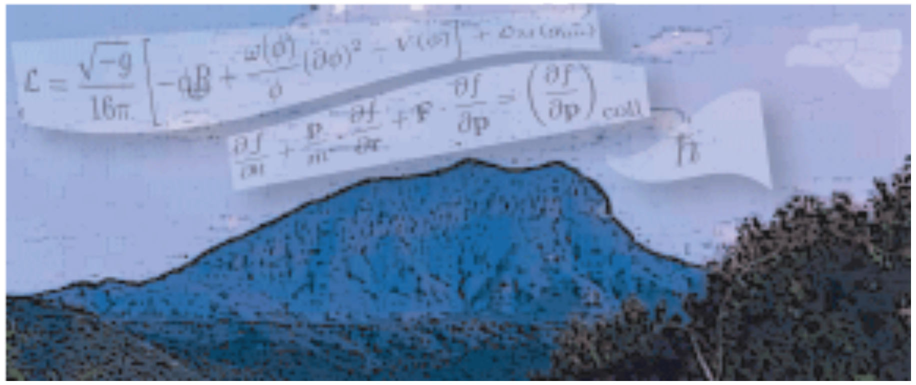
Large scale structure: a cosmological simulation review

Mario Alberto Rodríguez-Meza

Instituto Nacional de Investigaciones Nucleares

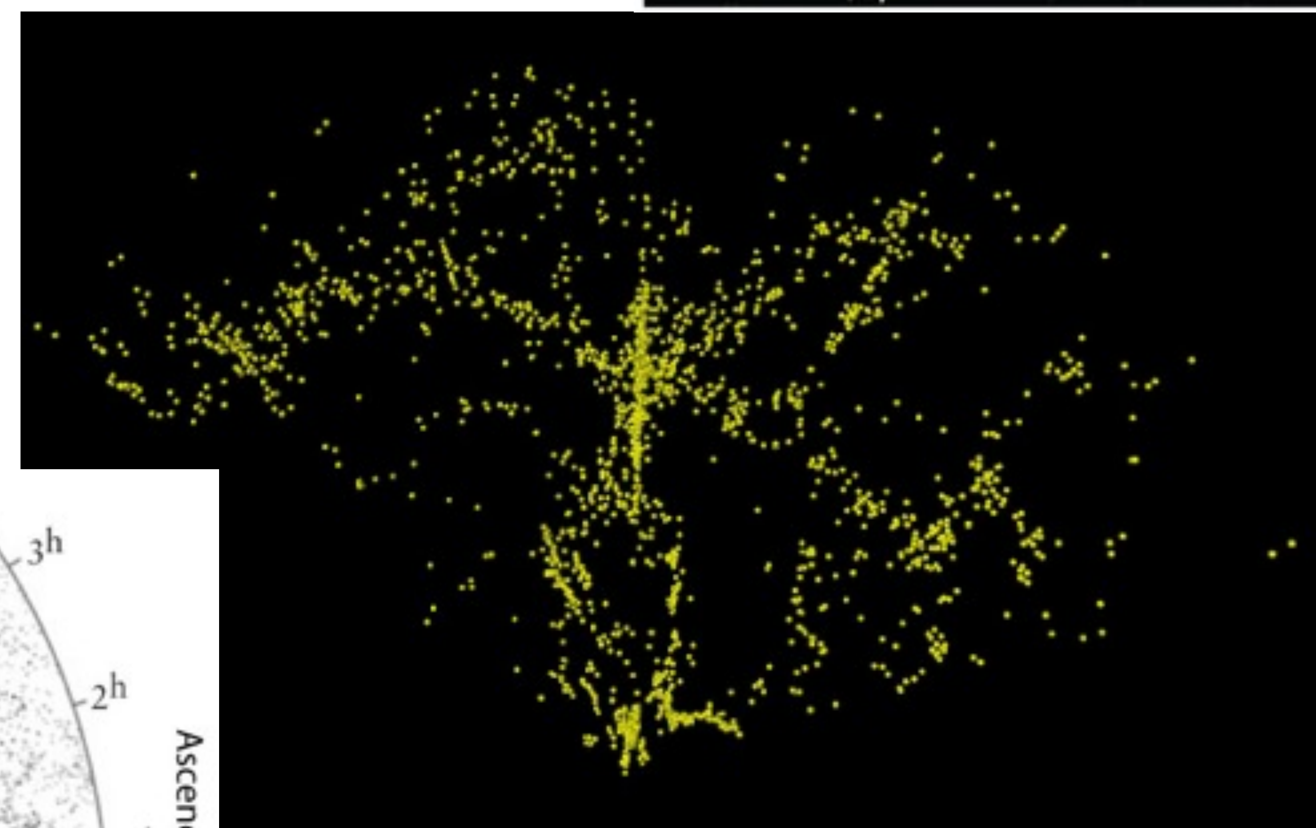
Correo Electrónico: marioalberto.rodriguez@inin.gob.mx

Segunda Reunión del
Grupo de Gravitación, Física de Altas Energías y Cosmología
Instituto de Ciencias Físicas, UNAM
Cuernavaca, Morelos. 4-6 de agosto de 2014

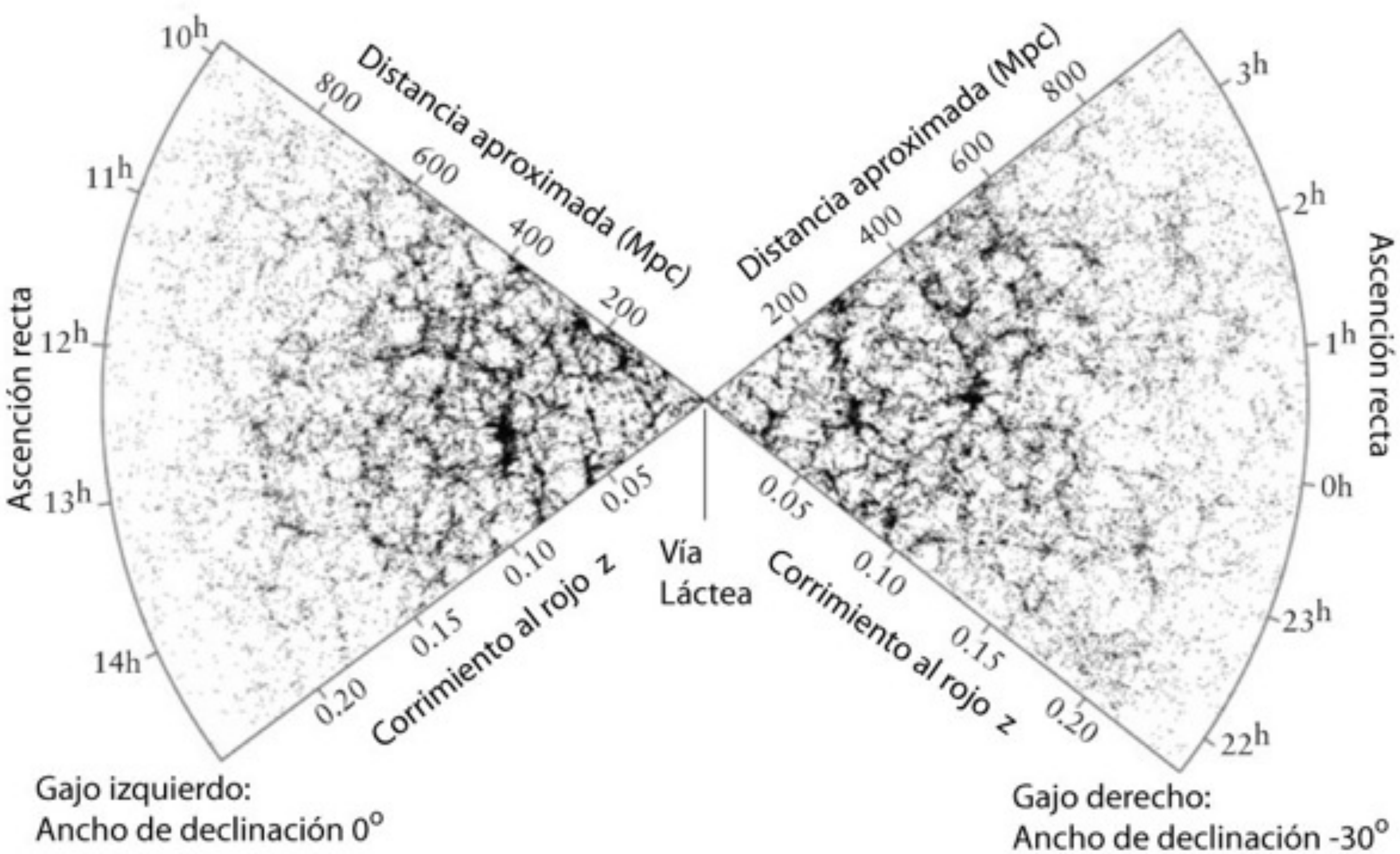


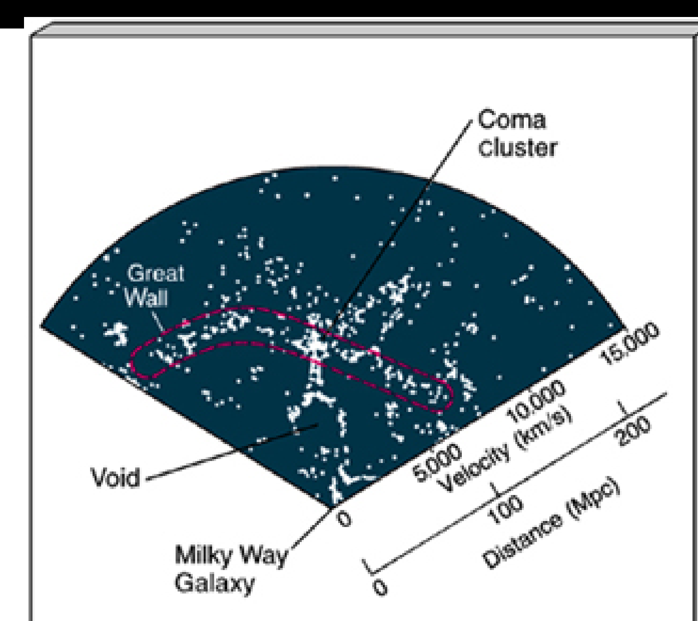
Observations: Large scale structure

- CfA
- 2dF (Redshift Survey Team/Anglo-Australian Observatory)
- 2MASS (Two Micron All Sky Survey)
- Durham/AAT redshift survey (included 200 galaxies at almost z=1/2!)
- SSDS (Sloan Sky Digital Survey)



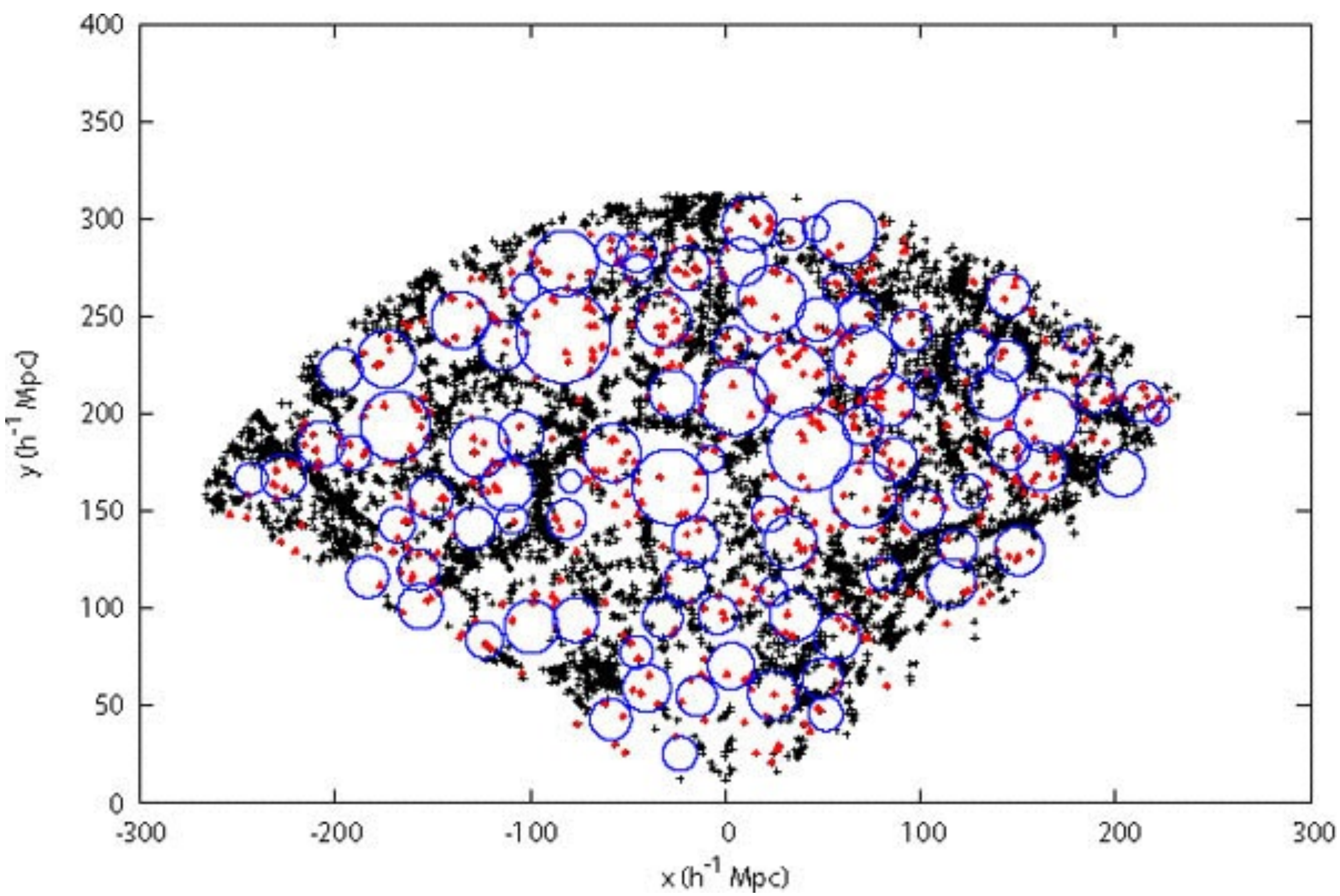
- 62,559 galaxias



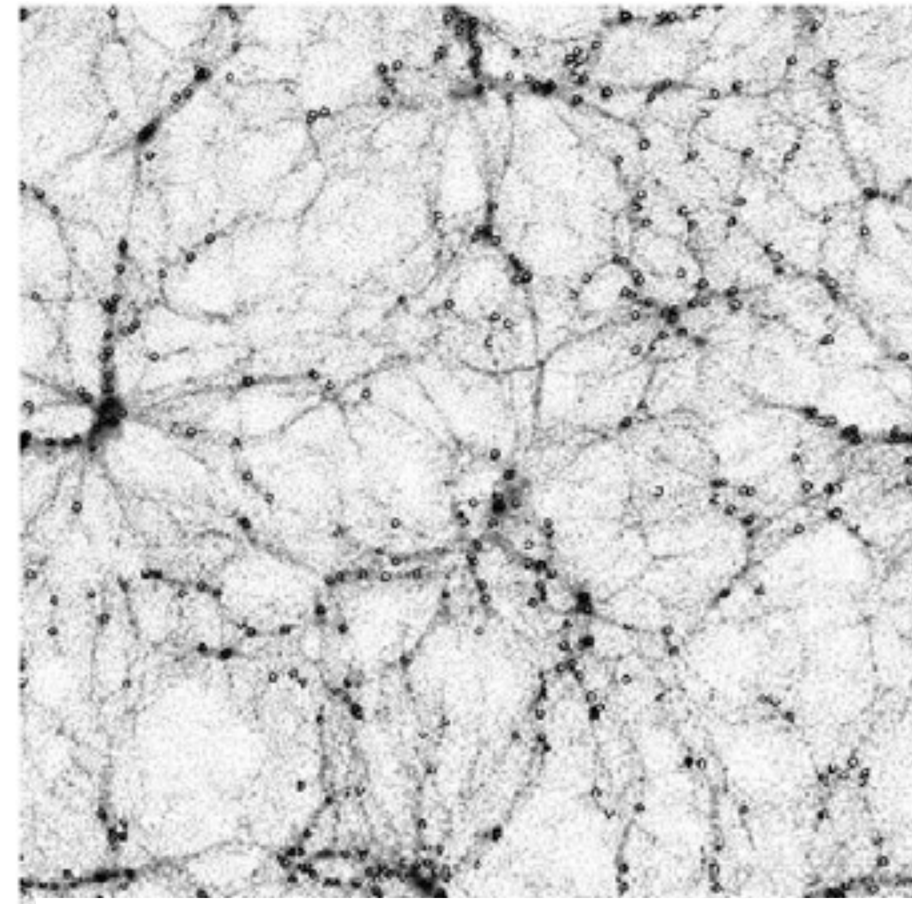


Observations: Voids

- Voids distribution in the SDSS survey. Shown is a projection of 10 Mpc/h slab with wall galaxies plotted as a black crosses and void galaxies plotted as red crosses. Blue circles indicate the intersection of the maximal sphere of each void with the midplane of the slab

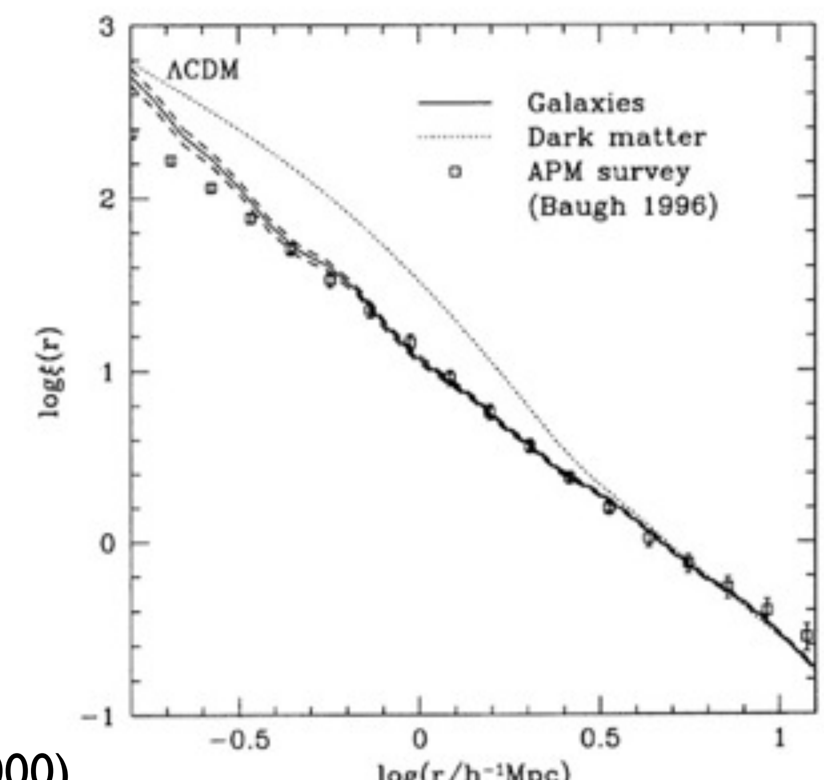


Simulations: characterising structure, voids, and clusters



- Top: The large scale structure obtained in a N-body simulation. Shown is only dark matter. Slab: 141x141x8 Mpc/h^3. The location of galaxies are shown by open circles. Galaxies were added by a semi-analytic model which assumes that dark matter haloes above a given mass threshold have at least one “central” galaxy located at the center of the halo. Higher mass haloes contain additional satellite galaxies.

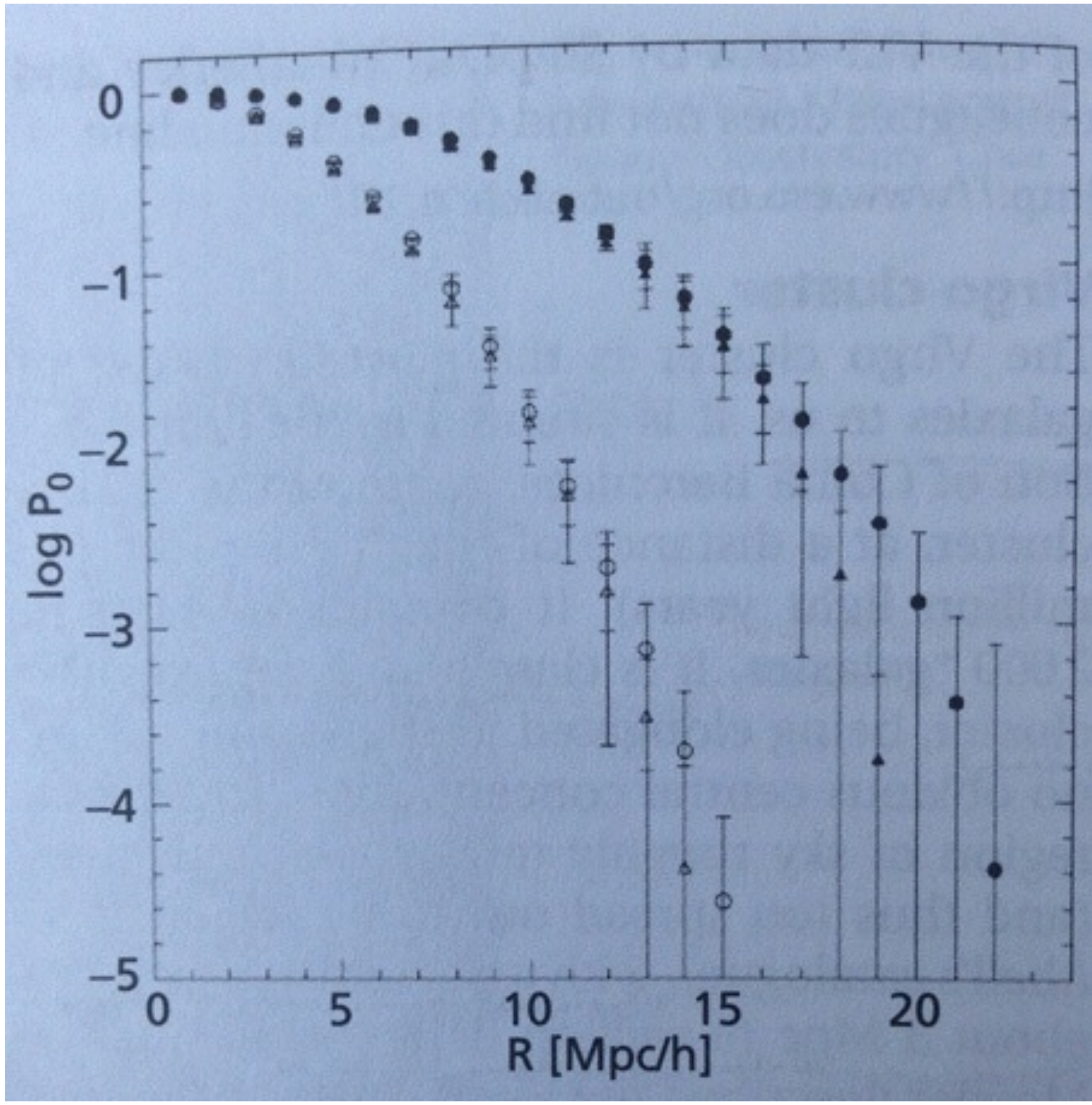
- Bottom: Two point correlation function for dark matter (dotted line); galaxies (solid lines, with dashed lines showing the Poisson error bars in the simulation. Open squares are APM survey.



- Benson et al (2000)
- Baugh (1996)

Simulations: characterising structure, voids, and clusters

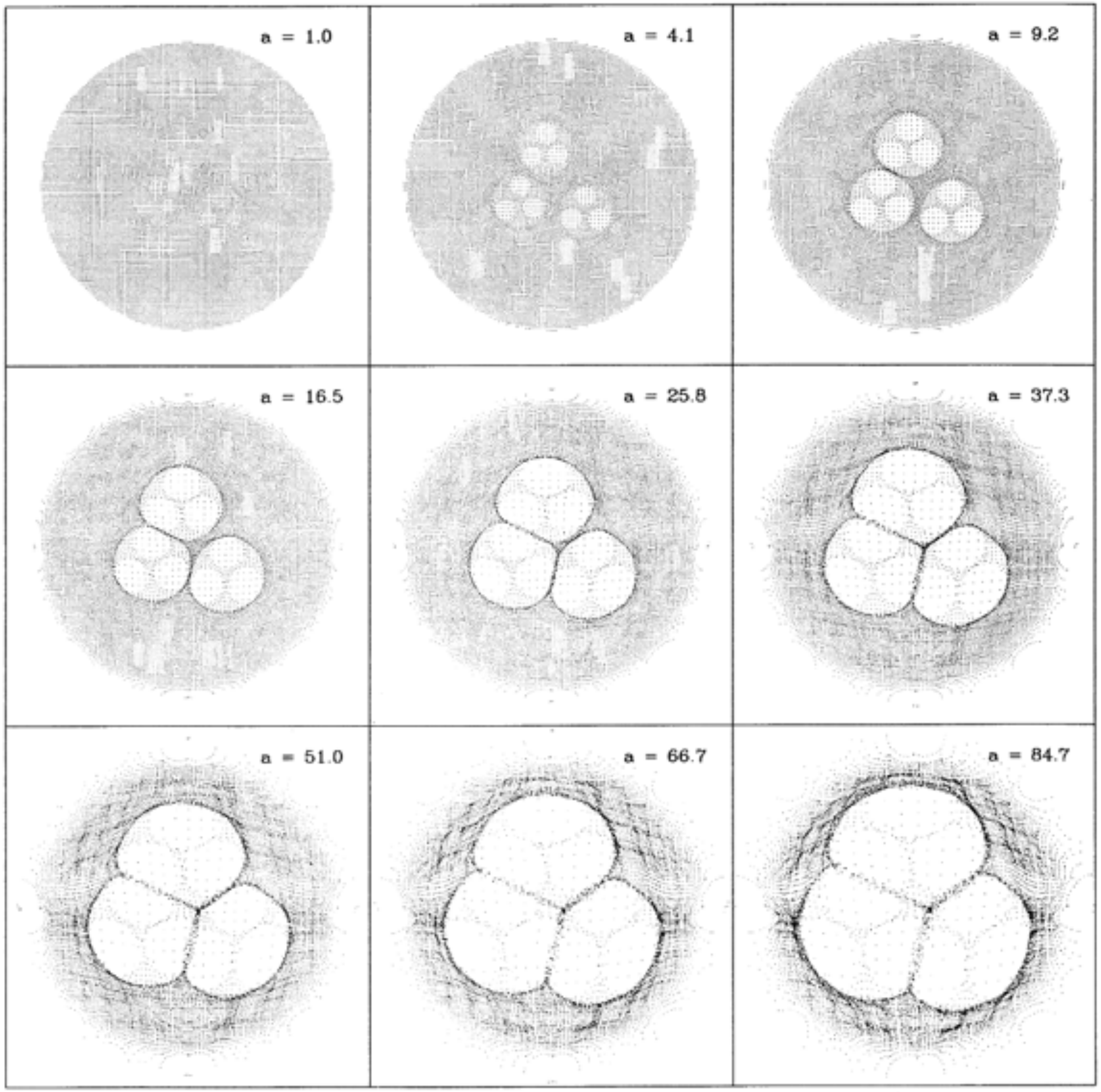
- The void probability function (VPF) measured from the 2dF galaxy redshift survey. Shown is the log probability that a randomly placed sphere of radius R contains no galaxies brighter than M=-19.3 (open symbols) and M=-20.2 (filled symbols). Circles and triangles show the VPF in the Southern and Northern galactic hemispheres respectively.



