi} + \dot{\phi}V\right] - 3\theta^2 a^6\left[\left(\frac{dV}{d\phi}\right)^2 - 6V^2\right];$$

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We also have the Hamiltonian constraint. (W. Guzmán, **M.S.,** J. Socorro PLB (2011))

NONCOMMUTATIVITY AND LAMBDA

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NONCOMMUTATIVITY AND LAMBDA

The results of the dynamical systems analysis have revealed is that, independently of the kind of self- interaction potential considered: i) exponential potential, or ii) cosh-like potential, the noncommutative effects of the kind considered here[9] affect not only the early-times dynamics but also modify the late-time behaviour.

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In the a more general case the noncommutative Friedmann equation for scalar field cosmology where derived, and it is found that noncommutativity could be relevant in the late time dynamics of the universe. W. Guzmán, M.S. and J.Socorro PLB (2011).

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$$\hat{x} = x + \frac{\theta}{2}p_y, \qquad \hat{y} = y - \frac{\theta}{2}p_x,$$

$$\hat{p}_x = p_x - \frac{\beta}{2}y, \qquad \hat{p}_y = p_y + \frac{\beta}{2}x,$$

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$$\widehat{H} = \frac{1}{2}\left\{\hat{p}_x^2 - \hat{p}_y^2 + \omega_1^2(\hat{x}\hat{p}_y + \hat{y}\hat{p}_x) + \omega_2^2(\hat{x}^2 - \hat{y}^2)\right\}.$$

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$$\omega_1^2 = \frac{4(\beta - \omega^2\theta)}{4 - \omega^2\theta^2}, \quad \omega_2^2 = \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2},$$

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$$\dot{x} = \{x, \widehat{H}\} = \frac{1}{2}[2\hat{p}_x + \omega_1^2 y], \quad \dot{y} = \{y, \widehat{H}\} = \frac{1}{2}[-2\hat{p}_y + \omega_1^2 x],$$

$$\dot{\hat{p}}_x = \{\hat{p}_x, \widehat{H}\} = \frac{1}{2}[-\omega_1^2\hat{p}_y - 2\omega_2^2 x], \quad \dot{\hat{p}}_y = \{\hat{p}_y, \widehat{H}\} = \frac{1}{2}[-\omega_1^2\hat{p}_x + 2\omega_2^2 y],$$

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$$\ddot{\eta} - \omega_1^2 \dot{\eta} + \frac{1}{4}(4\omega_2^2 + \omega_1^4)\eta = 0,$$
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This equations are very easy to solve for any value of Λ, θ and β.

NONCOMMUTATIVE COSMOLOGY

Let analyse the case when the cosmological constant is zero, as well as the parameter θ. In this case the solution is very simple, in particular the scale factor is (S. Perez, M.S, C. Yee,Phys. Rev D 2013)

$$a^3(t) = V_0 \cosh^2\left(\frac{1}{4}t\beta\right),$$

Comparing the to models we arrive to the relationship

$$\Lambda \sim \beta^2.$$

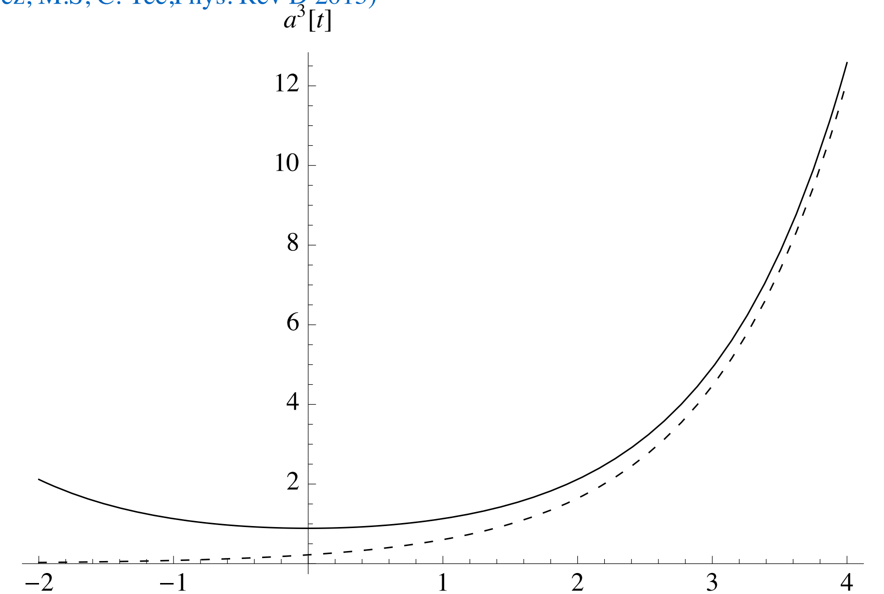

Figure 1: Dynamics of the phase space deformed model for the values $X_0 = Y_0 = 1, \delta_2 = \delta 1 = 0, \omega = 0$andβ = 1. Thesolidlinecorrespondstothe volume of the universe, calculated with the noncommutative model. The dotted line corresponds to the volume of the de Sitter spacetime. For large values of t the behavior is the same.

