GEOMETROTHERMODYNAMICS AND COSMOLOGY

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- The idea of GTD
- Fundamental cosmological relations
- Conclusions

THE IDEA OF EQUILIBRIUM GTD

GEOMETRY	$\Longleftrightarrow$	**THERMODYNAMICS**
Equilibrium manifold $\mathcal{E}$	$\Longleftrightarrow$	**Thermodynamic system**
Curvature of $\mathcal{E}$	$\Longleftrightarrow$	**Thermodynamic interaction**
Singularity of $\mathcal{E}$	$\Longleftrightarrow$	**Phase transition**
TD geodesic of $\mathcal{E}$	$\Longleftrightarrow$	**Free quasi-static process**
Diff. invariance of $\mathcal{E}$	$\Longleftarrow$	**Representation independent**
Legendre invariance of $\mathcal{T}$	$\Longleftrightarrow$	**TD potential independent**

EXAMPLE: $n = 2$

- **THERMODYNAMIC VARIABLES**

$$\Phi = U \ , \ E^a = \{S, V\} \ , \ I^a = \{T, P\} \ , \quad \text{Ex} : U_{ig} = \left(e^S V^{-1}\right)^{1/c_V}$$

- **FIRST LAW OF THERMODYNAMICS**

$$dU = TdS - PdV \Longleftrightarrow \qquad dS = \frac{1}{T}dU + \frac{P}{T}dV$$

Energy representation $\Longleftrightarrow$ Entropy representation

- **LEGENDRE TRANSFORMATIONS**

$F = U - TS$	partial Legendre transformation
$H = U + PV$	partial Legendre transformation
$G = U - TS + PV$	total Legendre transformation

EQUILIBRIUM MANIFOLD

Differential manifold	$\rightarrow$	$\mathcal{E}$
Coordinates of $\mathcal{E}$	$\rightarrow$	E^a
Conditions on $\mathcal{E}$	$\rightarrow$	$\Phi = \Phi(E^a),\ d\Phi = I_a dE^a,\ I_a = \frac{\partial \Phi}{\partial E^a}$
Metric on $\mathcal{E}$	$\rightarrow$	$g = ds^2_{\mathcal{E}} = g_{ab} dE^a dE^b$

- **CHANGE OF REPRESENTATION**

$$dU = TdS - PdV \iff dS = \frac{1}{T}dU + \frac{P}{T}dV$$

Energy representation $\iff$ Entropy representation

$$E^a = \{S, V\} \iff E^{a'} = \{U, V\} = \{U(S,V), V\}$$

$$J = \left| \frac{\partial E^{a'}}{\partial E^a} \right| = T \neq 0$$

- **CHANGE OF REPRESENTATION IN GENERAL**

  If

$$d\Phi = I_a dE^a \quad \Longleftrightarrow \quad d\Phi' = I_{a'} dE^{a'}$$

$$\Phi\text{−representation} \quad \Longleftrightarrow \quad \Phi'\text{−representation}$$

$$E^a \quad \Longleftrightarrow \quad E^{a'} = E^{a'}(E^a)$$

  then

$$J = \left| \frac{\partial E^{a'}}{\partial E^a} \right| \neq 0 \qquad g_{a'b'} = \frac{\partial E^a}{\partial E^{a'}} \frac{\partial E^b}{\partial E^{b'}} g_{ab}$$

$$g_{ab} = ???$$

LEGENDRE TRANSFORMATIONS IN GENERAL

Let the coordinates $\{Z^A\} = \{\Phi, E^a, I^a\}$ be independent:

$$\boxed{\{Z^A\} = \{\Phi, E^a, I^a\} \longrightarrow \{\widetilde{Z}^A\} = \{\tilde{\Phi}, \tilde{E}^a, \tilde{I}^a\}}$$

$$\boxed{\Phi = \tilde{\Phi} - \delta_{kl}\tilde{E}^k\tilde{I}^l \ , \quad E^i = -\tilde{I}^i, \ \ E^j = \tilde{E}^j, \quad I^i = \tilde{E}^i, \ \ I^j = \tilde{I}^j}$$

where $i \cup j$ disjoint decomposition of $\{1, ..., n\}$, and $k, l = 1, ..., i$.