Simulations: void evolution and interaction

- Evolution of a void hierarchy in a Einstein-de Sitter universe.
- Growth of a negative top-hat density perturbation.
- Adjacent voids merge to form larger voids. Thin planar walls form between the merging voids.
- Particle position comoving positions for one slice through the center of the sphere are plotted at different epochs as shown.
- At different epochs, different voids scales emerge based on their initial depths.
- As evolution proceeds, substructure freezes in as a network of walls.

- MODEL: A spherical top-hat perturbation with the density contrast constant in the spherical region and zero elsewhere.



- Dubinski et al (1993)

Old simulations (light bulbs and so…)

- Light bulbs!… A solution has been found by replacing gravitation by light (Holmberg, 1941: Clustering tendency among nebulae: A study of encounter between laboratory models of stellar systems by a new integration procedure)
- Star clusters. Aarseth (1970s). By 1979 used 4,000 particles.
- Galaxy interaction with a three body method: Toomre and Toomre (1972).
- N-body simulations by Peebles (1970) used 300 particles. In 1975 Groth & Peebles made a "cosmological" N-body simulations using 1,500 particles with Omega matter=1.
- Galactic dynamic. Barnes (1980s). Hernquist (1980s-1990s).
- Galaxy clusters. Aarseth (1970s).
- Cosmological simulations (1990s) by Park using 4 million particles. Very good agreement with CfA survey was obtained. Even with the a great wall.

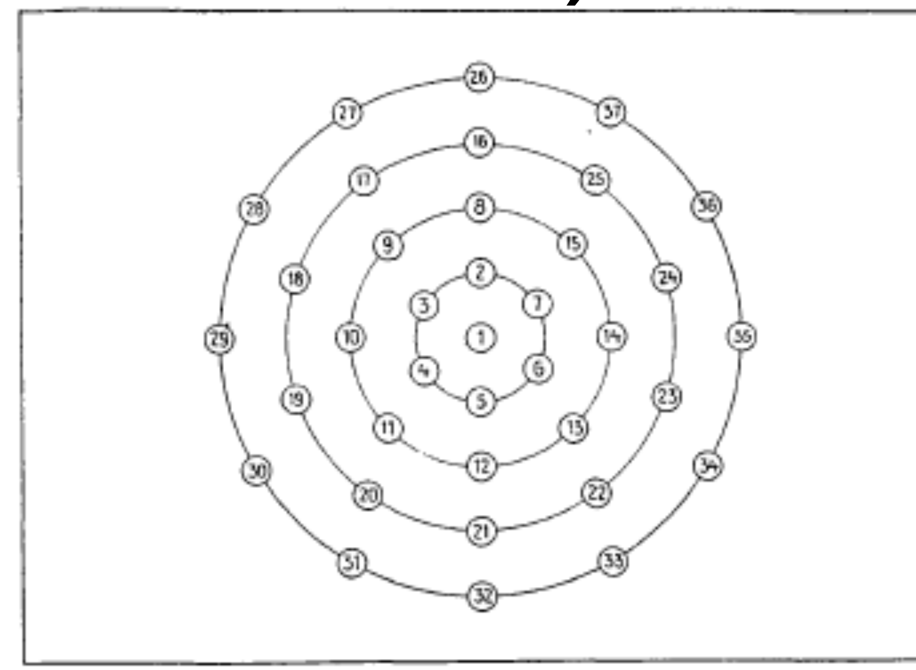
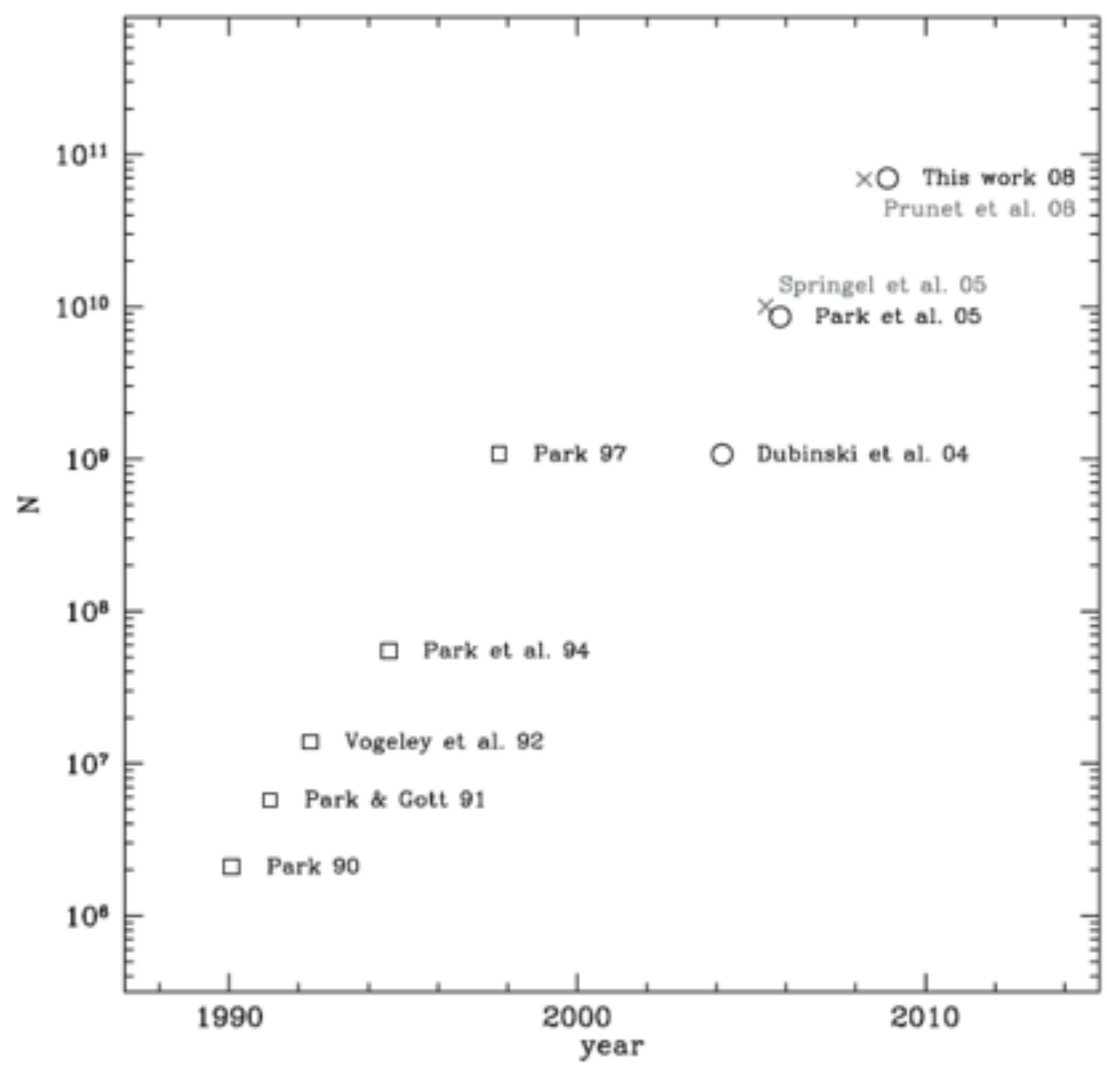


FIG. 3

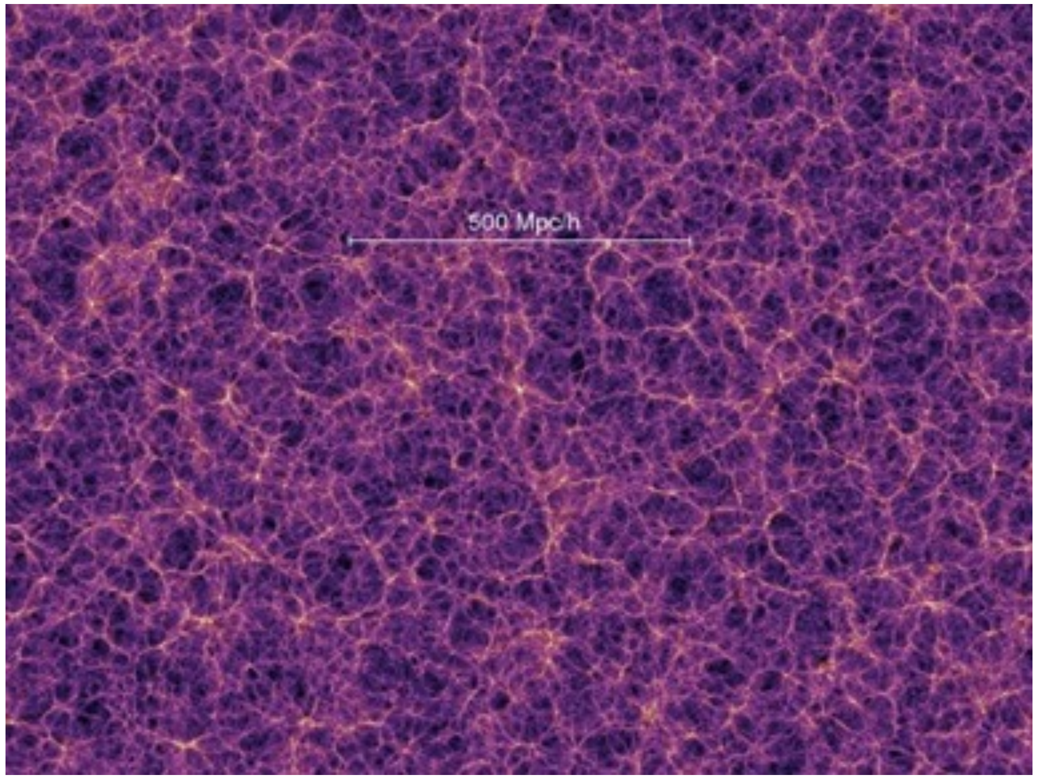


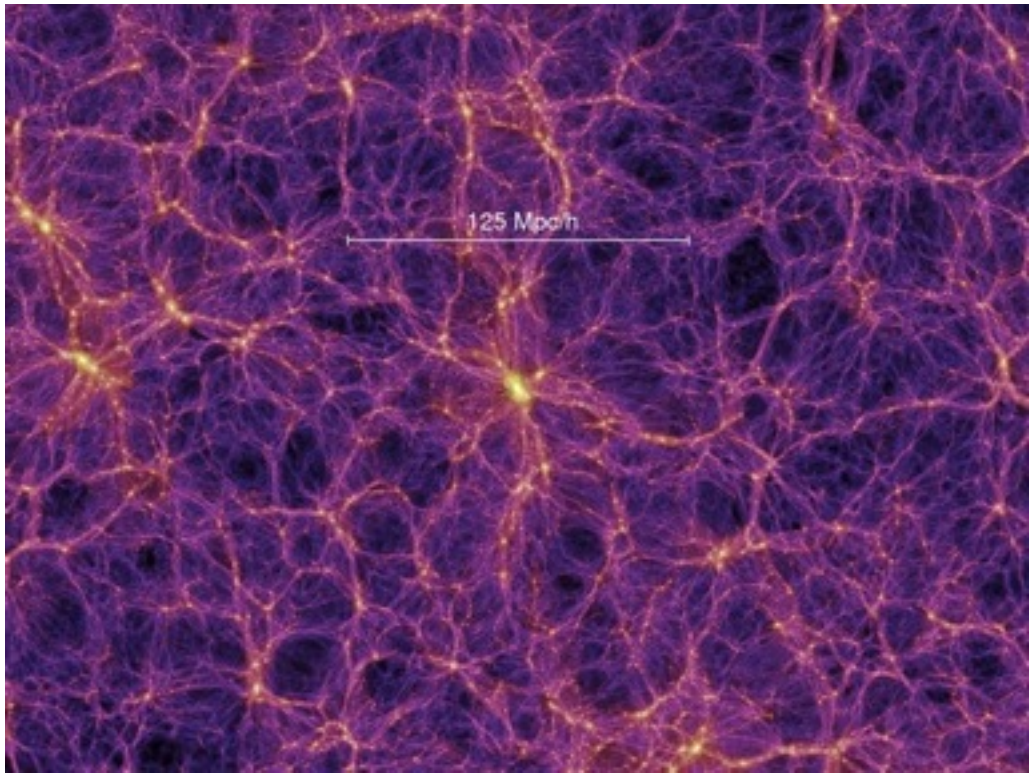
- Simulation number of particles in the Particle-Mesh (PM, squares) and PM-Tree (circles). Crosses are the Millennium and French collaboration simulations.

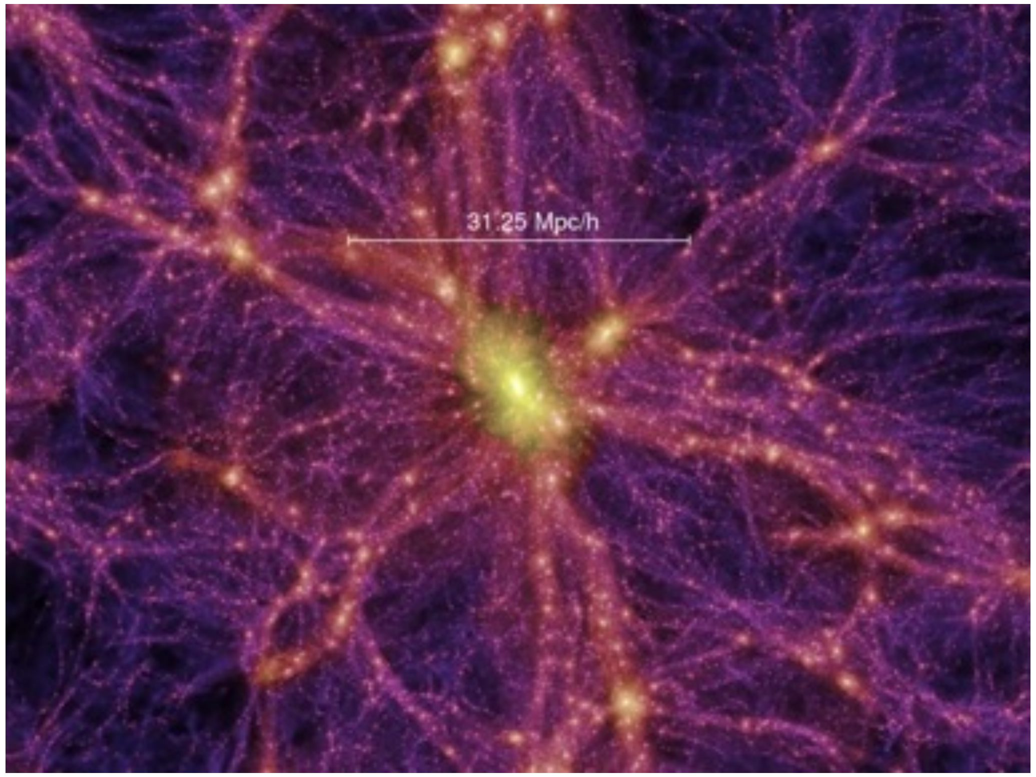
Millenium simulation I

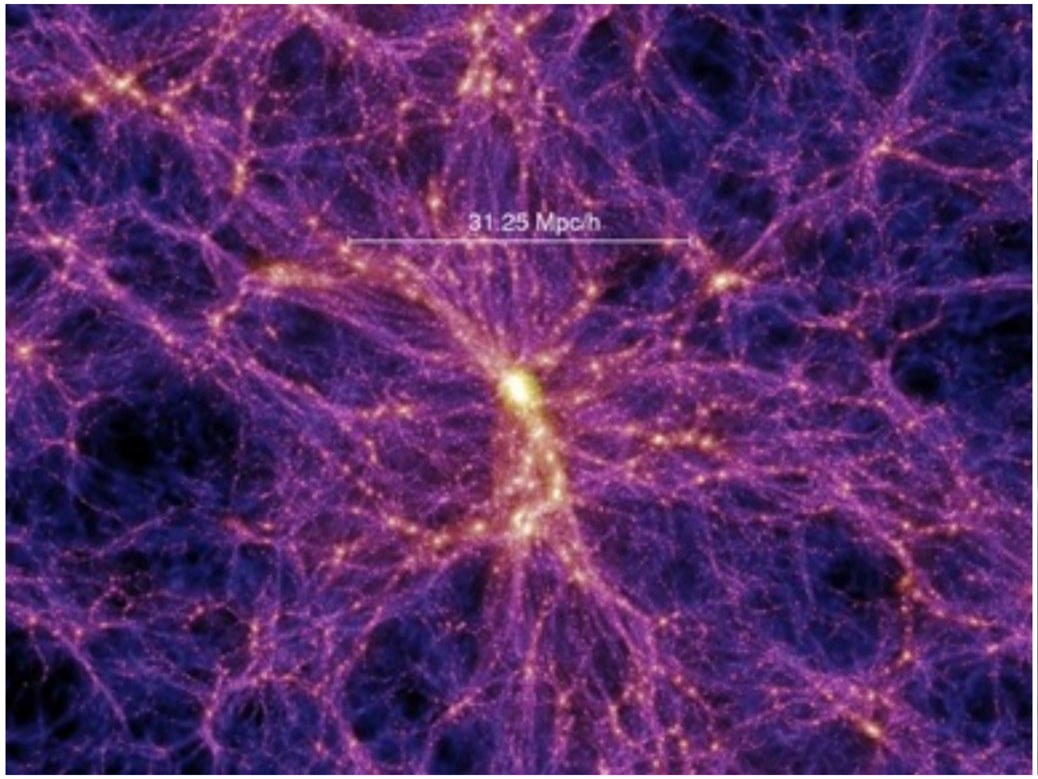
Nature 2005.

Simulation completed in summer 2004.





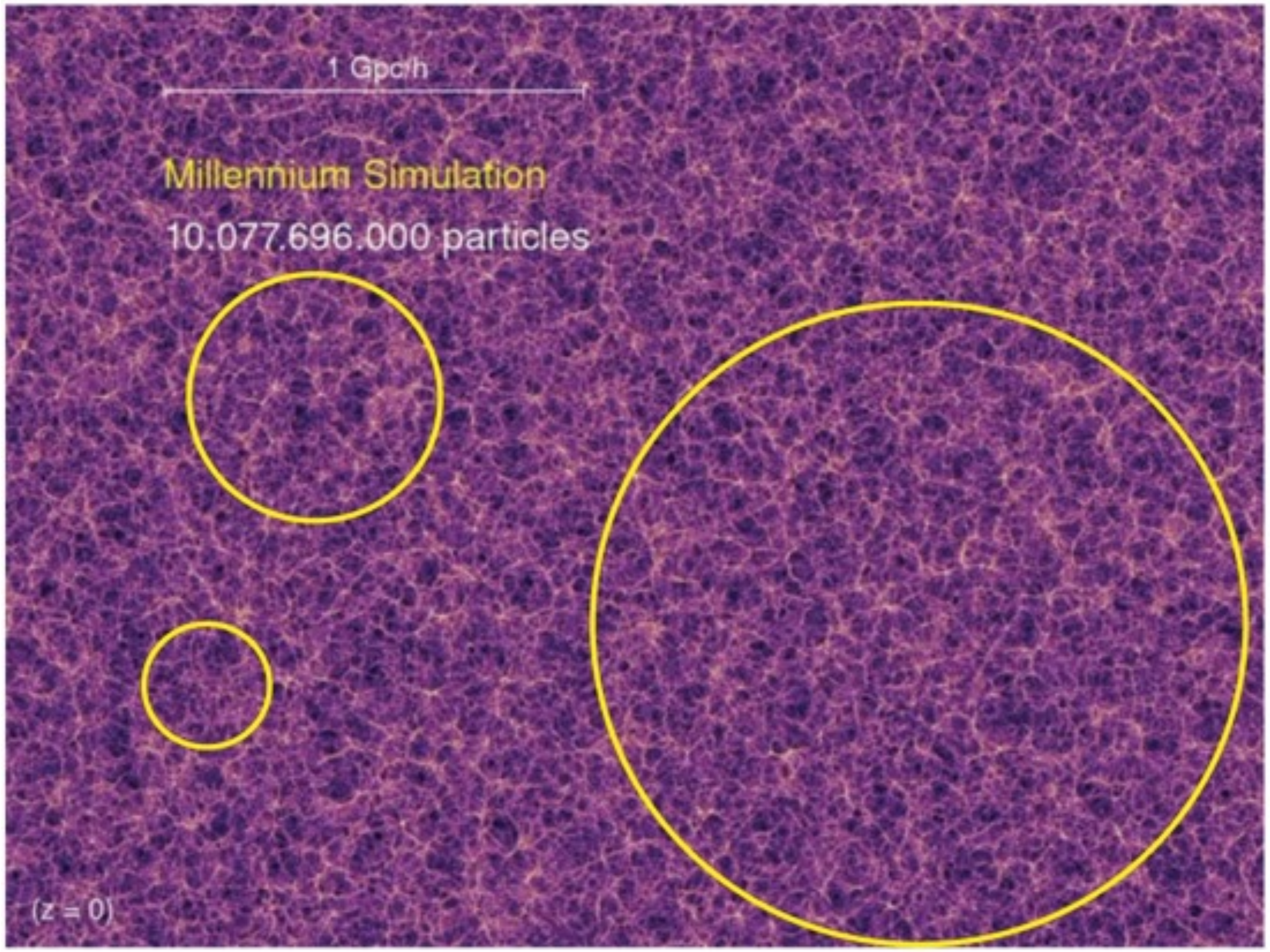


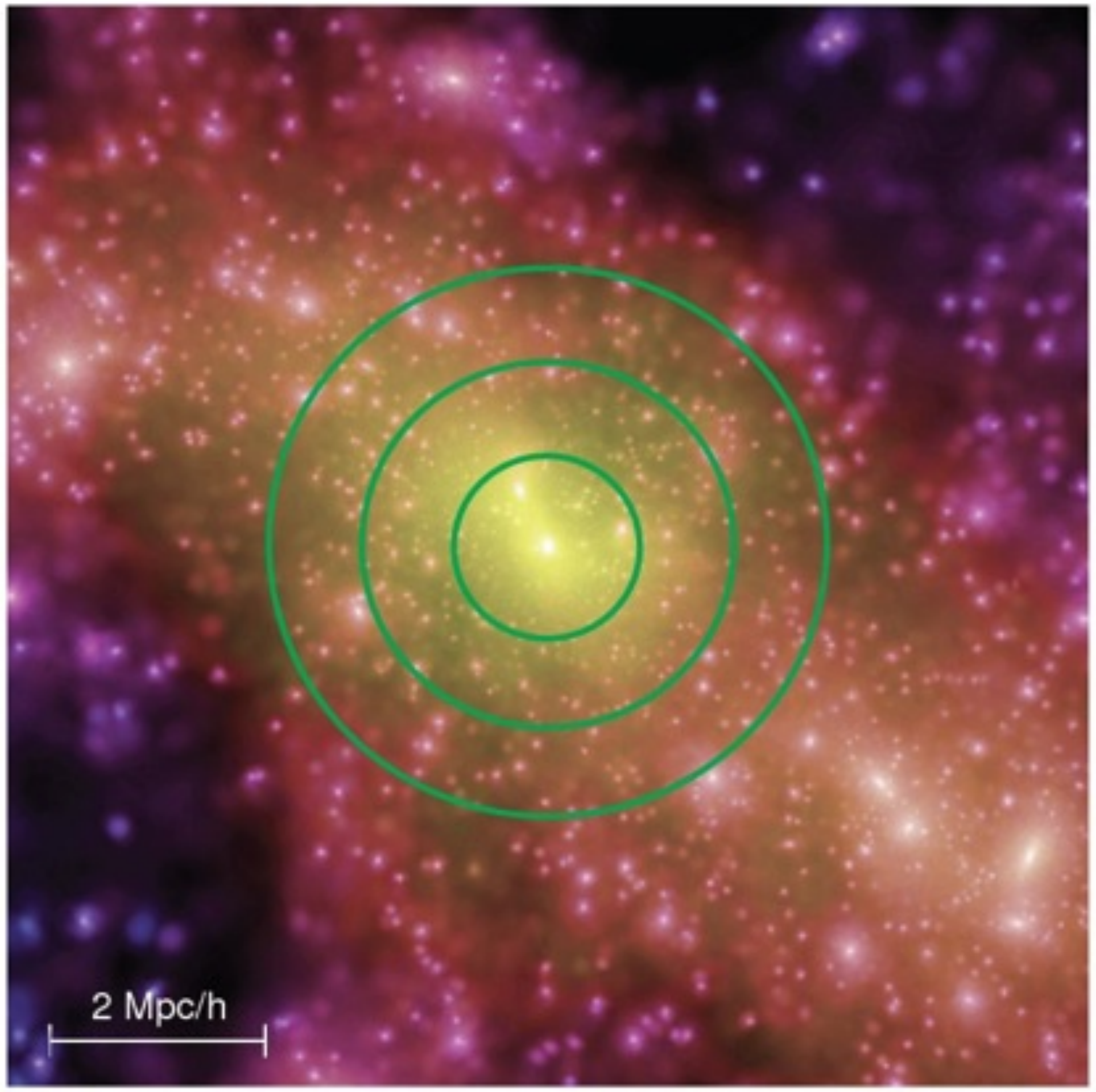


- 10,077,696,000 particles of masses 8.6 x 10^8 Msun/h. More than a month of CPU time of a supercomputer. 25 Tbytes data. Evolution history of 20 millions galaxies. Cubic region 500 Mpc/h on a side. Spatial resolution of 5 kpc/h. Initial z=127. Cosmology: 0.25 DM and baryons; 0.75 Lambda; sigma8=0.9; h=0.73, n=1. Gadget2 (modified version) with TreePM force calculation method.