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NONCOMMUTATIVITY AND LAMBDA

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$$H = \left\{ \left(\hat{p}_u - \frac{2(\beta - \theta\omega^2)}{4 - \omega^2\theta^2}\hat{v} \right)^2 - \left(\hat{p}_v + \frac{2(\beta - \theta\omega^2)}{4 - \omega^2\theta^2}\hat{u} \right)^2 \right.$$
$$\left. + \left(\frac{4(\beta - \theta\omega^2)^2}{(4 - \omega^2\theta^2)^2} + \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2} \right) \hat{u}^2 - \left(\frac{4(\beta - \theta\omega^2)^2}{(4 - \omega^2\theta^2)^2} + \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2} \right) \hat{v}^2 \right\}$$

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$$\omega'^2 \equiv \frac{4(\beta - \theta\omega^2)^2}{(4 - \omega^2\theta^2)^2} + \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2}$$

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$$B = \frac{4(\beta - \omega^2\theta)}{4 - \omega^2\theta^2}$$

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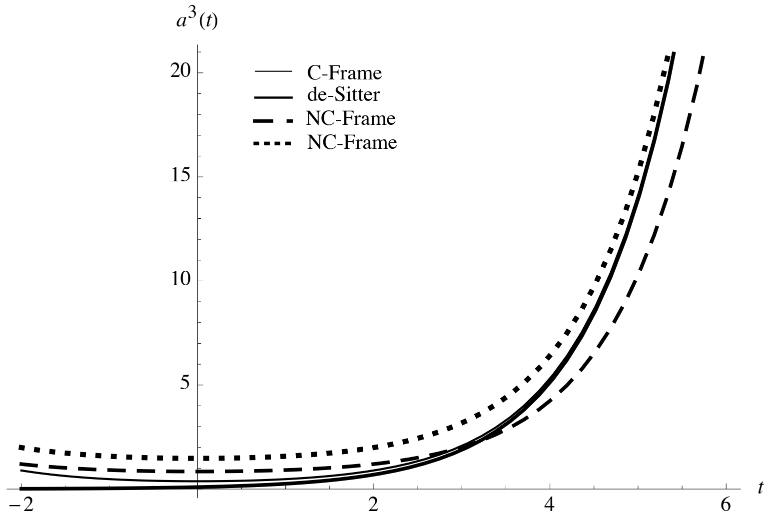

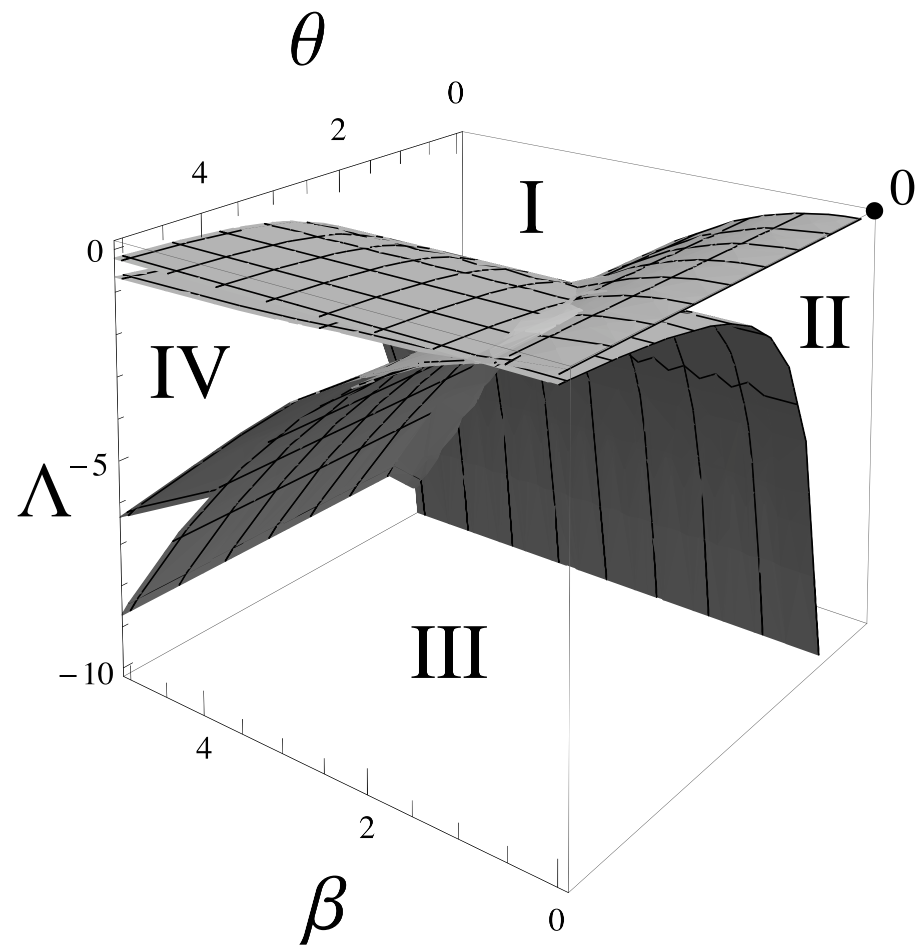

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- The late time influence of phase space deformations in cosmological evolution have been stablished.*

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- A simple 5 dimensional model was proposed, where the origin of Λ is related to the noncommutativity between the compact dimension and the radius of the non compact dimension.