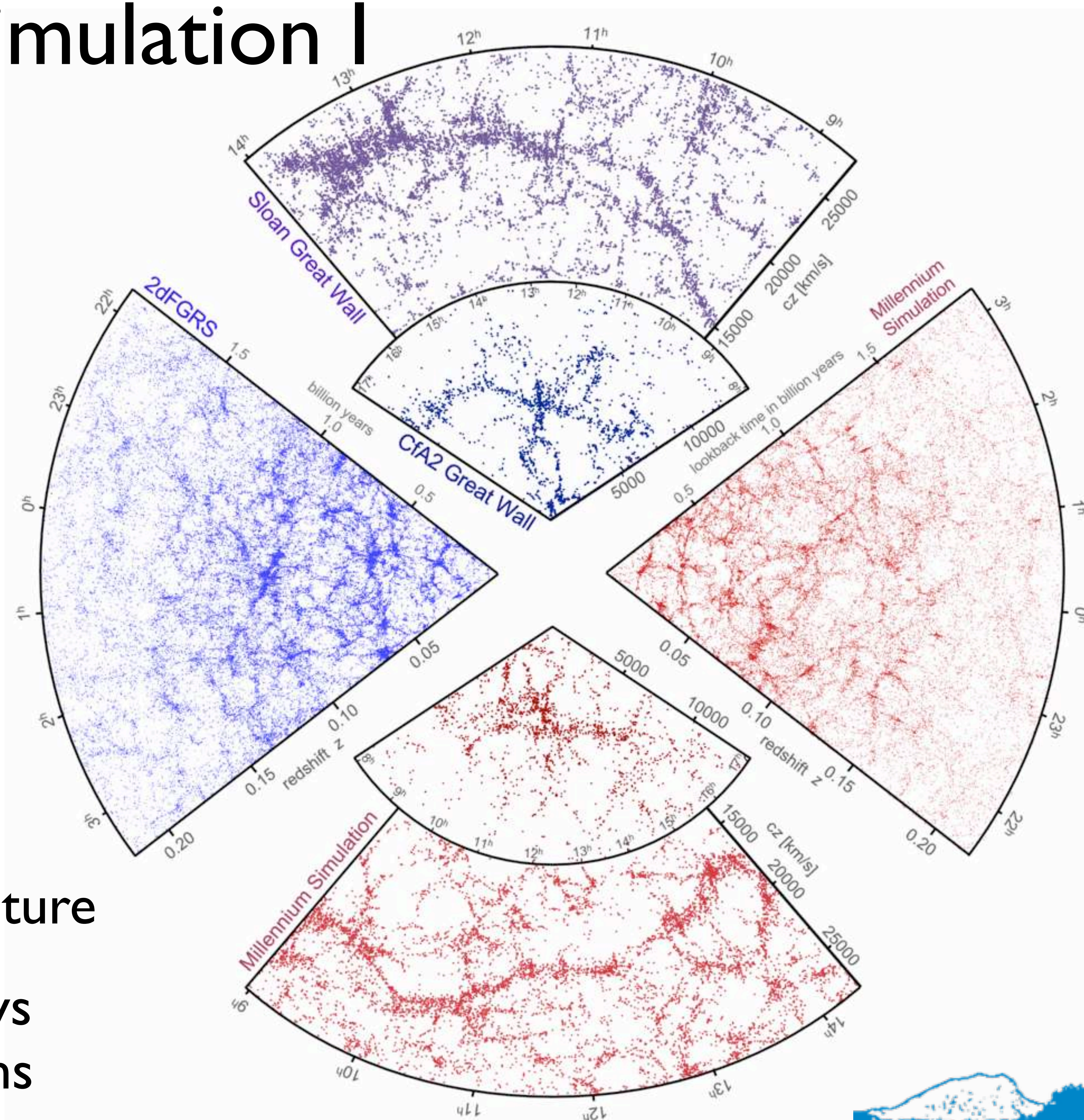
Millenium simulation I

- Cosmological principle
- NFW profile.





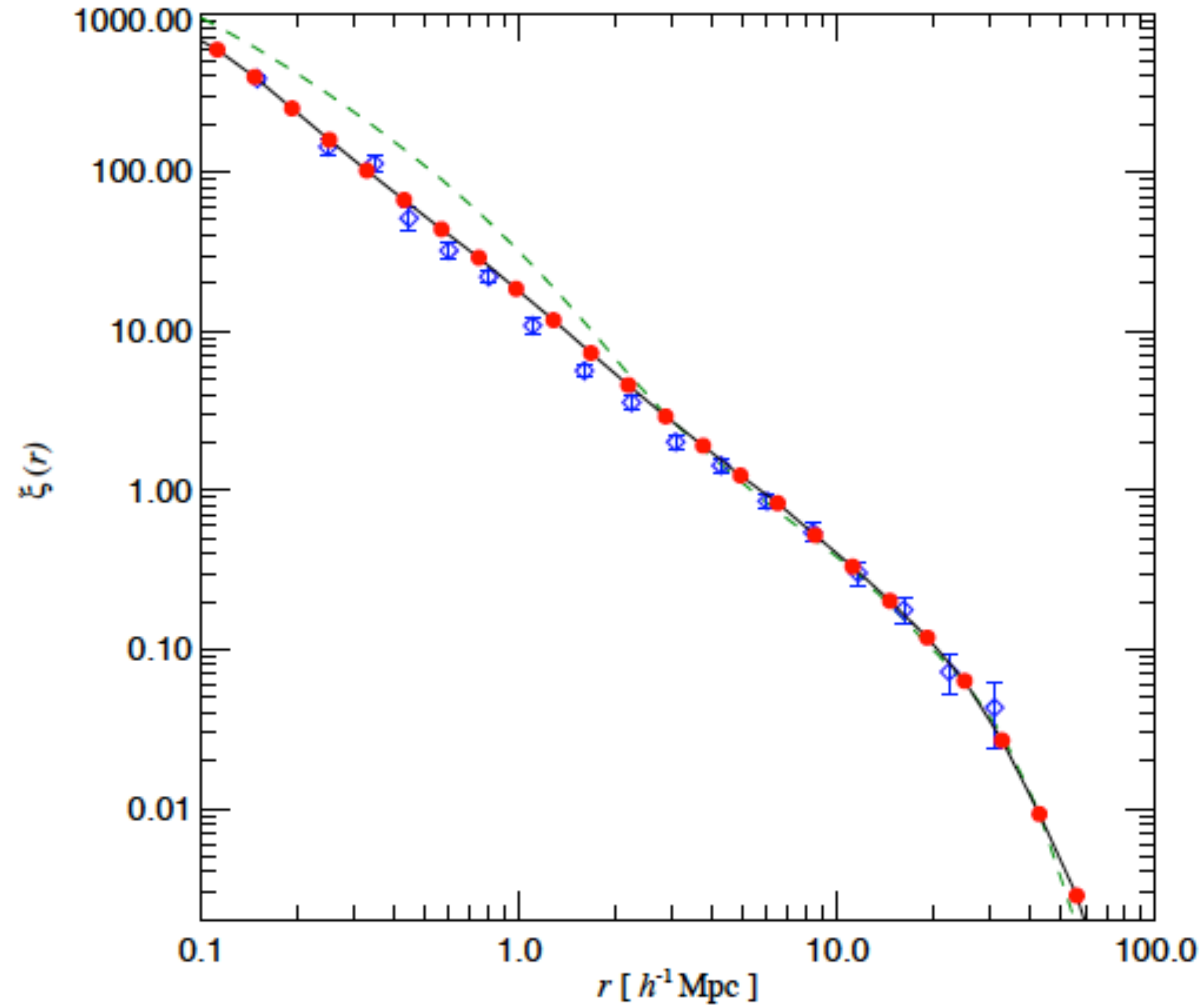
Millenium simulation I



- Large scale structure
- Observed surveys versus simulations

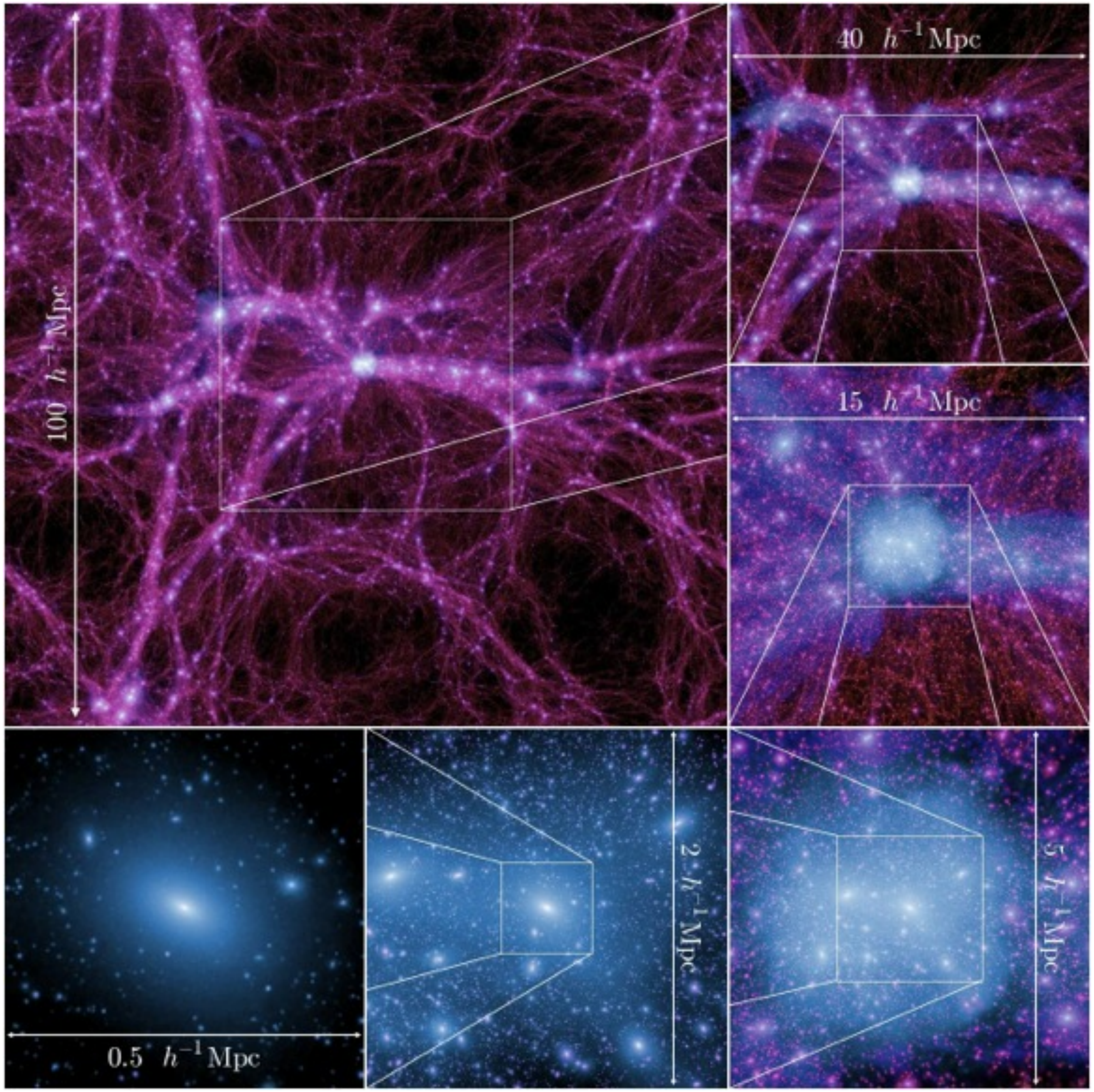
Millenium simulation I

- Galaxy 2-point correlation function at the present epoch. Red symbols (vanishingly small Poisson error-bars) show measurements for model galaxies brighter than M_K=-23. Data for the large spectroscopic redshift survey 2dFGRS are shown as blue diamonds. The SDSS and APM surveys give similar results. Both, for the observational data and for the simulated galaxies, the correlation function is very close to a power-law for r < 20 Mpc/h. By contrast the correlation function for the dark matter (dashed line) deviates strongly from a power-law



Millenium simulation II

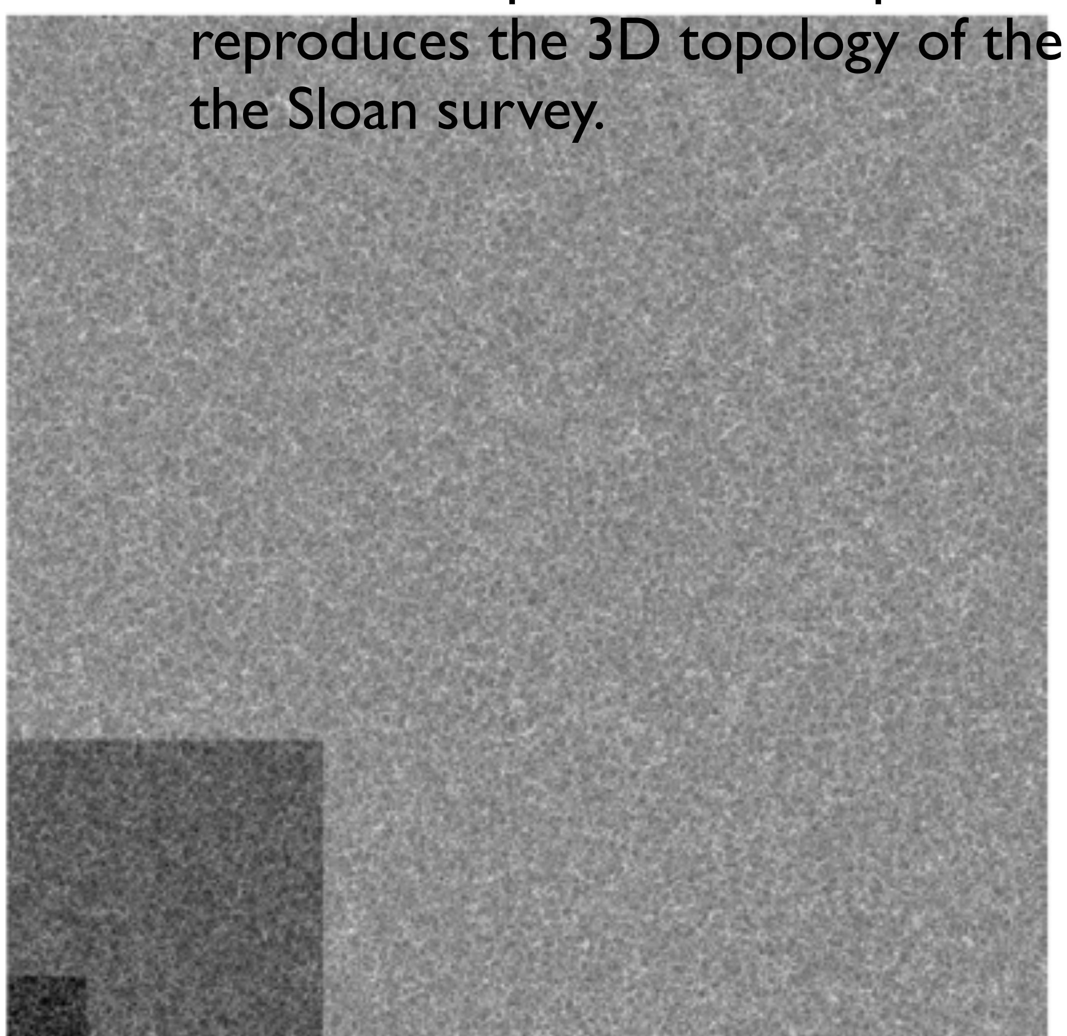
- It uses the same cosmological parameters and number of particles as Millennium I simulation in a cube one-fifth the size (100 Mpc/h), resulting in 5 times better spatial resolution and 125 times better mass resolution.

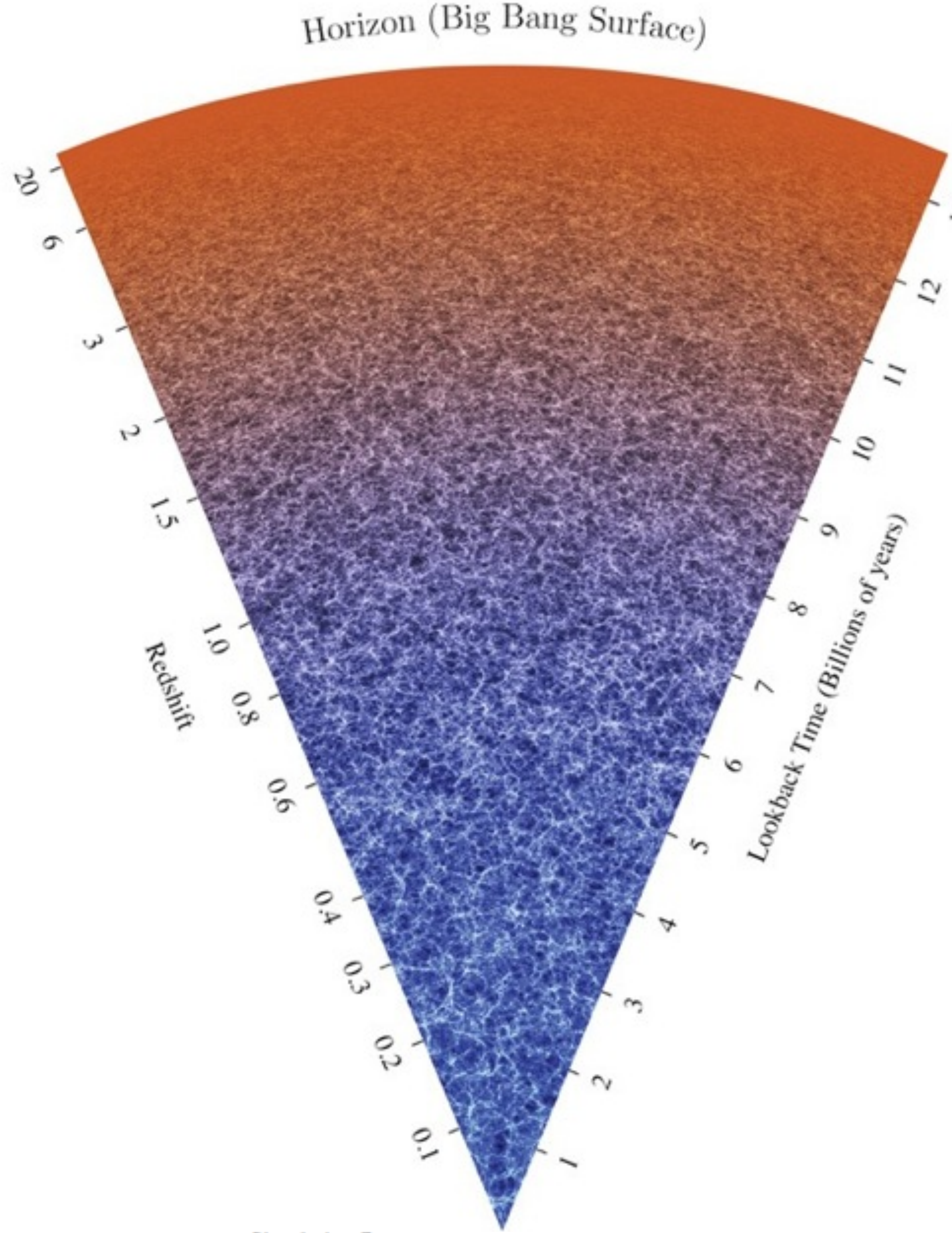


- Upper left panel is a 15 Mpc/h thick slice centered on the most massive halo. This FOF halo has M=8.2 x 10^14 Msun/h, similar to the mass of the Coma cluster and it contains 36,000 resolved subhalos spanning 6.7 decades in mass.

Horizon simulation

- Kim et al. 2009 ApJ: "BAO and topology of large-scale structure of the universe".
- N-body simulation using 4120^3=69.9 billion particles in a volumen (6.592 Gpc/h)^3, 2000 times the volume of the Millennium run.
- Luminous red galaxies (LRGs) are selected by finding the most massive gravitationally bound, cold dark matter subhalos, not subject to tidal disruption, a technique that correctly reproduces the 3D topology of the LRGs in the Sloan survey.





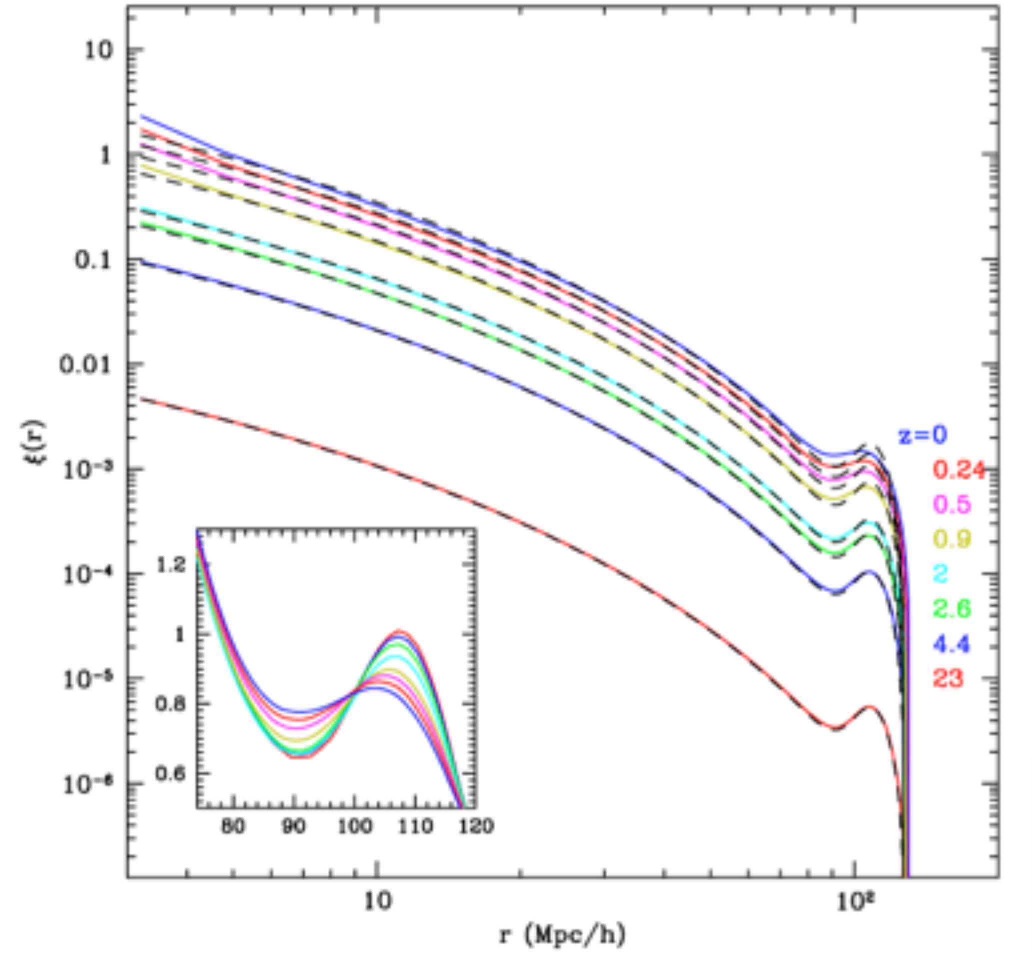
Simulation Parameters

N_p	N_m	L_{box}	N_{step}	z_i	h	n	Ω_m	Ω_b	Ω_Λ	b	m_p	f_ϵ
4120^3	4120^3	6592	400	23	0.72	0.96	0.26	0.044	0.74	1.26	2.96×10^{11}	$160\,h^{-1}$ kpc

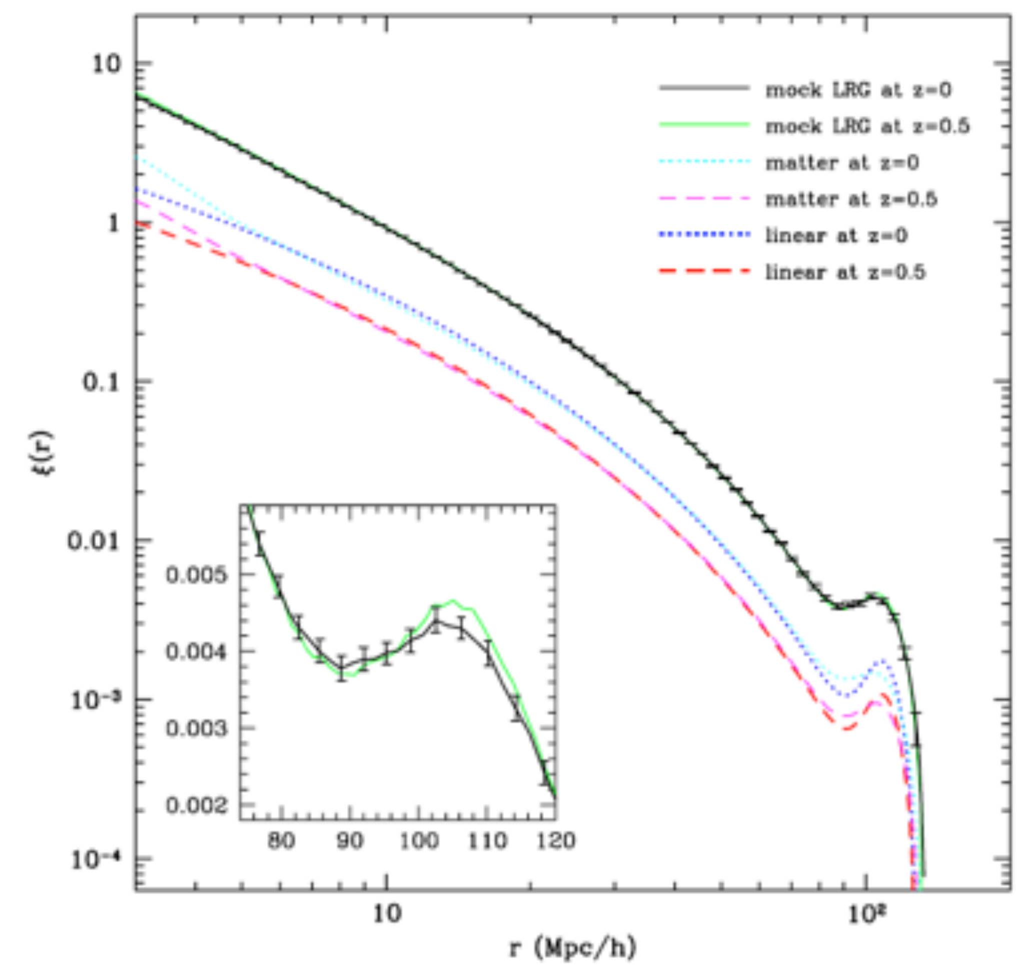
Notes. Coulmns: (1) number of particles, (2) Size of mesh. Number of initial conditions, (3) Simulation box size in h^{-1} Mpc, (4) Number of steps, (5) Initial redshift, (6) Hubble parameter in 100 km s^{-1} Mpc^{-1}, (7) Primordial spectral index of $P(k)$, (8) Matter density parameter at $z = 0$, (9) Baryon density parameter at $z = 0$, (10) Dark energy density parameter at $z = 0$, (11) Bias factor, (12) Particle mass in $h^{-1} M_\odot$, and (13) Gravitational force softening length.

Horizon simulation

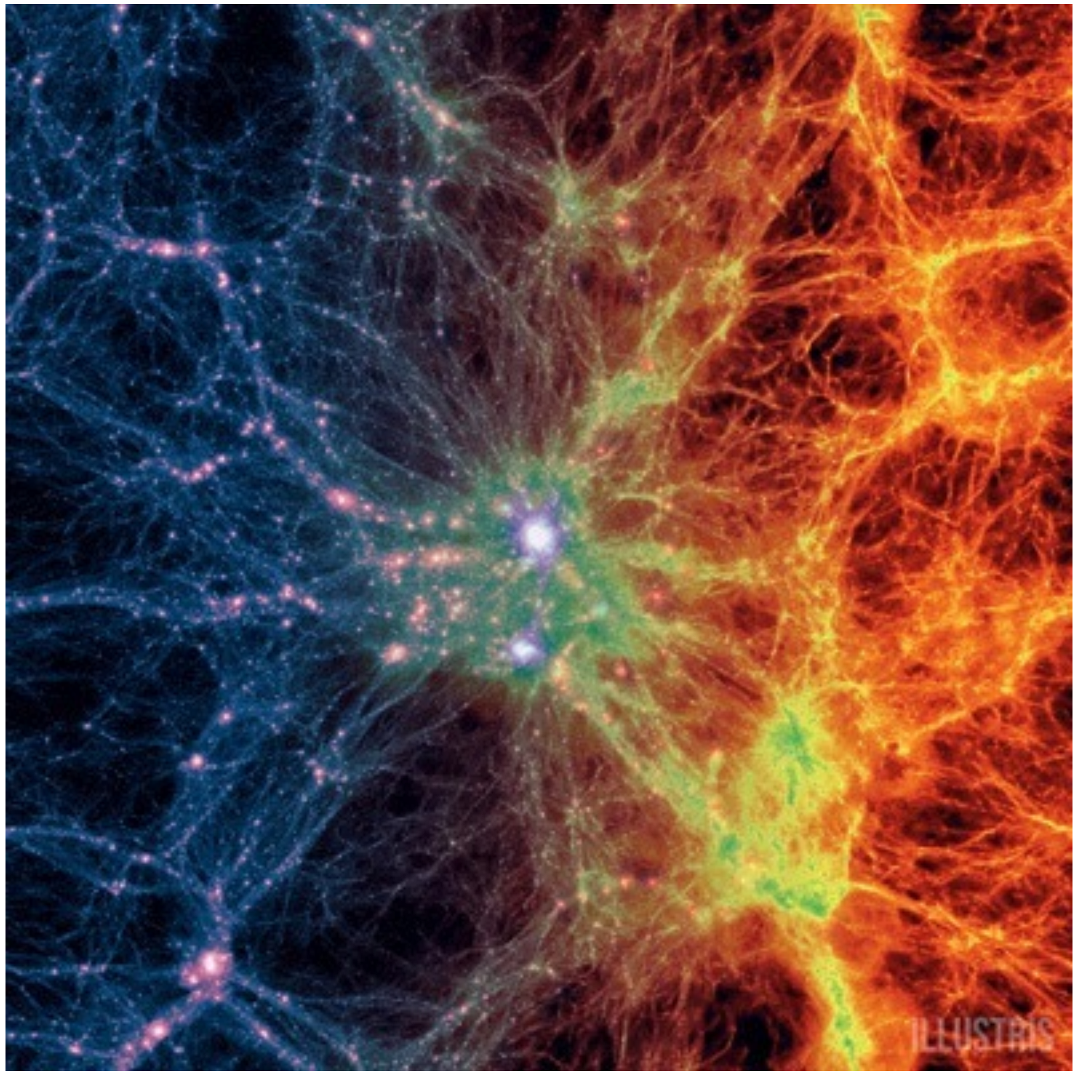
- Evolution of the correlation function of the matter density field at the epochs from z=0 to 23. Dashed curves are the linearly evolved correlation functions, and the coloured ones are the matter correlation functions measured from the horizon simulation. The inset box magnifies the matter correlation functions near the baryon oscillation bump with amplitudes to match at r=48 Mpc/h after scaling the peak of the baryonic bump of matter correlation at z=23 to unity.



- Top curves: the real space correlation functions of the mock LRGs in the whole cube at z=0 and 0.5. 3sigma error bars are attached to the correlation function at z=0. The matter density correlation functions and the linear theory correlation functions at z=0 and 0.5 (bottom curves), are also shown.



Illustris simulation

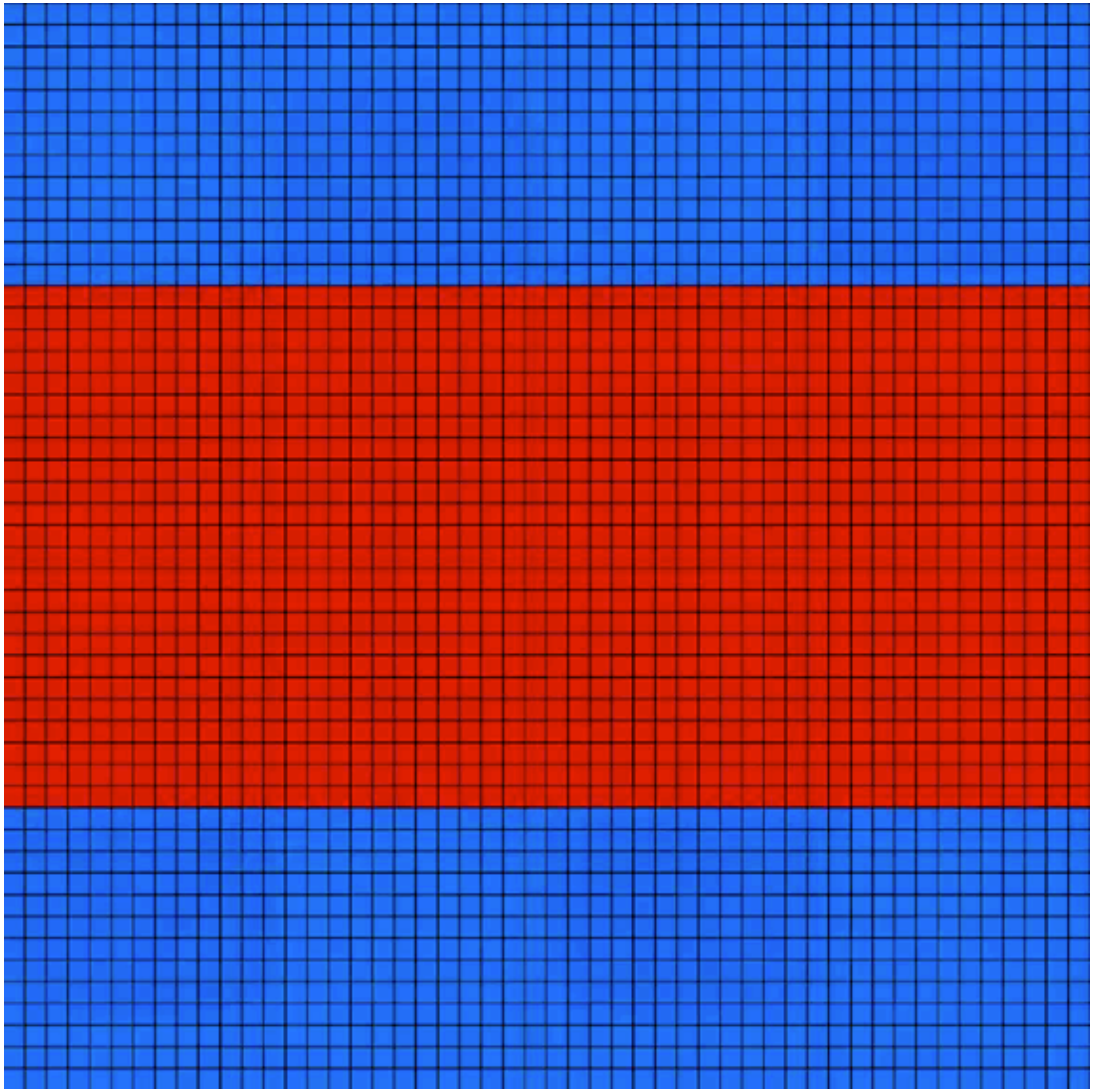


- Cosmology and IC: It was used CAMB to compute the linear power spectrum of a LCDM cosmology with 0.2726 DM and baryons, 0.7274 Lambda, 0.0456 Baryons, sigma8=0.809, n_s=0.963 and H0=100 h km/s/Mpc with h=0.704. These parameters are consistent with the latest WMAP-9 measurement.

- The simulation box is a cube with a side length of 75 Mpc/h. It was used FFT to calculate the displacement field and use Lagrangian Perturbation Theory (Zeldovich approximation) to displace particles. Initial condition were generated at z=127 with mesh-generating points added to the initial conditions by splitting each original particle into a dark matter and gas cell pair, displacing them with respect to each other such that two interleaved grids are formed, keeping the centre-of-mass of each pair fixed. The initial gas temperature at z=127 was set to 245 K based on RECFAST calculation.

- The simulation volume contains initially 6,028,568,000 hydrodynamic cells and the same number of dark-matter particles resulting in a dark-matter mass resolution of 6.26 x 10^6 Msun, and a baryonic mass resolution of 1.26x10^6 Msun. The gravitational softening length was 1 kpc/h. The smalles cells in Illustris have a typical extent of 48 pc. For the least massive cells they achieve a mass resolution of 1.5 x 10^4 Msun.

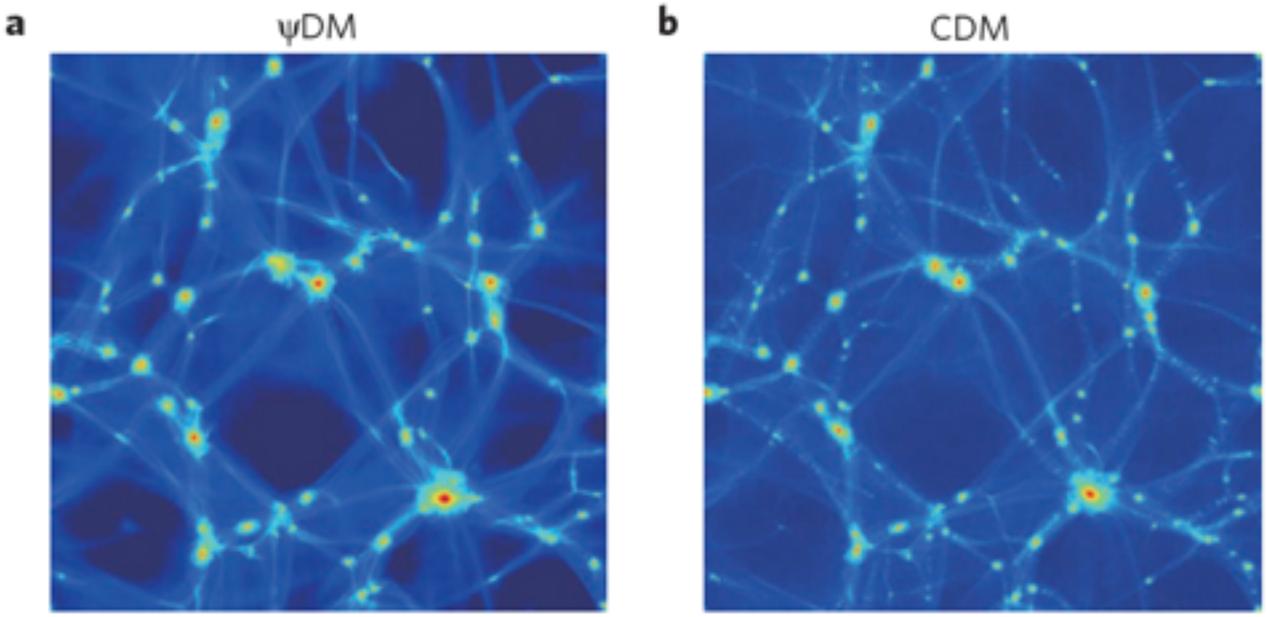
Arepo code

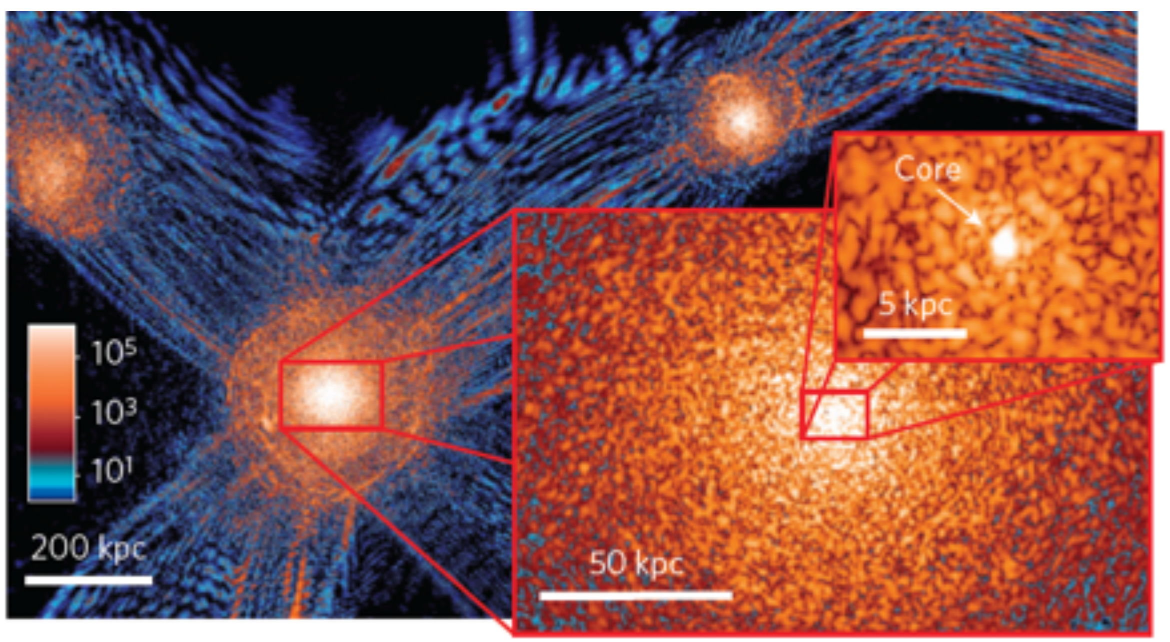


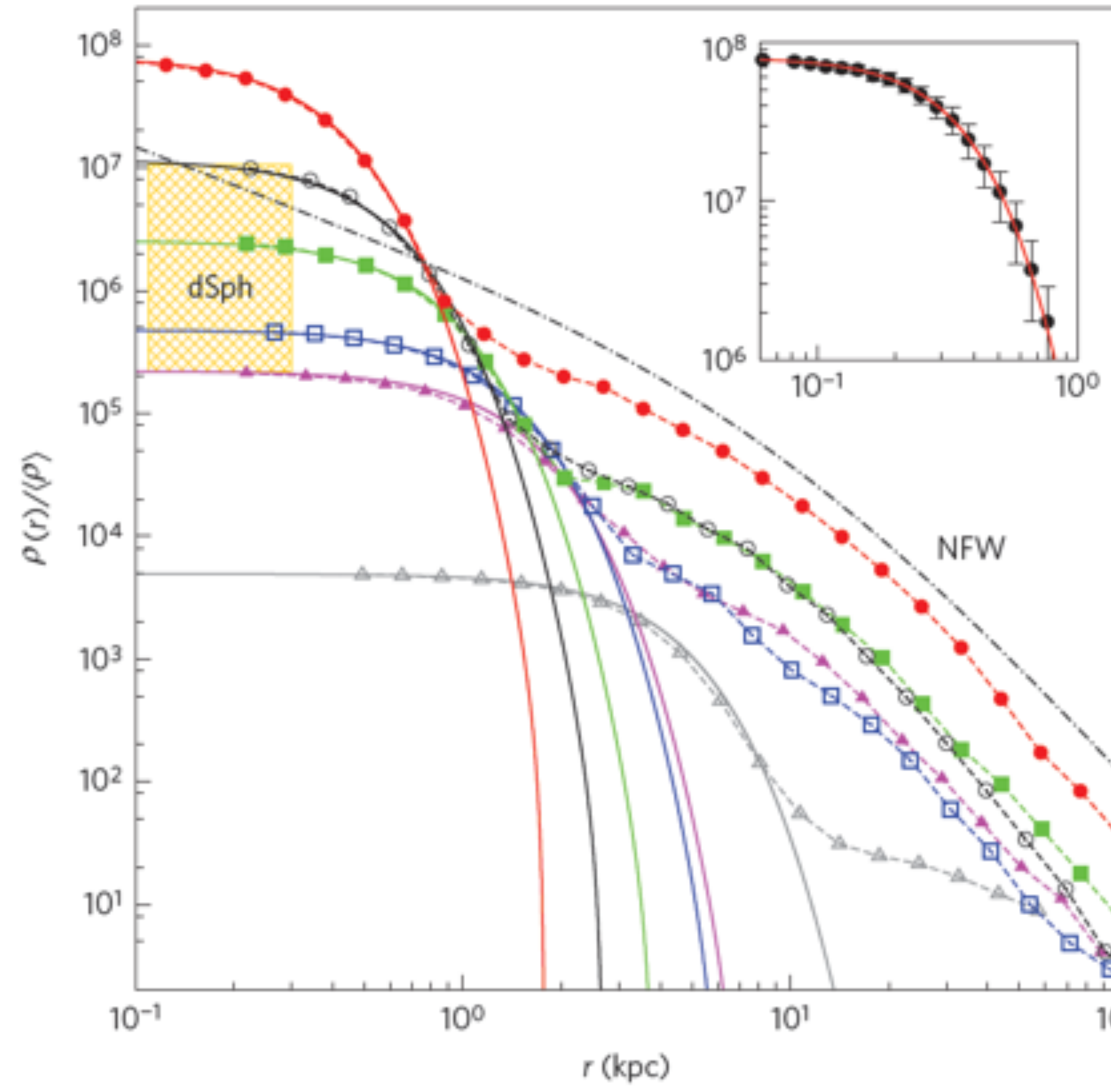
- Arepo code uses an unstructured Voroni tessellation of the simulation volume, where the mesh-generating points of this tessellation are moved with the gas flow. The adaptive mess is used to solve the equations of ideal hydrodynamics with a finite volume approach using a second-order unsplit Godunov scheme with an exact Riemann solver. In short better than SPH and Eulerian adaptive mesh refinement methods (AMR). This scheme naturally produces extended disk galaxies without invoking extreme forms of stellar feedback or star formation, which was a major problem of previous galaxy formation simulations.

- Gravity calculation employs a Tree-PM scheme where long-range forces are determined with a particle-mesh method (PM) while short-range forces are computed via a hierarchical tree algorithm.

Scalar field cosmological simulation







- 80 % of the halos in the simulation have an average density within 300 pc, in the range 5.3x10^-3 -6.1x10^-1 Msun/pc^3, consisten with dSph satellites around MW

- LCDM problems with common dwarf galaxies with surprisingly uniform central masses and shallow density profiles. Galaxies predicted by CDM extend to much lower masses, with steeper, singular profiles.

- This is the motivation of cold, wavelike dark matter composed of a non-relativistic BEC, such that the uncertainty principle counters gravity below a Jeans scale.

- This is a simulation capable to resolve dwarf galaxies with only one parameter, the boson mass that result equal to 8 x 10^-23 eV. The onset of galaxy formation is substantially delayed relative to CDM, appearing in z<13 (aprox.).

- The structure are created by evolving a single coherent wave function for LpsiDM computed with an AMR grids.

The N-body problem

- Boltzmann
- Jeans
- Focker-Planck
- Newton second law
- And Poisson equation or geodesics

The N-body problem: numerical issues

- Force smoothing length, number of particles, time step size.
- Relaxation and dynamical friction
- Consistency, convergence and stability

Initial conditions

- Zeldovich
- 1LPT
- 2LPT

N-body cosmological data analysis

- Halo catalogs (FOF)
- Voids and VPF

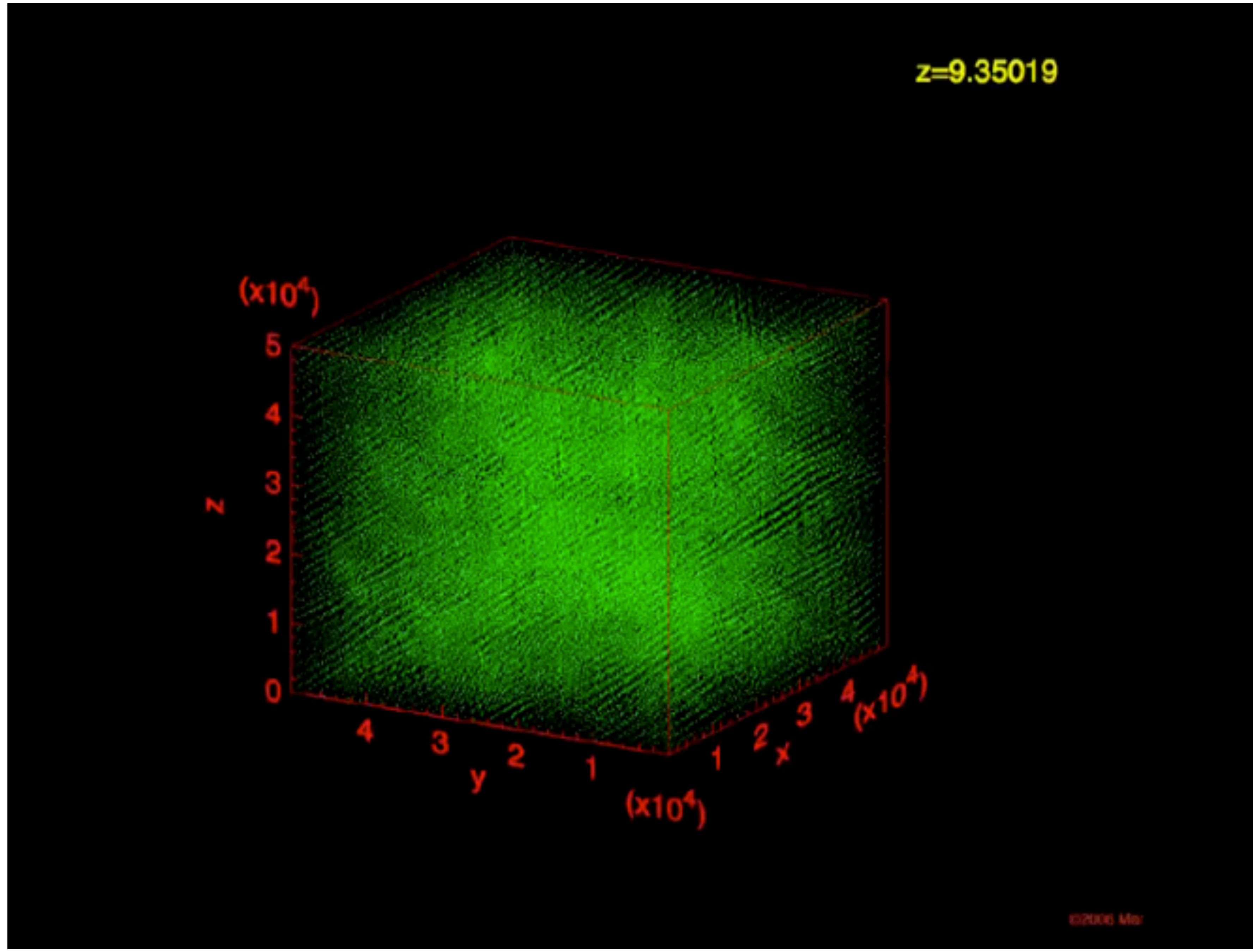
The N-body simulations: Comparison of numerical codes

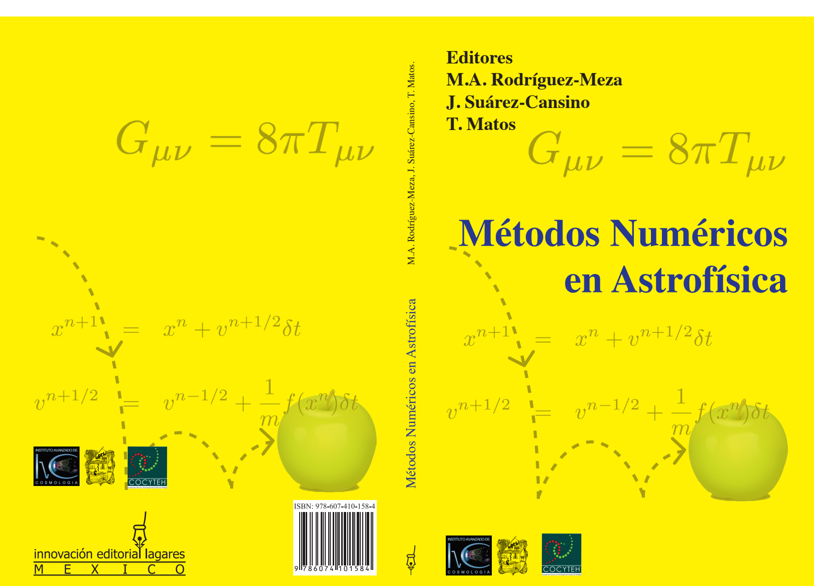
- Santa Barbara comparison project
- Heitmann et al. comparision

Una simulación numérica ...

$$\mathbf{F} = m_i \mathbf{a}_i$$

$$\mathbf{F} = -\frac{G m_i m_j}{r_{ij}^2} \mathbf{r}_{ij}$$





Scalar field N-body simulations: coupled scalar-field cosmology

- N-body simulations for coupled-scalar-field models, including background cosmology and the generation of initial conditions (with different couplings to different matter species taken into account).
- Class of coupled-scalar-field models with an inverse power-law potential and negative coupling constant, for which the chameleon mechanism does not work.
- For such cosmological models the scalar-field potential plays a negligible role except in the background expansion, and the fifth force that is produced is proportional to gravity in magnitude, justifying the use of a rescaled gravitational constant G in some N-body simulations made for similar models.
- It is studied the effects of the scalar coupling on the nonlinear matter power spectra and compare with linear perturbation calculations to see the agreement and places where the nonlinear treatment deviates from the linear approximation.
- Virialization is also modified by the scalar-field coupling.
- It is found that the net effect of the scalar coupling helps produce more heavy halos in the simulations and suppresses the inner density profile of halos compared with the LCDM, while the suppression weakens as the coupling between the scalar field and dark-matter particles increases in strength.

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: The equations

The Lagrangian for our coupled scalar field model is

$$\mathcal{L} = \frac{1}{2}\left[\frac{R}{\kappa} - \nabla^a\varphi\nabla_a\varphi\right] + V(\varphi) - C(\varphi)\mathcal{L}_{\mathrm{CDM}} + \mathcal{L}_{\mathrm{S}} \ , \tag{1}$$

where R is the Ricci scalar, $\kappa = 8\pi G$, with G Newton's constant, φ is the scalar field, $V(\varphi)$ its potential energy, and $C(\varphi)$ its coupling to the dark matter, which is assumed to be cold and described by the Lagrangian $\mathcal{L}_{\mathrm{CDM}}$; $\mathcal{L}_{\mathrm{S}}$ includes all other matter species, including the *baryons*. The contributions from photons and neutrinos in the N-body simulations (for late times, $z \sim \mathcal{O}(1)$) is negligible, but should be included when generating the matter power-spectrum at redshift $z \sim \mathcal{O}(50)$, which depends on the early evolution of the universe, from which the initial conditions for our N-body simulations are obtained.

Scalar field N-body simulations: coupled scalar-field cosmology: The equations

The dark-matter Lagrangian for a point particle with (bare) mass m_0 is

$$\mathcal{L}_{\rm CDM}(\mathbf{y}) = -\frac{m_0}{\sqrt{-g}}\delta(\mathbf{y}-\mathbf{x}_0)\sqrt{g_{ab}\dot{x}_0^a\dot{x}_0^b}, \tag{1}$$

where $\mathbf{y}$ is the general coordinate and $\mathbf{x}_0$ is the coordinate of the centre of the particle. From this equation we derive the corresponding energy-momentum tensor:

$$T^{ab}_{\rm CDM} = \frac{m_0}{\sqrt{-g}}\delta(\mathbf{y}-\mathbf{x}_0)\dot{x}_0^a\dot{x}_0^b. \tag{2}$$

Also, because $g_{ab}\dot{x}_0^a\dot{x}_0^b \equiv g_{ab}u^a u^b = 1$, where u^a is the four-velocity of the dark-matter particle centred at x_0, the Lagrangian can be rewritten as

$$\mathcal{L}_{\rm CDM}(\mathbf{y}) = -\frac{m_0}{\sqrt{-g}}\delta(\mathbf{y}-\mathbf{x}_0), \tag{3}$$

which will be used below. Eq. (2) is just the energy-momentum tensor for a single dark matter particle.

Scalar field N-body simulations: coupled scalar-field cosmology: The equations

For a fluid with many particles, the energy-momentum tensor will be

$$T^{ab}_{\mathrm{CDM}} = \frac{1}{\mathcal{V}}\int_{\mathcal{V}} d^4y\sqrt{-g}\frac{m_0}{\sqrt{-g}}\delta(y-x_0)\dot{x}^a_0\dot{x}^b_0$$
$$= \rho_{\mathrm{CDM}}u^a u^b, \quad (1)$$

in which $\mathcal{V}$ is a volume that is microscopically large but macroscopically small, and we have extended the 3-dimensional δ function to a 4-dimensional one by adding a time component. Here, u^a is the averaged four-velocity of the dark-matter fluid, which is *not* necessarily the same as the four-velocity of the observer.

Scalar field N-body simulations: coupled scalar-field cosmology: The equations

The energy-momentum tensor for the scalar field is

$$T^{\varphi ab} = \nabla^a\varphi\nabla^b\varphi - g^{ab}\left[\frac{1}{2}\nabla_c\varphi\nabla^c\varphi - V(\varphi)\right]. \quad (1)$$

Therefore the total energy-momentum tensor is

$$\boxed{\begin{aligned} T_{ab} \quad = \quad & \nabla_a\varphi\nabla_b\varphi - g_{ab}\left[\frac{1}{2}\nabla_c\varphi\nabla^c\varphi - V(\varphi)\right] \\ & +C(\varphi)T^{\mathrm{CDM}}_{ab} + T^{\mathrm{S}}_{ab} \end{aligned}} \quad (2)$$

where $T^{\mathrm{CDM}}_{ab} = \rho_{\mathrm{CDM}}u_a u_b$, T^{S}_{ab} is the energy-momentum tensor for all other matter species including baryons, and the Einstein equations are

$$\boxed{G_{ab} = \kappa T_{ab}} \quad (3)$$

where G_{ab} is the Einstein tensor.

Scalar field N-body simulations: coupled scalar-field cosmology: The equations

Note that because of the extra coupling between the scalar field, φ, and the dark matter, the energy-momentum tensors for either will not be separately conserved, and we have

$$\boxed{\nabla_b T^{\mathrm{CDM}ab} = -\frac{C_\varphi(\varphi)}{C(\varphi)}\left(g^{ab}\mathcal{L}_{\mathrm{CDM}} + T^{\mathrm{CDM}ab}\right)\nabla_b\varphi,} \qquad (1)$$

where throughout this presentation we shall use subscript $_\varphi$ to denote a derivative with respect to φ. However, the total energy-momentum tensor is conserved.

Finally, the scalar field equation of motion (EOM) is

$$\boxed{\Box\varphi + \frac{\partial V(\varphi)}{\partial\varphi} + \rho_{\mathrm{CDM}}\frac{\partial C(\varphi)}{\partial\varphi} = 0.} \qquad (2)$$

where $\Box \equiv \nabla^a\nabla_a$.

Scalar field N-body simulations: coupled scalar-field cosmology: The equations

We will consider an inverse power-law potential energy for the scalar field,

$$V(\varphi) = \frac{\Lambda^4}{(\sqrt{\kappa}\varphi)^\alpha}, \tag{1}$$

where α is a dimensionless constant and Λ is a constant with dimensions of mass. This potential has also been adopted in various background or linear perturbation studies of scalar fields (either minimally or non-minimally coupled); the tracking behaviour its produces makes it a good dark energy candidate and for that purpose we shall choose $\alpha \sim \mathcal{O}(0.1 - 1)$. Meanwhile, the coupling between the scalar field and dark matter particles is chosen as

$$C(\varphi) = \exp(\gamma\sqrt{\kappa}\varphi), \tag{2}$$

where $\gamma < 0$ is another dimensionless constant characterizing the strength of the coupling. As we shall see below, $2|\gamma|^2$ is roughly the ratio of the magnitudes of the fifth force and gravity on the dark matter particles.

The bare potential Eq. (1) and the coupling function Eq. (2) form an effective total potential

$$V_{eff}(\varphi) = V(\varphi) + \rho_{\mathrm{CDM}}C(\varphi) \tag{3}$$

for the scalar field φ. However, both $V(\varphi)$ and $C(\varphi)$ decrease as φ increases here, there is no finite global minimum for $V_{eff}(\varphi)$, and the scalar field will always continue rolling down the potential given appropriate initial condition. As a result the scalar field almost always resides around the minimum of $V_{eff}(\varphi)$, where it acquires a heavy mass to become a chameleon, this does not necessarily happen here. Instead, the rolling of the scalar field can be quite rapid, introducing interesting new dynamics in both background cosmology and perturbation evolution.

Scalar field N-body simulations: coupled scalar-field cosmology:

For the scalar field equation of motion, we denote by $\bar{\varphi}$ the background value of φ and $\delta\varphi \equiv \varphi - \bar{\varphi}$. Then scalar field Eq. can be rewritten as

$$\begin{aligned} \ddot{\delta\varphi} + 3H\dot{\delta\varphi} + \vec{\nabla}^2_{\mathbf{r}}\varphi + V_{,\varphi}(\varphi) - V_{,\varphi}(\bar{\varphi}) & \\ +\rho_{\mathrm{CDM}}C_{,\varphi}(\varphi) - \bar{\rho}_{\mathrm{CDM}}C_{,\varphi}(\bar{\varphi}) &= 0 \end{aligned}$$

by removing the background part. Here, $\vec{\nabla}_{\mathbf{r}a}$ is the covariant spatial derivative with respect to the physical coordinate $\mathbf{r} = a\mathbf{x}$, $\mathbf{x}$ is the comoving coordinate, and $\vec{\nabla}^2_{\mathbf{r}} = \vec{\nabla}_{\mathbf{r}a}\vec{\nabla}^a_{\mathbf{r}}$. $\vec{\nabla}_{\mathbf{r}a}$ is strictly speaking non-Euclidian as the spacetime is not completely flat, but because we are working in the weak field limit we approximate it as Euclidian, that is $\vec{\nabla}^2_{\mathbf{r}} \doteq -\left(\partial^2_{r_x} + \partial^2_{r_y} + \partial^2_{r_z}\right)$; the minus sign is because our metric convention is $(+,-,-,-)$.

In our simulations we also work in the quasi-static limit, and assume that the spatial gradients are much larger than the time derivatives, $|\vec{\nabla}_{\mathbf{r}}\varphi| \gg |\frac{\partial\delta\varphi}{\partial t}|$. Therefore, the above equation is simplified to

$$\begin{aligned} & c^2\partial^2_{\mathbf{x}}(a\delta\varphi) \\ = \quad & a^3\left[V_{,\varphi}(\varphi) - V_{,\varphi}(\bar{\varphi}) + \rho_{\mathrm{CDM}}C_{,\varphi}(\varphi) - \bar{\rho}_{\mathrm{CDM}}C_{,\varphi}(\bar{\varphi})\right], \end{aligned} \qquad (1)$$

in which $\partial^2_{\mathbf{x}} = -\vec{\nabla}^2_{\mathbf{x}} = +\left(\partial^2_x + \partial^2_y + \partial^2_z\right)$ is with respect to $\mathbf{x}$, with $\vec{\nabla}_{\mathbf{x}} = a\vec{\nabla}_{\mathbf{r}}$, and we have restored the factor c^2 in front of $\vec{\nabla}^2_{\mathbf{x}}$ (the φ here and in the remaining of this paper is c^{-2} times the φ in the original Lagrangian unless otherwise stated). Note that here V and ρ_{CDM} both have the dimensions of *mass* density rather than *energy* density.

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: The equations:: Poisson (modified)

Next consider the Poisson equation, which is obtained from the Einstein equation in the weak-field and slow-motion limits. Here the metric can be written as

$$ds^2 = (1 + 2\phi)dt^2 - (1 - 2\psi)\delta_{ij}dr^i dr^j \tag{1}$$

from which we find that the time-time component of the Ricci curvature tensor $R^0{}_0 = -\vec{\nabla}^2_{\mathbf{r}}\phi$. The Einstein equation gives

$$\boxed{R^0{}_0 = -\vec{\nabla}^2_{\mathbf{r}}\phi = \frac{\kappa}{2}(\rho_{\mathrm{TOT}} + 3p_{\mathrm{TOT}})} \tag{2}$$

where ρ_{TOT} and p_{TOT} are the total energy density and pressure, respectively. The quantity $\vec{\nabla}^2_{\mathbf{r}}\phi$ can be expressed in terms of the comoving coordinate $\mathbf{x}$ as

$$\begin{aligned}\vec{\nabla}^2_{\mathbf{r}}\phi &= \frac{1}{a^2}\vec{\nabla}^2_{\mathbf{x}}\left(\frac{\Phi}{a} - \frac{1}{2}a\ddot{a}\mathbf{x}^2\right) \\ &= \frac{1}{a^3}\vec{\nabla}^2_{\mathbf{x}}\Phi - 3\frac{\ddot{a}}{a}\end{aligned} \tag{3}$$

where we have defined a new Newtonian potential

$$\Phi \equiv a\phi + \frac{1}{2}a^2\ddot{a}\mathbf{x}^2 \tag{4}$$

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: The equations:: Poisson (modified)

Thus,

$$\begin{aligned}\vec{\nabla}^2_{\mathbf{x}}\Phi &= a^3\left(\vec{\nabla}^2_{\mathbf{r}}\phi + 3\frac{\ddot{a}}{a}\right) \qquad (1)\\ &= -a^3\left[\frac{\kappa}{2}(\rho_{\mathrm{TOT}} + 3p_{\mathrm{TOT}}) - \frac{\kappa}{2}(\bar{\rho}_{\mathrm{TOT}} + 3\bar{p}_{\mathrm{TOT}})\right]\end{aligned}$$

where in the second step we have used Eq. (2) (last slide) and the Raychaudhuri equation, and an overbar labels the background value of a quantity. Because the energy-momentum tensor for the scalar field is given by Eq. (1) (slides back, energy-momentum tenor for the scalar field), it is easy to show that $\rho^{\varphi} + 3p^{\varphi} = 2\left[\dot{\varphi}^2 - V(\varphi)\right]$ and so

$$\begin{aligned}\vec{\nabla}^2_{\mathbf{x}}\Phi &= -4\pi G a^3\left\{\rho_{\mathrm{CDM}}C(\varphi) + \rho_{\mathrm{B}} + 2\left[\dot{\varphi}^2 - V(\varphi)\right]\right\}\\ &\quad +4\pi G a^3\left\{\bar{\rho}_{\mathrm{CDM}}C(\bar{\varphi}) + \bar{\rho}_{\mathrm{B}} + 2\left[\dot{\bar{\varphi}}^2 - V(\bar{\varphi})\right]\right\}.\end{aligned}$$

In this equation $\dot{\varphi}^2 - \dot{\bar{\varphi}}^2 = 2\dot{\varphi}\delta\dot{\varphi} + \delta\dot{\varphi}^2 \ll (\vec{\nabla}_{\mathbf{r}}\varphi)^2$ in the quasi-static limit, and so could be dropped safely. Therefore, we have

$$\begin{aligned}\partial^2_{\mathbf{x}}\Phi &= 4\pi G a^3\left[\rho_{\mathrm{CDM}}C(\varphi) - \bar{\rho}_{\mathrm{CDM}}C(\bar{\varphi})\right]\\ &\quad +4\pi G a^3\left[\rho_{\mathrm{B}} - \bar{\rho}_{\mathrm{B}}\right] - 8\pi G a^3\left[V(\varphi) - V(\bar{\varphi})\right]. \qquad (2)\end{aligned}$$

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: The equation of motion (geodesics)

Finally, for the equations of motion of the dark matter particles, consider the energy-momentum dark-matter and scalar field combined that can be reduced to

$$\ddot{x}_0^a + \Gamma^a_{bc}\dot{x}_0^b\dot{x}_0^c \quad = \quad \left(g^{ab} - u^a u^b\right)\frac{C_\varphi(\varphi)}{C(\varphi)}\nabla_b\varphi. \tag{1}$$

In this equation, the left-hand side is the conventional geodesic equation of general relativity, and the right-hand side is the new fifth force contribution from the coupling to the scalar field.

We will always use the $\hat{\nabla}_a\varphi$ measured in the fundamental observer's frame (in which the density perturbation is obtained more directly), and so the equation above should be changed to

$$\ddot{x}_0^a + \Gamma^a_{bc}\dot{x}_0^b\dot{x}_0^c \quad = \quad \frac{C_\varphi(\varphi)}{C(\varphi)}\left(\hat{\nabla}^a\varphi - \dot{\varphi}v^a\right). \tag{2}$$

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: The equation of motion (geodesics)

In the non-relativistic limit, the spatial components of Eq. (2) (last slide) can be written as

$$\frac{d^2\mathbf{r}}{dt^2} = -\vec{\nabla}_{\mathbf{r}}\phi - \frac{C_\varphi(\varphi)}{C(\varphi)}\vec{\nabla}_{\mathbf{r}}\varphi - \frac{C_\varphi(\varphi)}{C(\varphi)}a\frac{d(\mathbf{r}/a)}{dt}\dot{\varphi} \tag{1}$$

where t is the physical time coordinate. If we use the comoving coordinate $\mathbf{x}$ instead, then this becomes

$$\ddot{\mathbf{x}} + 2\frac{\dot{a}}{a}\dot{\mathbf{x}} = -\frac{1}{a^3}\vec{\nabla}_{\mathbf{x}}\Phi - \frac{C_\varphi(\varphi)}{C(\varphi)}\left(\frac{1}{a^2}\vec{\nabla}_{\mathbf{x}}\varphi + \dot{\varphi}\dot{\mathbf{x}}\right) \tag{2}$$

where we have used

$$\Phi \equiv a\phi + \frac{1}{2}a^2\ddot{a}\mathbf{x}^2 \tag{3}$$

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: The equation of motion (geodesics)

The canonical momentum conjugate to $\mathbf{x}$ is $\mathbf{p} = a^2\dot{\mathbf{x}}$ so from the equation above we have

$$\frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{a^2}, \tag{1}$$

$$\frac{d\mathbf{p}_{\rm CDM}}{dt} = -\frac{1}{a}\vec{\nabla}_{\mathbf{x}}\Phi - \frac{C_\varphi(\varphi)}{C(\varphi)}\left(\vec{\nabla}_{\mathbf{x}}\varphi + a^2\dot{\varphi}\dot{\mathbf{x}}\right)$$

$$= -\frac{1}{a}\vec{\nabla}_{\mathbf{x}}\Phi - \frac{C_\varphi(\varphi)}{C(\varphi)}\left(\vec{\nabla}_{\mathbf{x}}\varphi + \dot{\varphi}\mathbf{p}_{\rm CDM}\right), \tag{2}$$

$$\frac{d\mathbf{p}_{\rm B}}{dt} = -\frac{1}{a}\vec{\nabla}_{\mathbf{x}}\Phi, \tag{3}$$

where Eq. (2) is for CDM particles and Eq. (3) is for baryons. Note that according to Eq. (2) the quantity $a\log[C(\varphi)]$ acts as an effective potential for the fifth force. This is an important observation and we will return to it later when we calculate the escape velocity of CDM particles within a virialized halo.

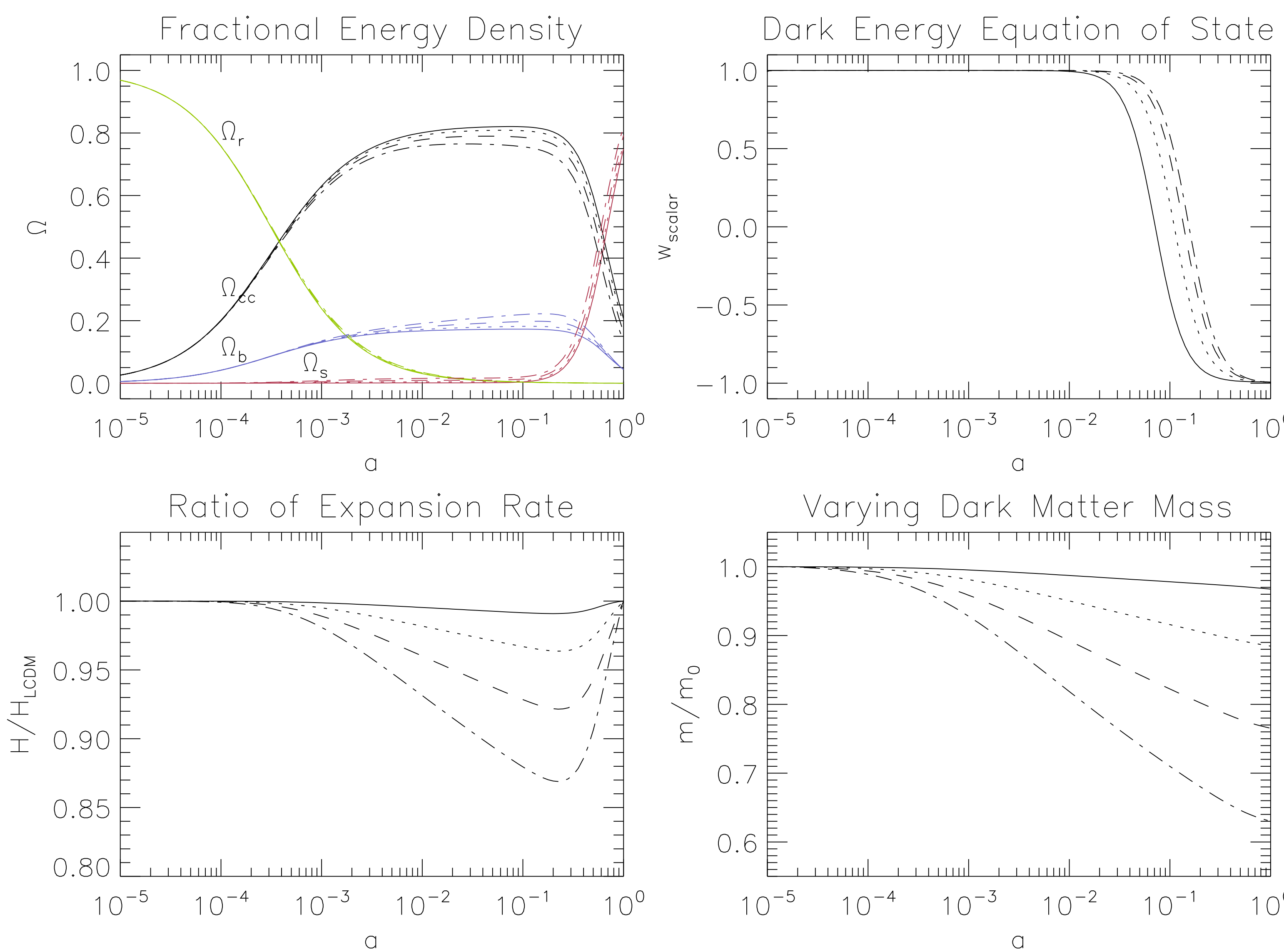
- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: Code and simulation details

- Used a Multi-Level Adaptive Particle Mesh code.
- Domain grid 128^3. Number of particles: 256^3.
- DE: 0.743; DM+B: 0.257; H0=71.9 km/s/Mpc; ns=0.963; sigma8=0.761.
- Size of the box: 64 Mpc/h.
- Simulations consider 4 parameters model with alpha=0.1 and gamma=-0.05, -0.10, -0.15, -0.20. These parameters values have been chosen in order to the results are no too different from the LCDM cosmology.

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: Background evolution



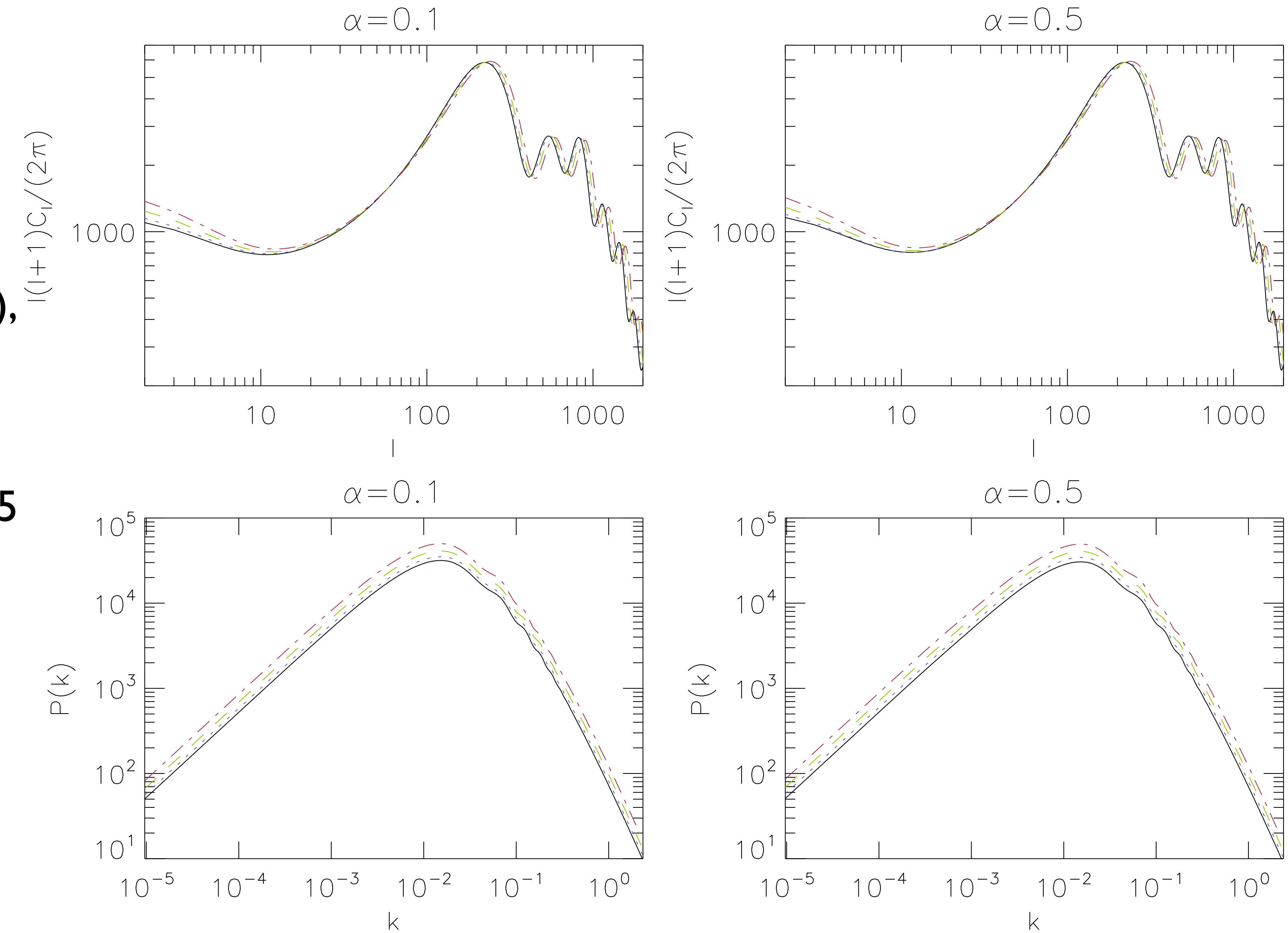
- ...

$$w \equiv p_\varphi / \rho_\varphi \text{ where } p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$m = m_0 \exp(\gamma\varphi)$$

- Barrow & Li (2011)

Scalar field N-body simulations: coupled scalar-field cosmology: Perturbaciones



- In all figures: solid (black), dotted (blue), dashed (green) and dot-dashed (red) correspond to: gamma=-0.05, -0.10, -0.15 and -0.20 respectively.

- Barrow & Li (2011)

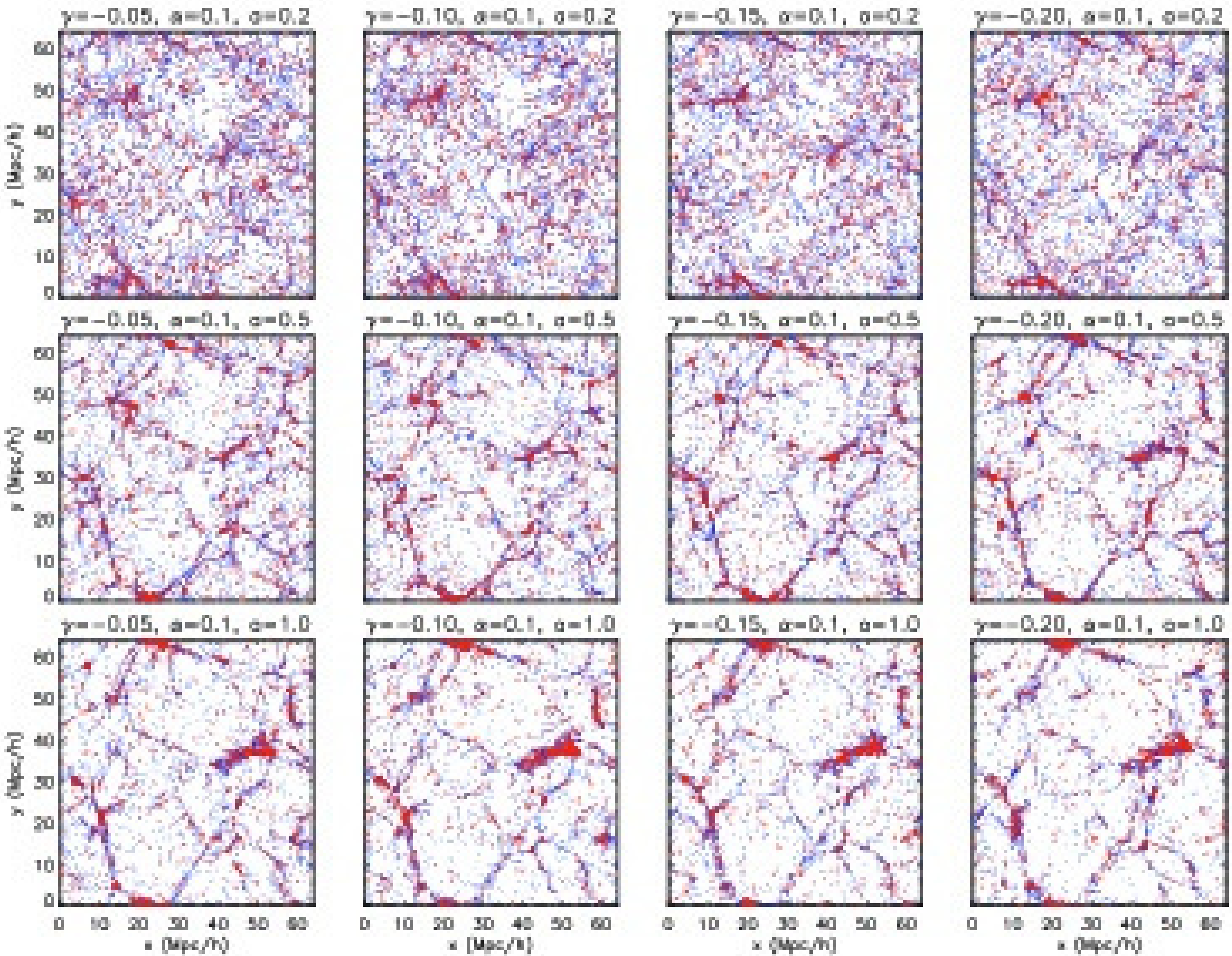
Scalar field N-body simulations: coupled scalar-field cosmology :: Initial conditions

- Scalar field dark matter coupling begins to take effect at rather high redshift.
- Then, initial condition for the N-body simulations of the coupled scalar field models (z=50) will be different from LCDM IC.
- One way to proceed is to use modified GRAFIC2, CAMB or CLASS to produce a linear matter power spectra at z=50 and utilise them to produce gaussian random density fluctuation field and displace the particles. In GRAFIC2 a spectrum is generated at time z=0 and then evolved back to z=50, and use to displace the particles.
- The other way is to use same initial condition in CAMB at z=10^6 for all models including the LCDM and evaluate the power spectrum at z=50. Results here use this approach.

- Barrow & Li (2011)

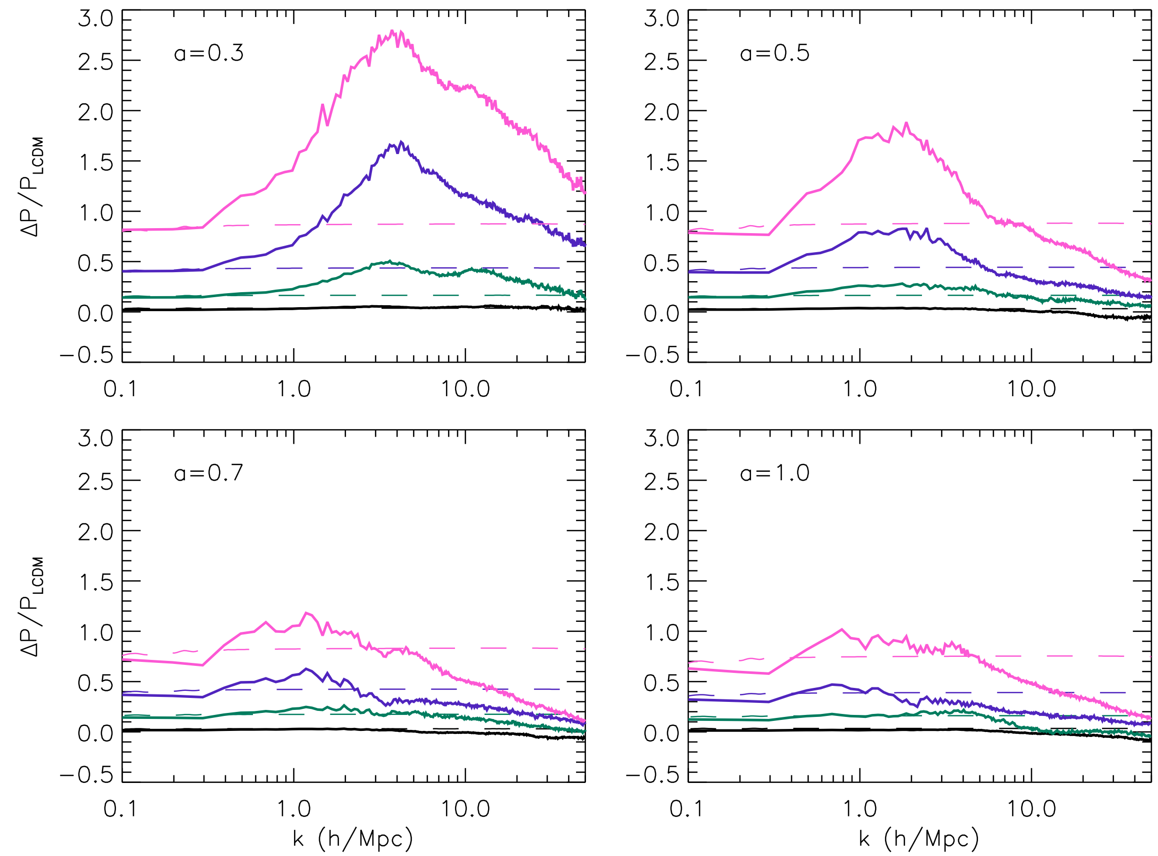
Scalar field N-body simulations: coupled scalar-field cosmology :: LSSF

- …

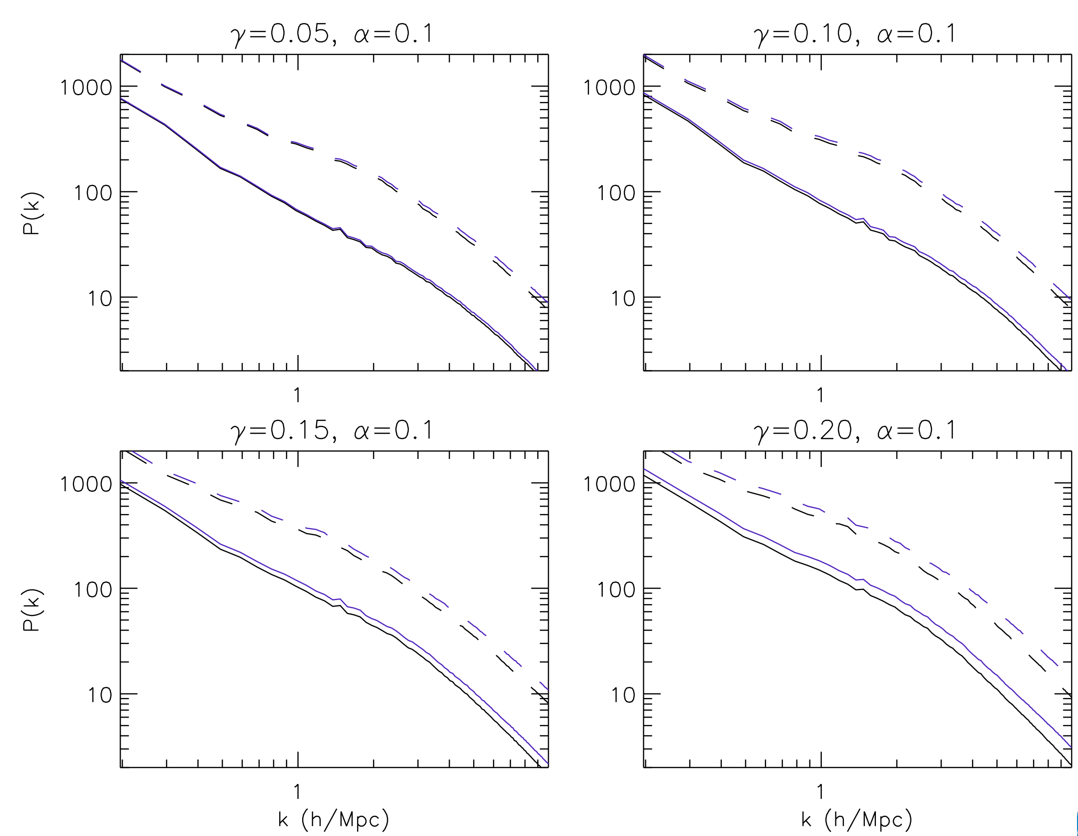


- Barrow

Scalar field N-body simulations: coupled scalar-field cosmology :: LSSF



- alpha=0.1 and gamma= -0.05 (black); -0.10 (green); -0.15 (blue); -0.20 (purple).



- Solid curves are output time a=0.5; and dashed for a=1.0. Lower curves are for baryons and upper ones for DM.

- Barrow & Li (2011)

Scalar-tensor theory N-body simulations: Newtonian limit

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi G} R \longrightarrow \Phi_N = -G\frac{m}{r}$$

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \phi R \longrightarrow \Phi_N = -G\frac{m}{r}(1 + \alpha e^{-r/\lambda})$$

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \left[\phi R + \frac{\omega(\phi)}{\phi} (\partial\phi)^2 - V(\phi) \right] + \mathcal{L}_M(g_{\mu\nu})$$

Ecuaciones de Vlasov-Poisson-Helmoltz para *f*(x,p,*t*)

- Vlasov: $$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{ma^2} \cdot \frac{\partial f}{\partial \mathbf{x}} - m \nabla \Phi_N(\mathbf{x}) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$
- con: $$\Phi_N = \Psi - \frac{1}{2} \frac{G_N}{1+\alpha} \bar{\phi}$$
- Poisson: $$\nabla^2 \Psi(\mathbf{x}) = 4\pi G_N \, a^2 [\rho(\mathbf{x}) - \rho_b]$$
- Helmholtz: $$\nabla^2 \bar{\phi}(\mathbf{x}) - \frac{1}{\lambda^2} \bar{\phi}(\mathbf{x}) = -8\pi\alpha \, a^2 [\rho(\mathbf{x}) - \rho_b]$$

Método de *N*-cuerpos

- Distribución discreta de partículas:

$$f(\mathbf{x},\mathbf{p},t)=\sum_{i=1}^{N}\delta(\mathbf{x}-\mathbf{x}_i)\delta(\mathbf{p}-\mathbf{p}_i)$$

- Ec. de movimiento:

$$\ddot{\mathbf{x}}_i+2\frac{\dot{a}}{a}\mathbf{x}_i=-\frac{1}{a^3}\frac{G_N}{1+\alpha}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{m_j(\mathbf{x}_i-\mathbf{x}_j)}{|\mathbf{x}_i-\mathbf{x}_j|^3}F_{SF}(|\mathbf{x}_i-\mathbf{x}_j|,\alpha,\lambda)$$

- donde: $$F_{SF}(r,\alpha,\lambda)=1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}$$

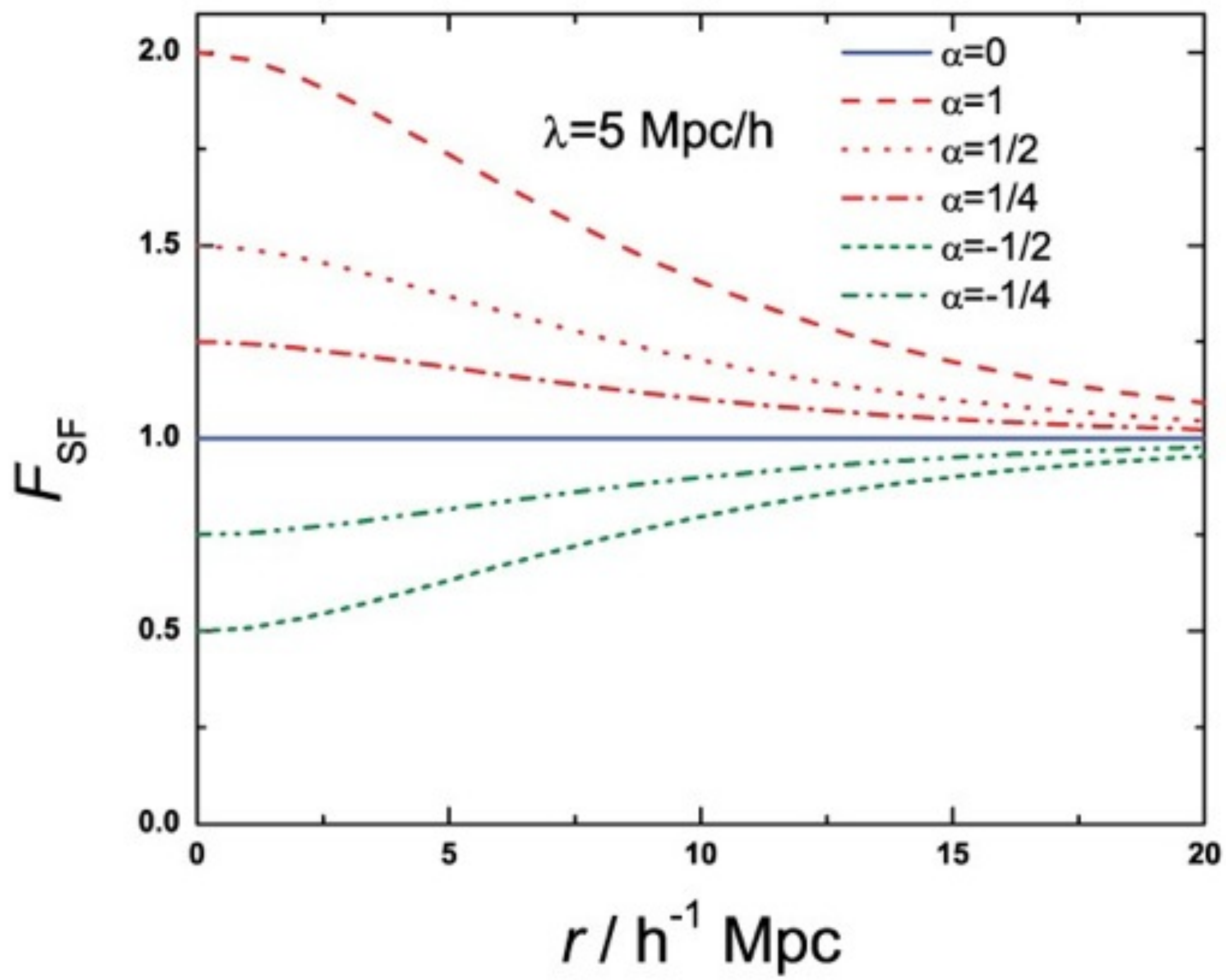
- Límites:

$$F_{SF}(r\ll\lambda,\alpha,\lambda)\approx 1+\alpha\left(1+\frac{r}{\lambda}\right)$$

$$F_{SF}(r\gg\lambda,\alpha,\lambda)\approx 1$$

La modificación ...

$$F_{SF}(r, \alpha, \lambda) = 1 + \alpha \left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda}$$

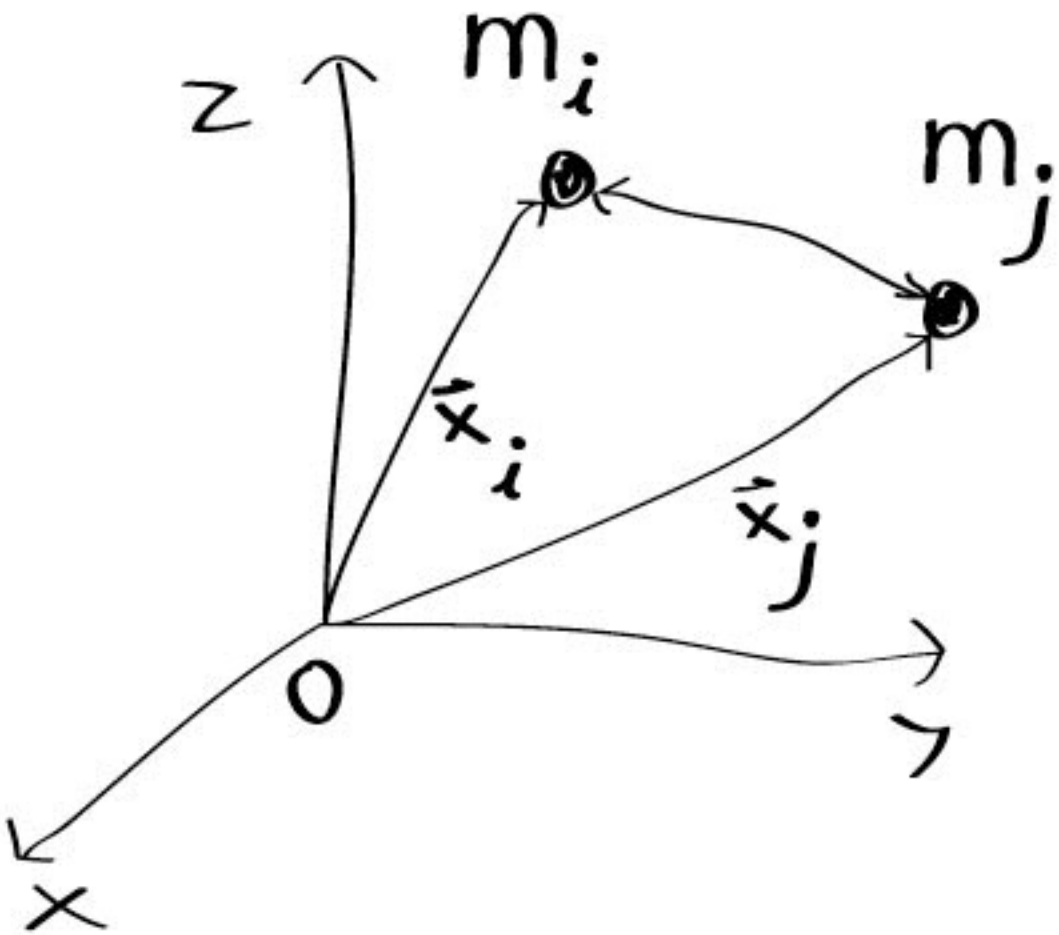


Ecuaciones de movimiento ...

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\mathbf{x}_i = -\frac{1}{a^3}\frac{G_N}{1+\alpha}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{m_j(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3}F_{SF}(|\mathbf{x}_i - \mathbf{x}_j|, \alpha, \lambda)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right)$$



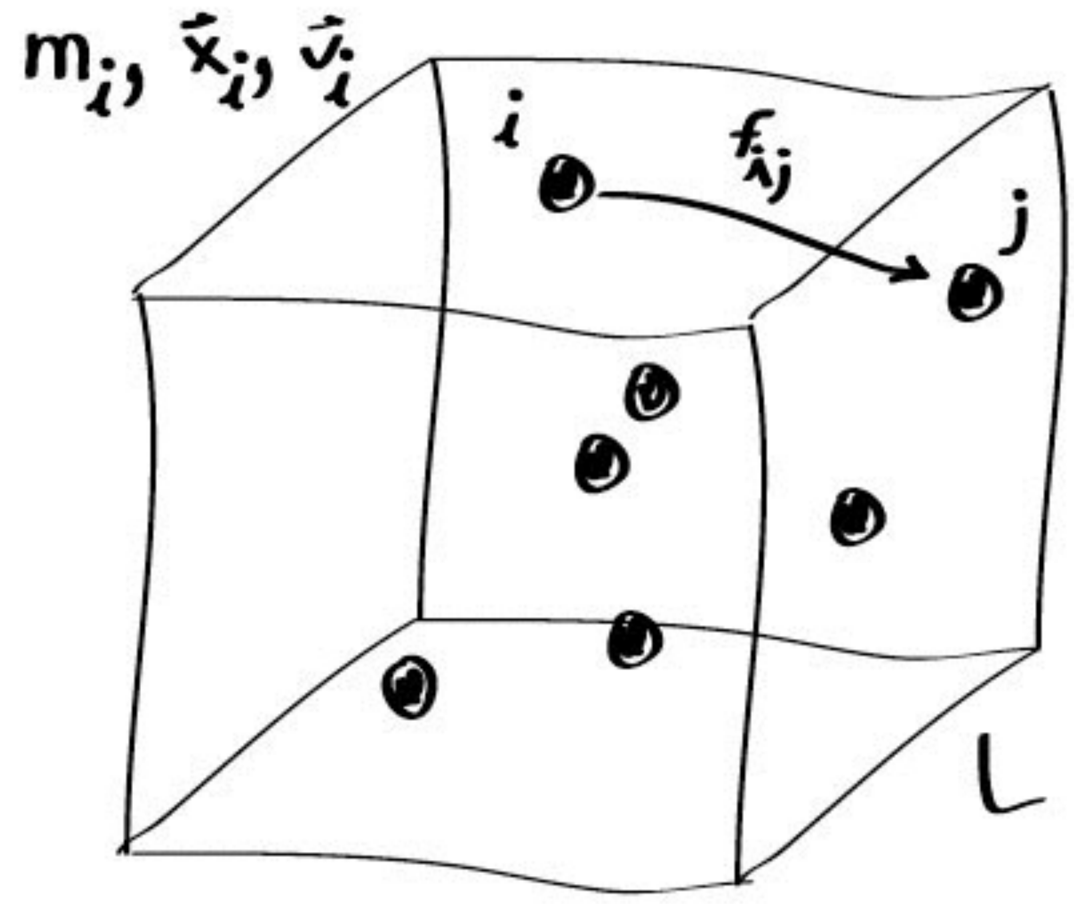
Simulaciones

- A un redshift inicial de *z* = 50 escogemos la siguiente cosmología:

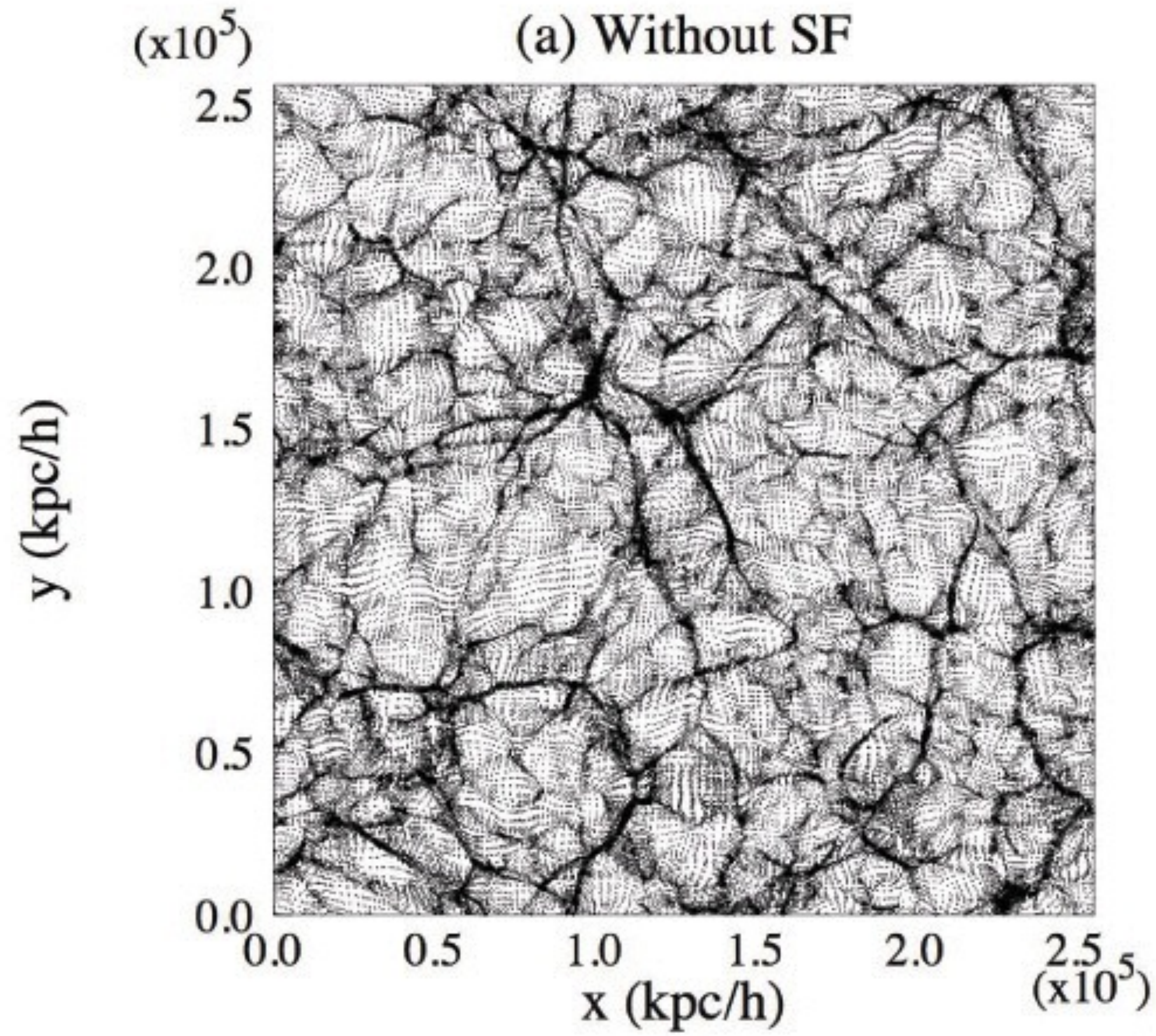
$$\Omega_{DM} = 0.314,\ \Omega_\Lambda = 0.686,\ H_0 = 71\ \mathrm{km/s/Mpc},\ \sigma_8 = 0.84,\ n = 0.99$$

- Una caja de simulación de lado *L* = 250 Mpc/*h*.
- La caja contiene 16,777,216 distribuidas en una malla uniforme, a las cuales se les ha desplazado ligeramente y con velocidades gaussianas, de acuerdo a la aproximación de Zeldovich.
- La masa de las partículas es: $1 \times 10^{10} M_\odot$

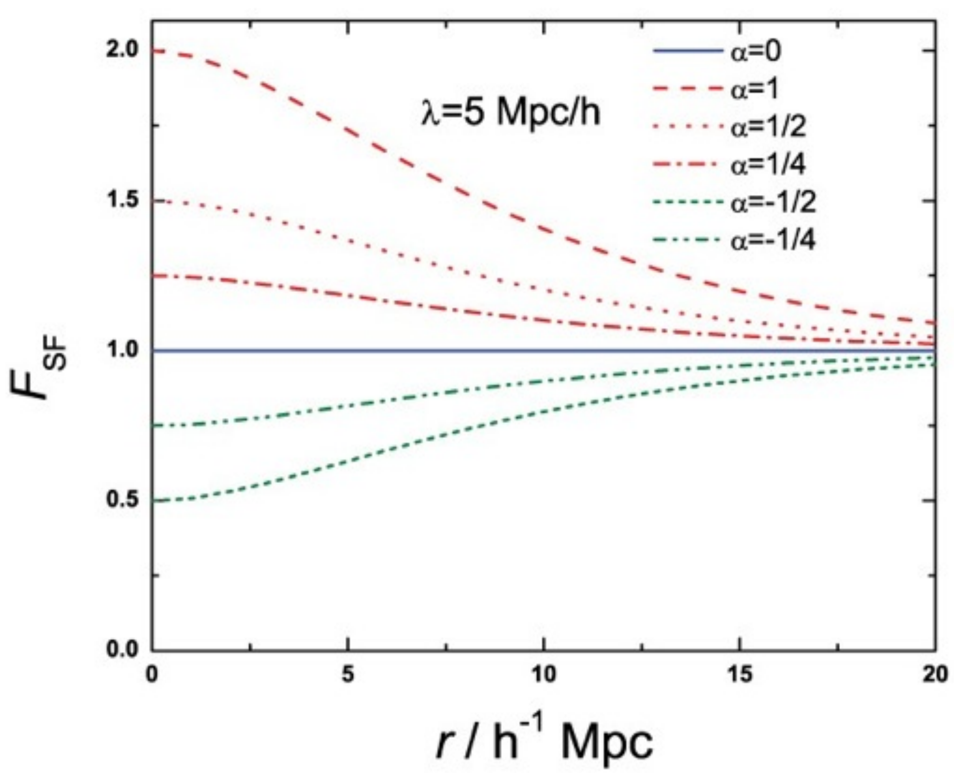
$$\mathbf{f}_{ij} = -\frac{G_N}{1+\alpha}\frac{m_j(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3}$$
$$\times F_{SF}(|\mathbf{x}_i - \mathbf{x}_j|, \alpha, \lambda$$

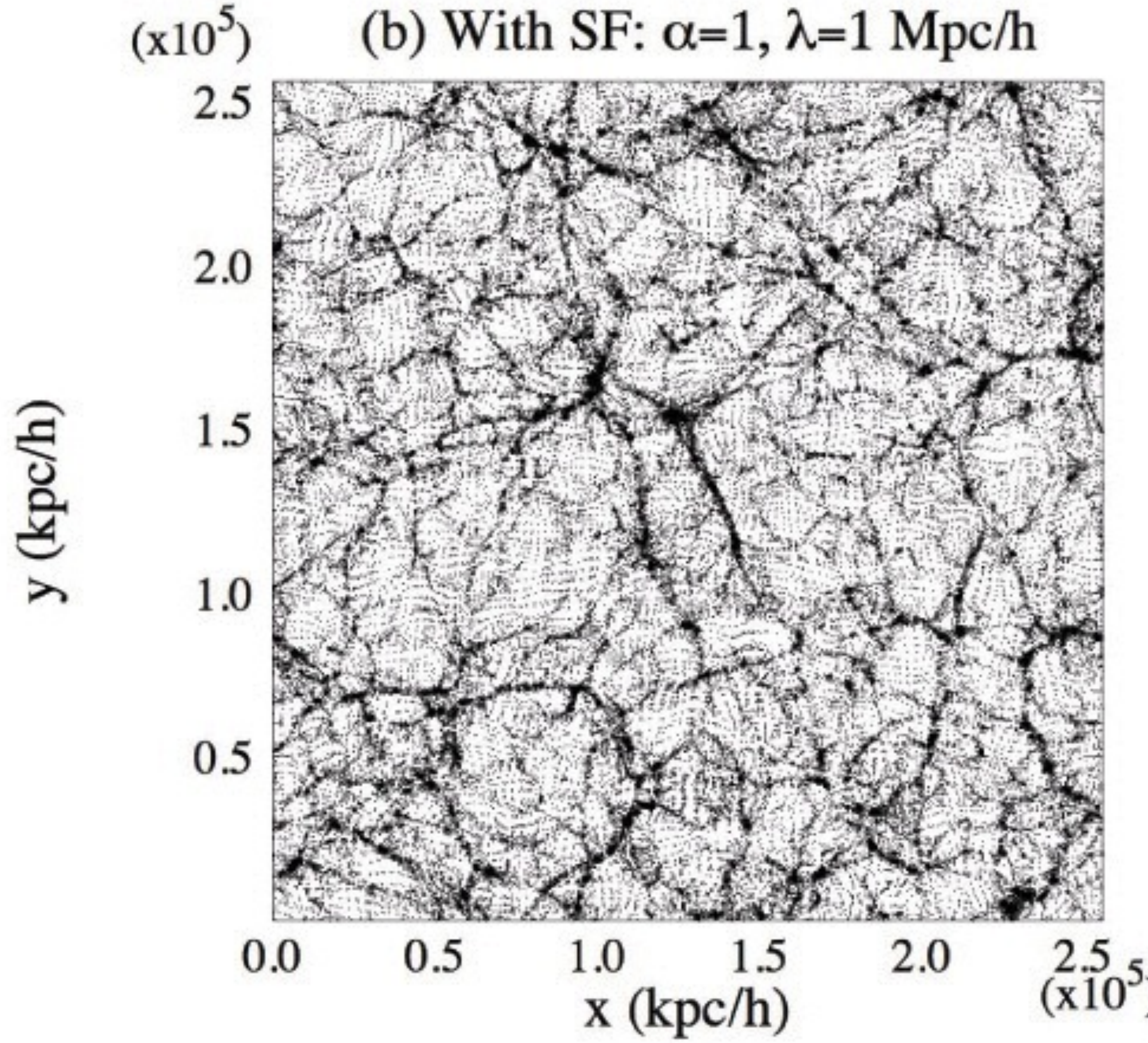


Formación de estructura en el Universo

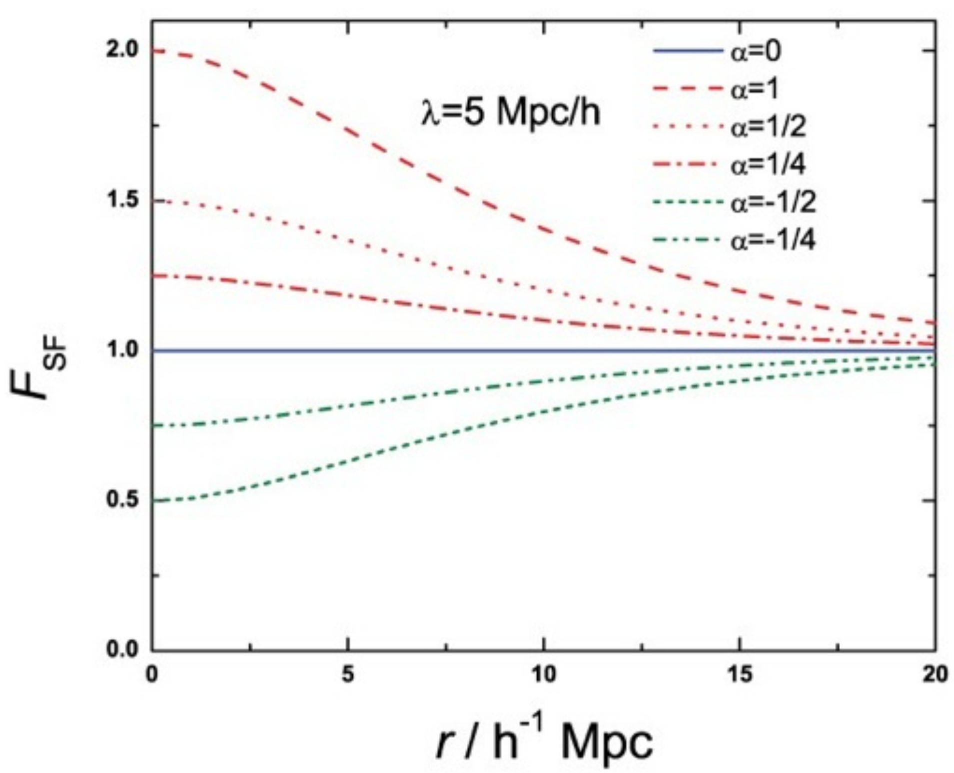


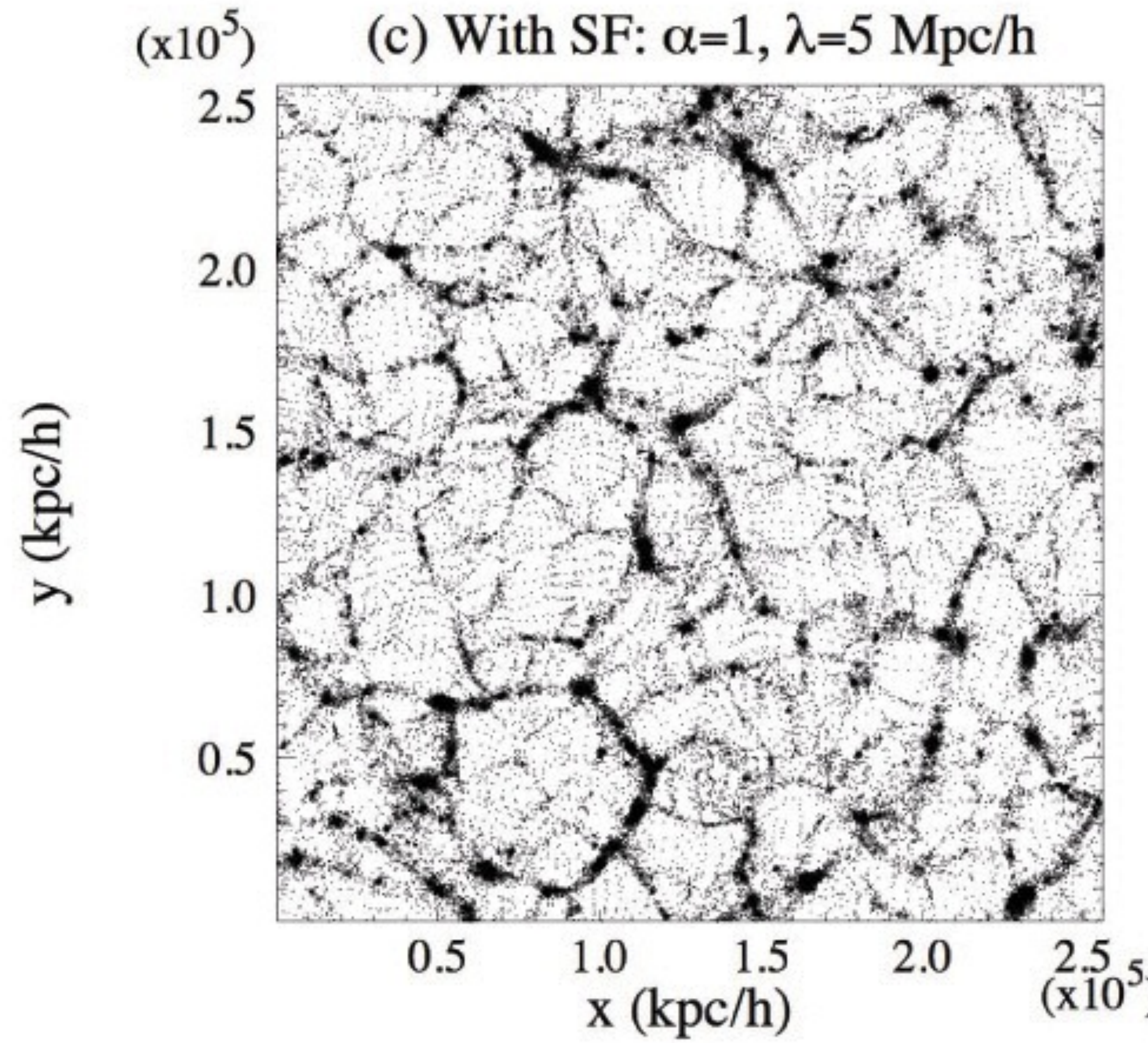
Formación de estructura en el Universo



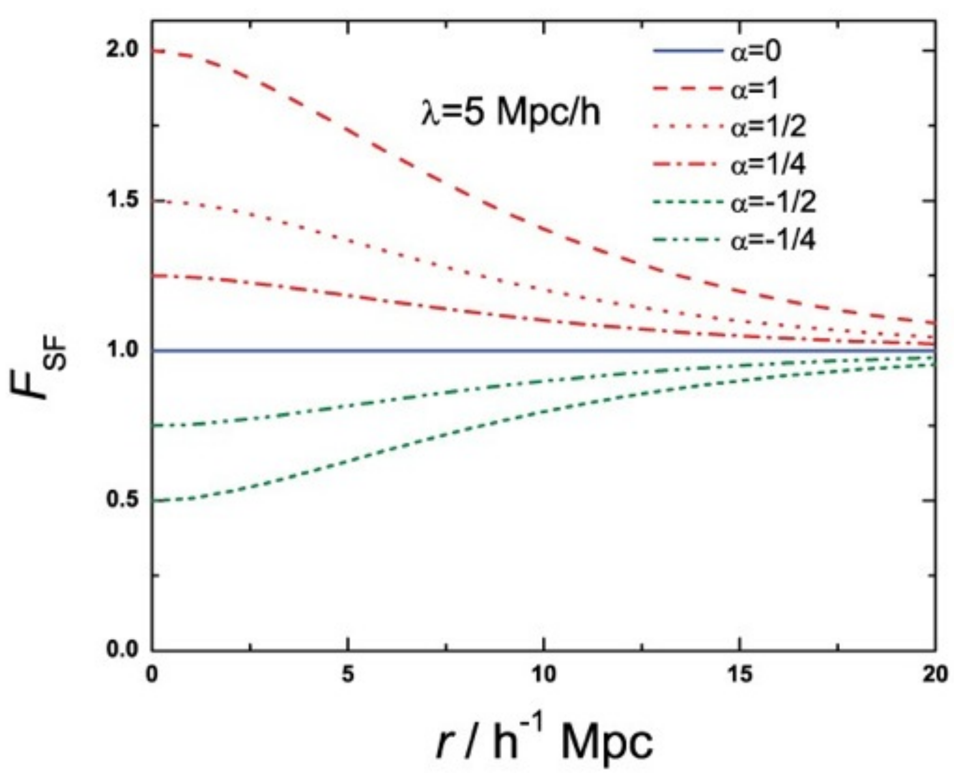


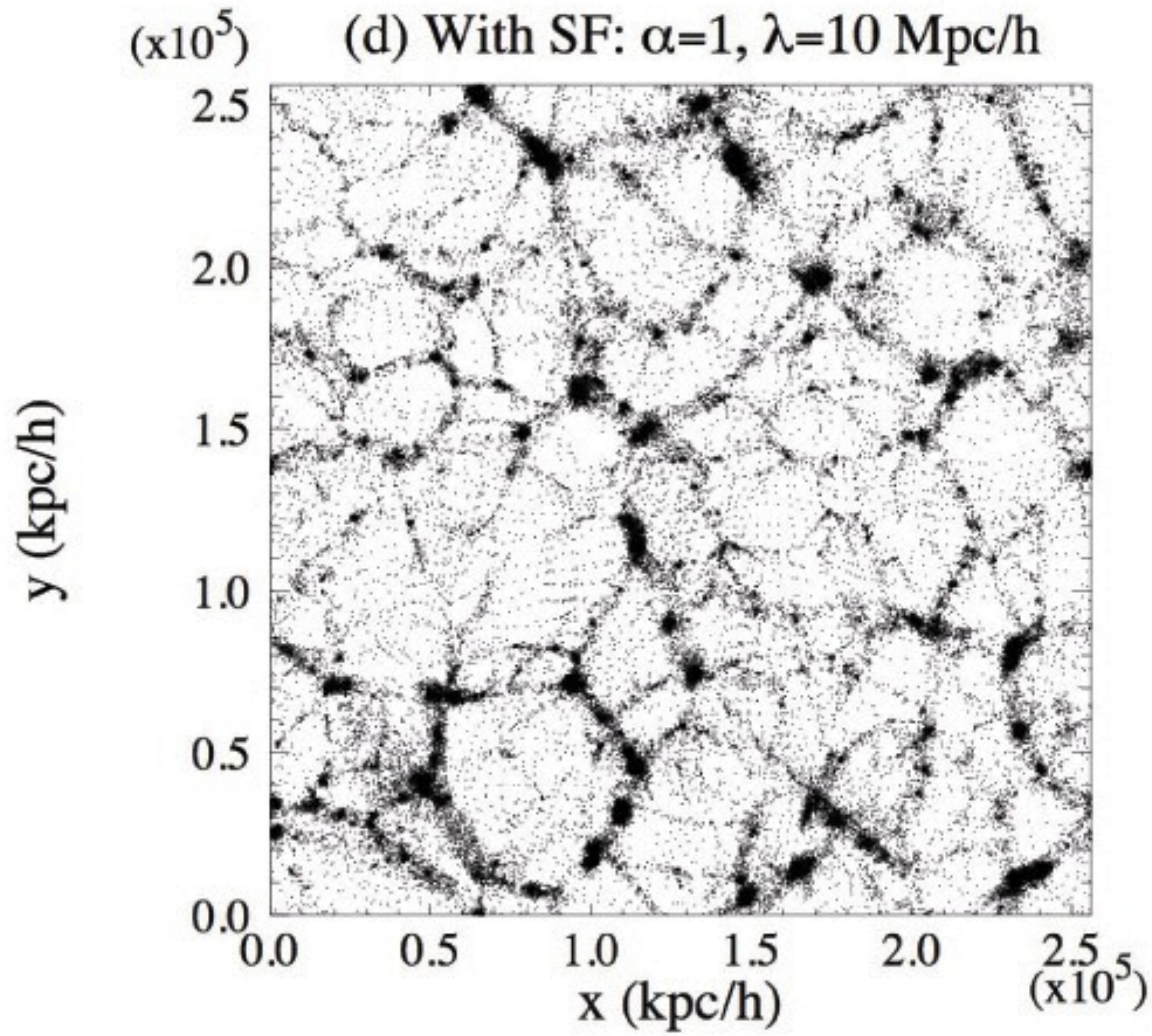
Formación de estructura en el Universo



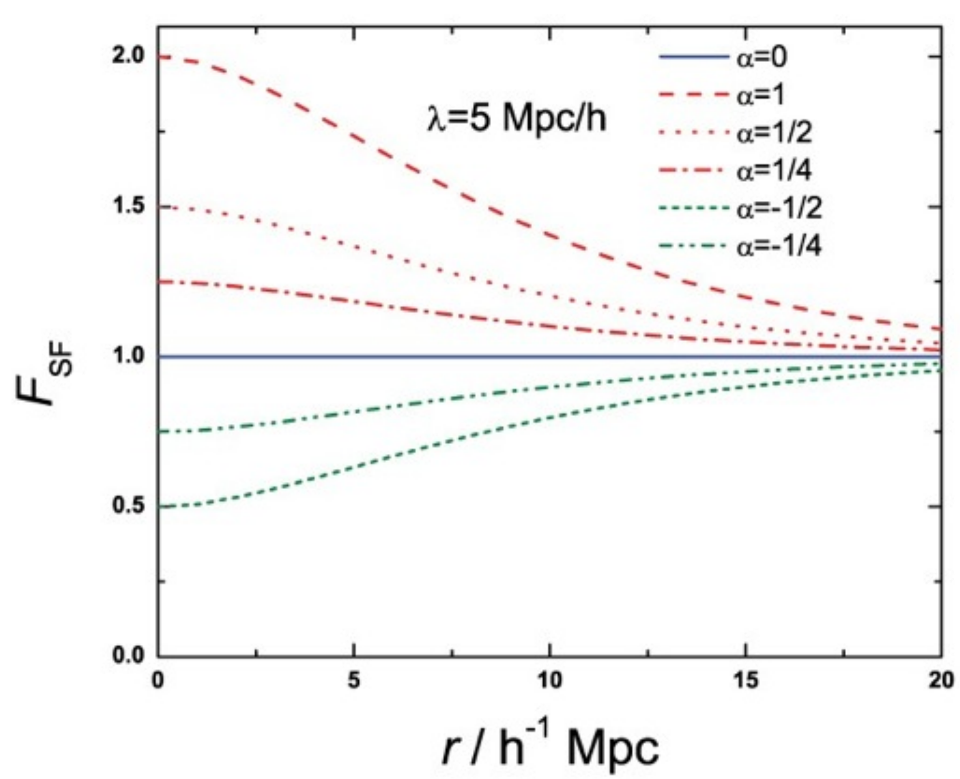


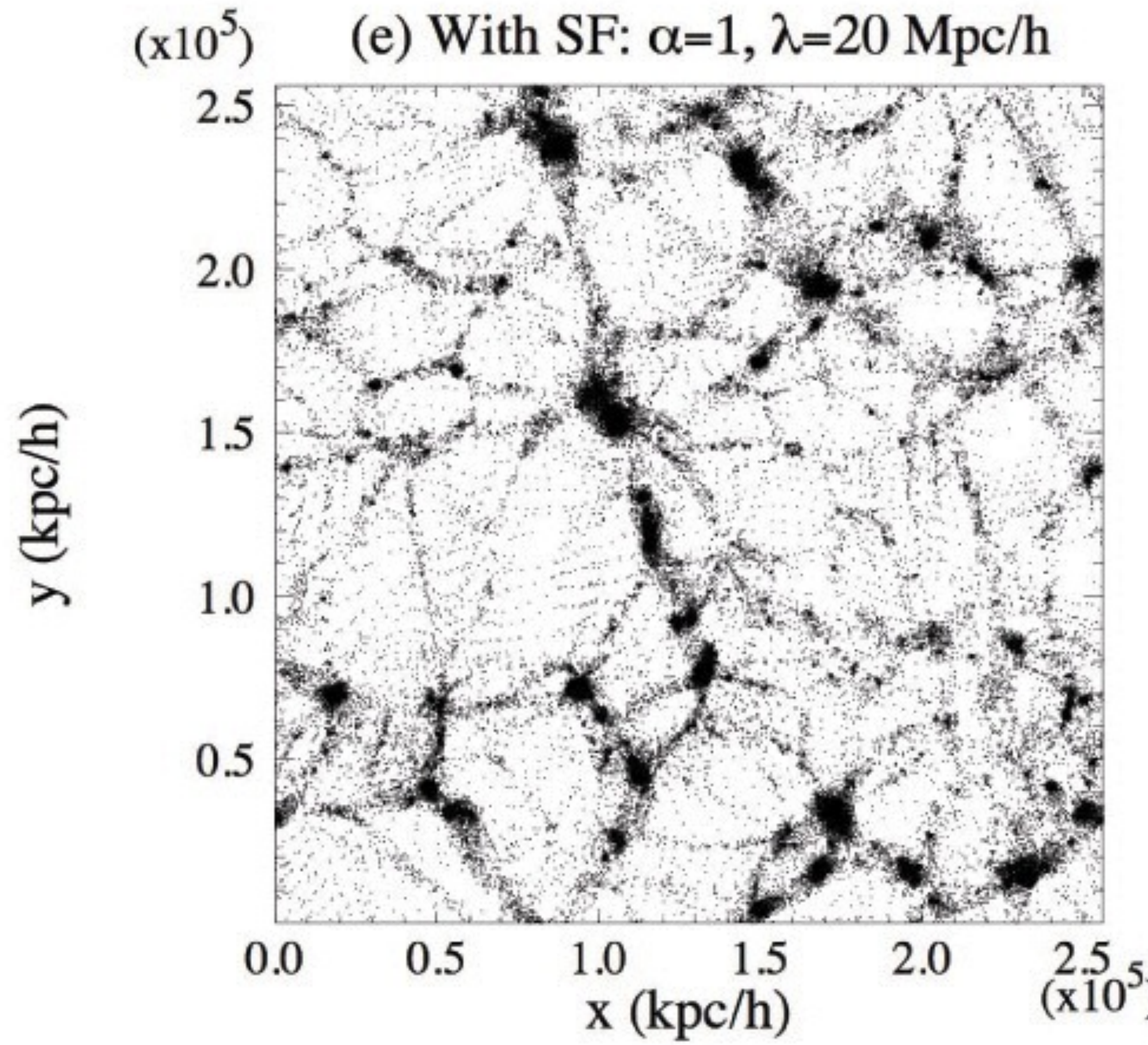
Formación de estructura en el Universo



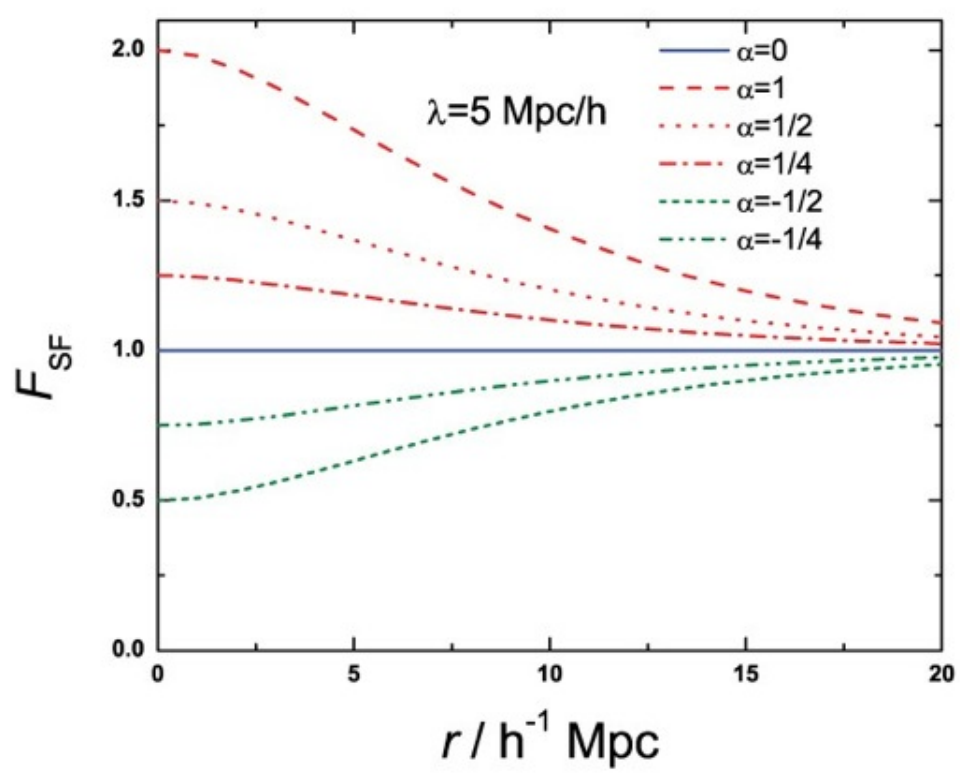


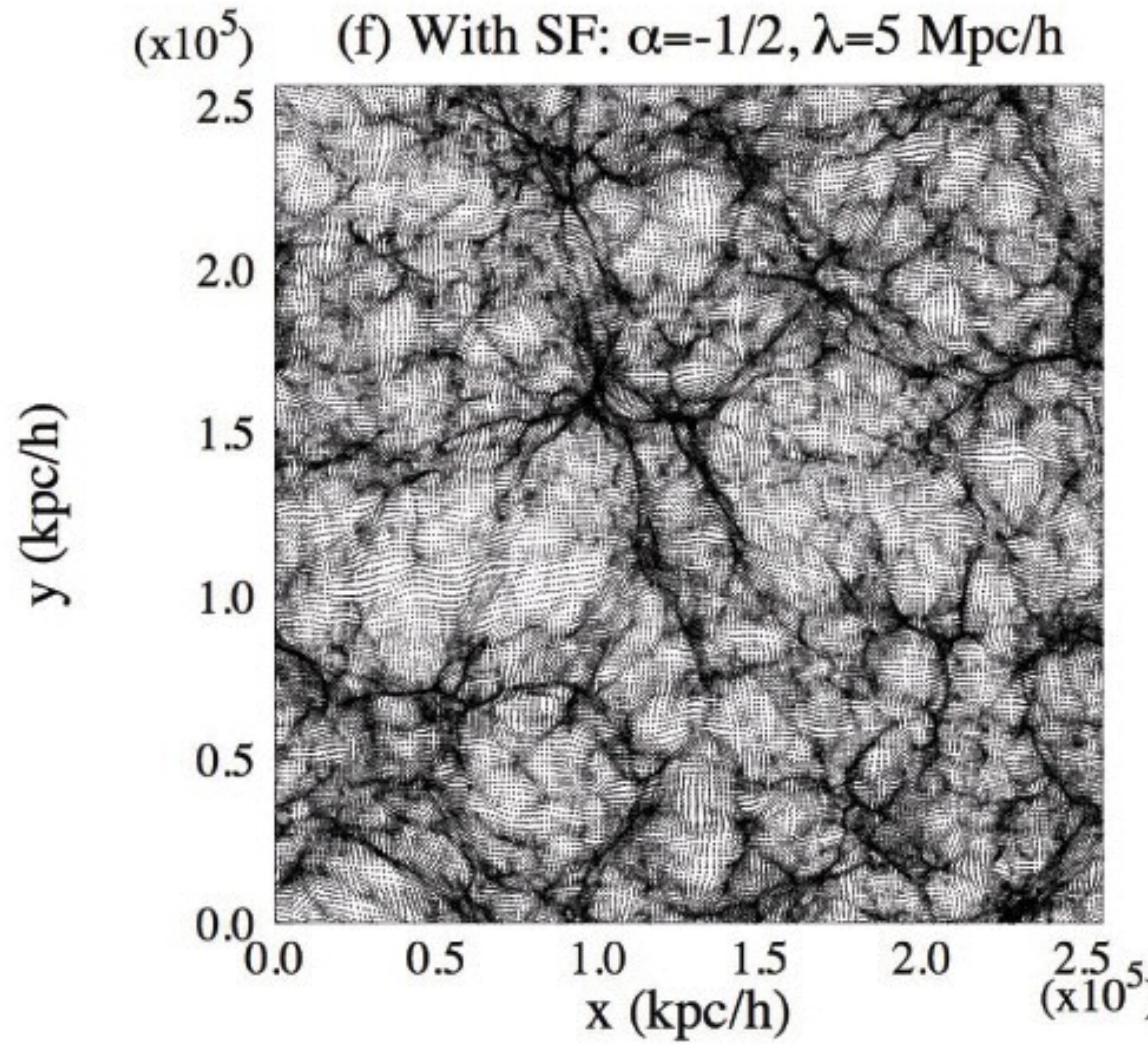
Formación de estructura en el Universo



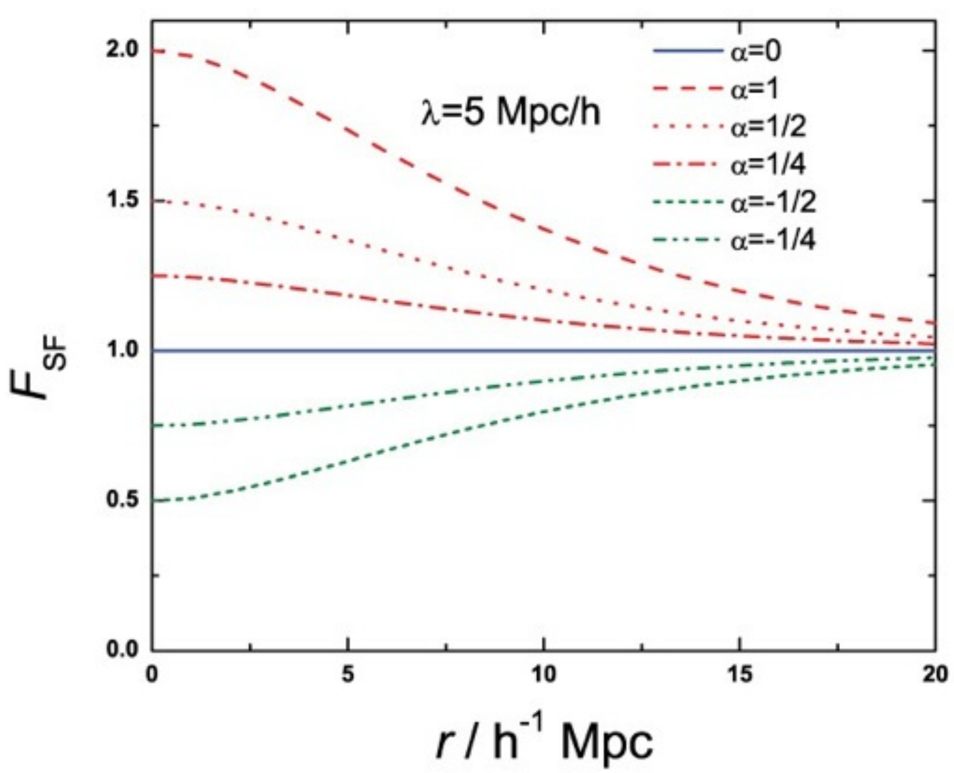


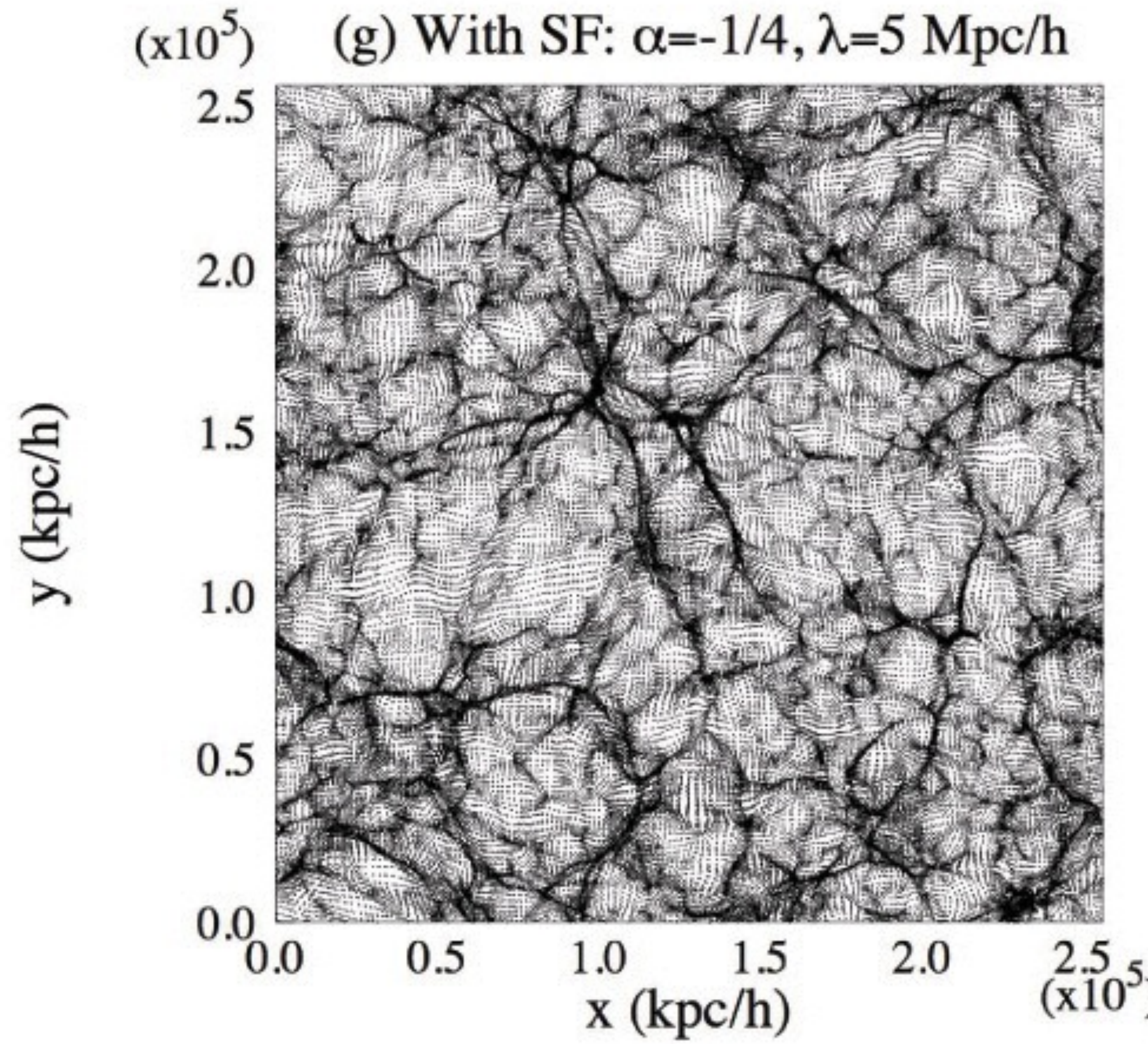
Formación de estructura en el Universo





Formación de estructura en el Universo





Formación de estructura: función de correlación y espectro de potencias

- Para estudiar la formación de estructura en el Universo calculamos la sobre-densidad:

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x})}{\rho_0} - 1$$

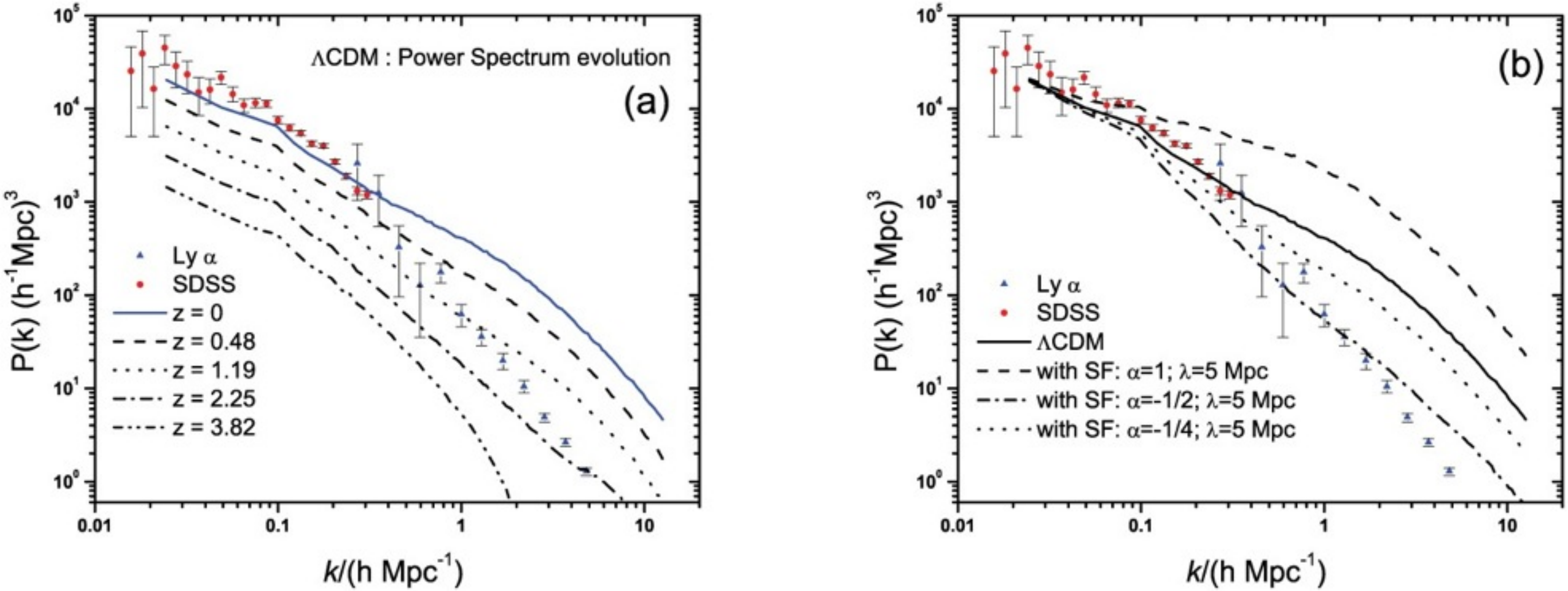
- La función de correlación es:

$$\xi(\mathbf{x}) \equiv \langle \delta(\mathbf{x}')\delta(\mathbf{x}' + \mathbf{x}) \rangle$$

- El espectro de potencias es la transformada de Fourier de la función de correlación:

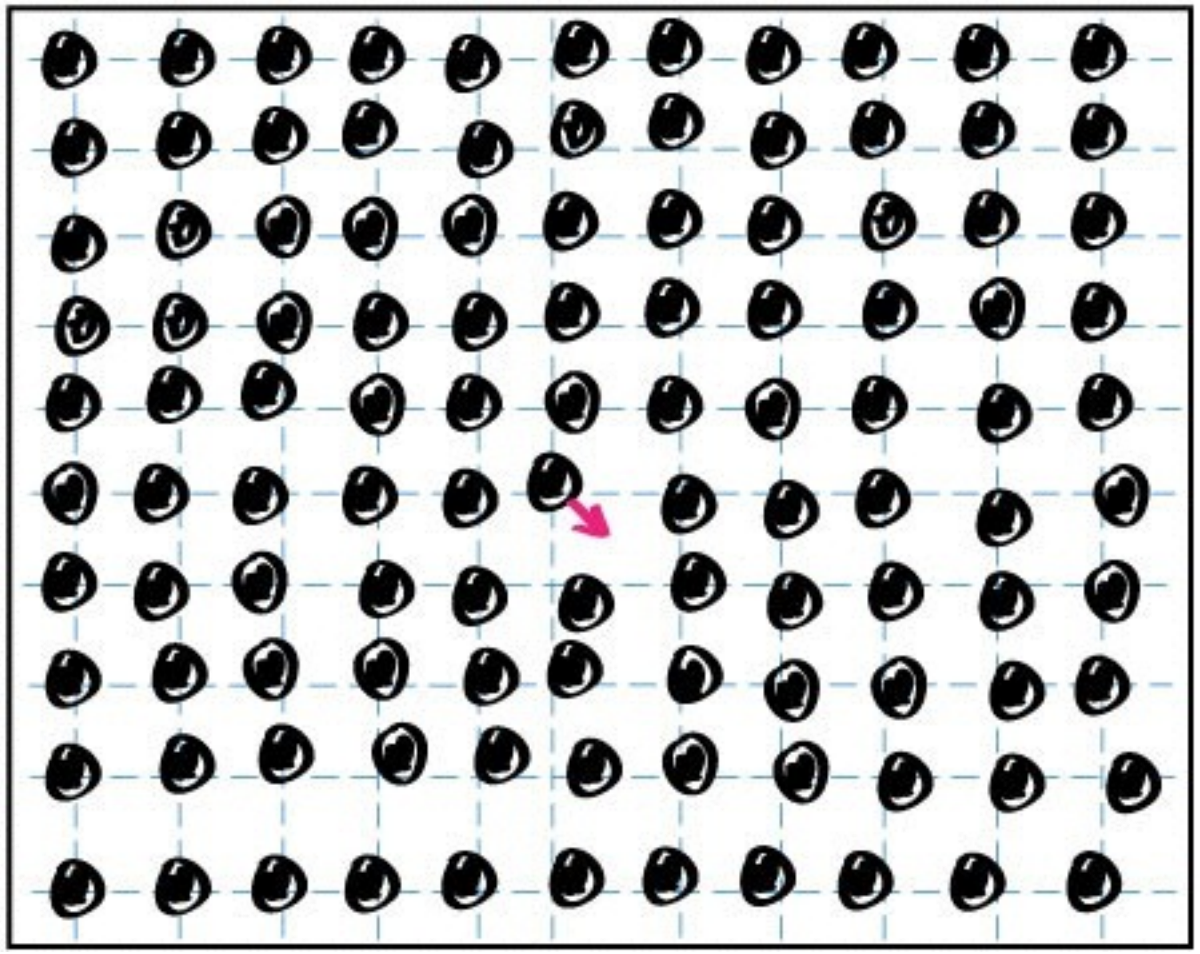
$$\xi(x) = \frac{1}{V} \sum_{\mathbf{k}} P(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

El espectro de potencia de la estructura



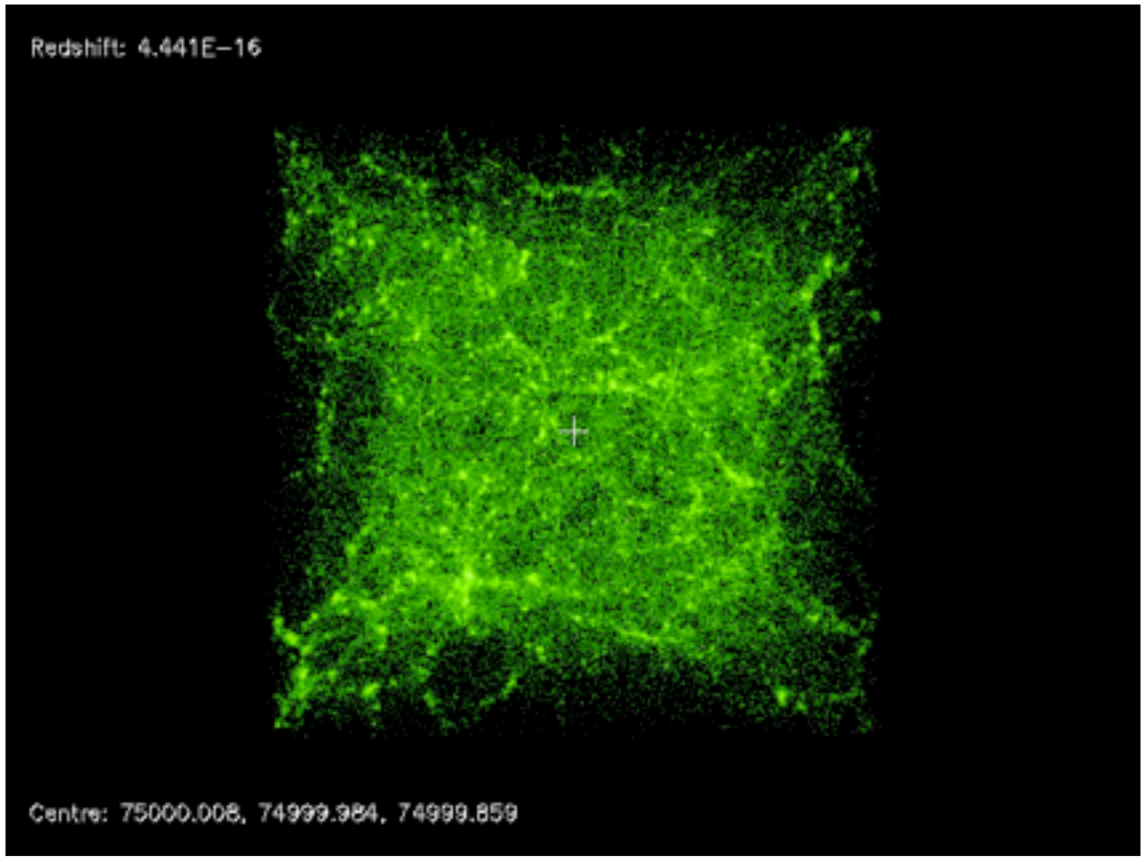
Cosmological simulations: initial conditions

- Harrison-Zel'dovich
- Efstatiou
- Power spectrum given
- 1LPT
- 2LPT

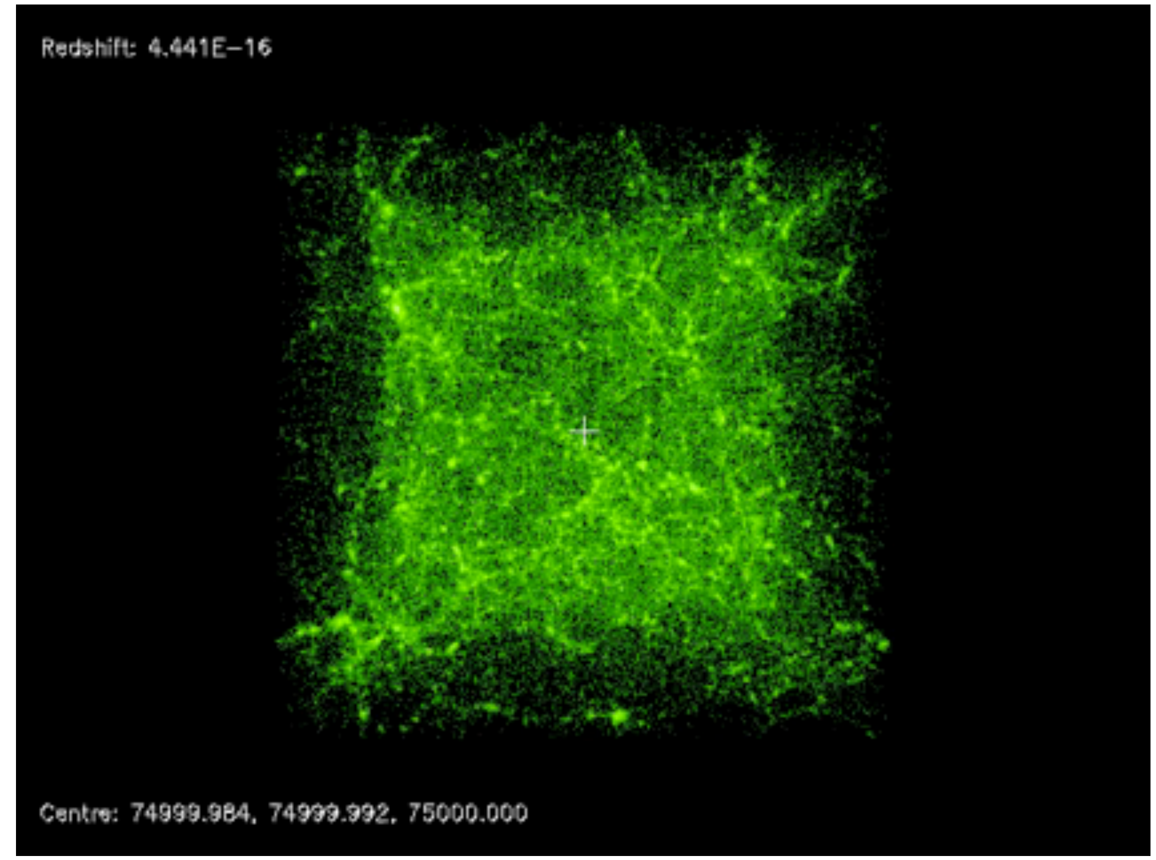


Standard IC versus 2LPTC

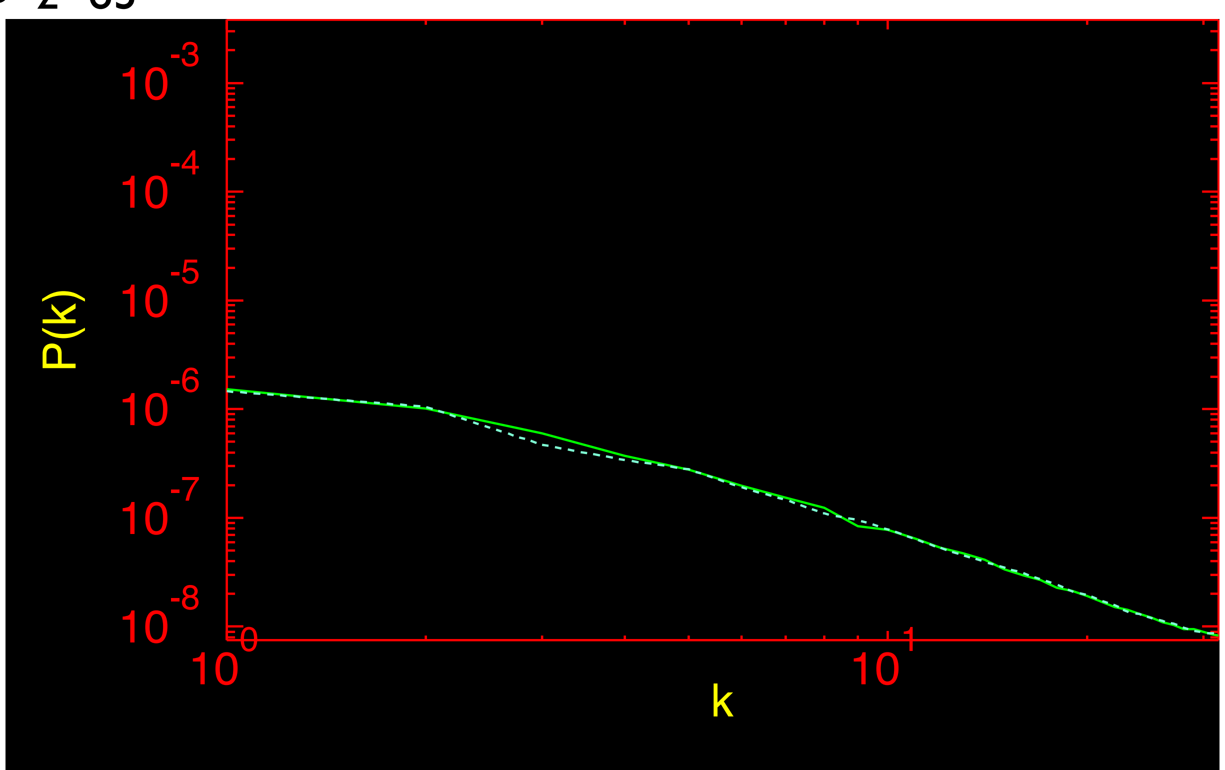
- Standar IC



- 2LPT IC



- z=63



- z=0