Total Legendre transformation if $i = \{1, ..., n\}$.

Identity transformation $i = \emptyset$.

Phase space = Riemannian contact manifold $(\mathcal{T}, \Theta, G)$

Differential manifold	$\rightarrow$	$\mathcal{T}$
Coordinates of $\mathcal{T}$	$\rightarrow$	$Z^A = \{\Phi, E^a, I^a\}$
Contact 1-form	$\rightarrow$	$\Theta = d\Phi - I_a dE^a$
Metric on $\mathcal{T}$	$\rightarrow$	$G = ds^2_{\mathcal{T}} = G_{AB} dZ^A dZ^B$

Equilibrium space = Riemannian submanifold $(\mathcal{E}, g)$, $\mathcal{E} \subset \mathcal{T}$

$$\varphi : \mathcal{E} \longrightarrow \mathcal{T} \qquad \varphi^*(\Theta) = 0$$

$$\varphi : \{E^a\} \longmapsto Z^A = \{\Phi(E^a), E^a, I^a(E^a)\}$$

Let:

$$g = \varphi^*(G) \longleftrightarrow g_{ab} = \frac{\partial Z^A}{\partial E^a} \frac{\partial Z^B}{\partial E^b} G_{AB}$$

LEGENDRE INVARIANT METRICS

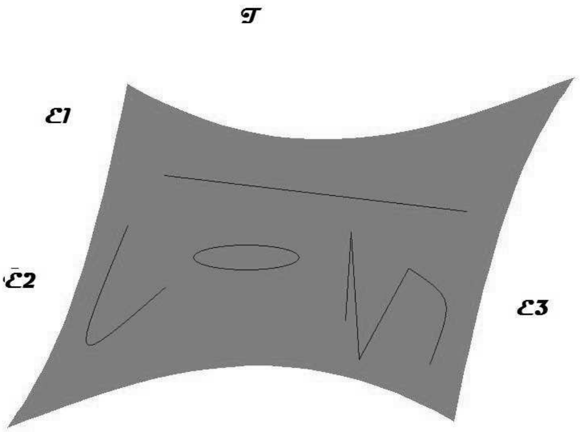
$$G^{I} = (d\Phi - I_a dE^a)^2 + \Lambda\,(\chi_{ab} E^a I^b)(\delta_{ab} dE^a dI^b)$$ (total inv.)

$$G^{II} = (d\Phi - I_a dE^a)^2 + \Lambda\,(\chi_{ab} E^a I^b)(\eta_{ab} dE^a dI^b)$$ (total inv.)

$$G^{III} = (d\Phi - I_a dE^a)^2 + \Lambda\,(E_a I_a)^{2k+1} dE^a dI^a$$ (total and partial inv.)

$$\Lambda = \Lambda(Z^A) \qquad \chi_{ab} = \mathrm{diag}(c_{11}, c_{22}, ..., c_{nn})\ , \quad c_{bb} = const.$$

GEOMETROTHERMODYNAMICS



VDW-FLUID

Fluid $(n = 2)$: $\Phi = S \;, E^a = (U, V)$

$$S = c_1 \ln\left(U + \frac{A}{V}\right) + c_2 \ln(V - B)$$

$$R_{\mathcal{E}}^{I/III} \neq 0 \quad \text{Phase transitions (I)}$$

For $A \to 0$ **and** $B \to 0 \Longrightarrow \; R_{\mathcal{E}}^{I/III} = 0$

$c_1, \; c_2$: No-interaction constants

$A, \; B$: Interaction constants

Friedman-Robertson-Walker spacetime:

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right]$$

Einstein equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}(P, \rho)$

$$\boxed{\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi}{3}\rho \qquad \frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3P)}$$

Equation of State:

$$\boxed{P = P(\rho)}$$

ad hoc

Let EoS = GTD-fluid

GTD-inspired cosmological model

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\rho\left[1 - \frac{27c_2}{8c_1}\rho_* + \frac{9c_2}{c_1(3-\rho_*)}\right]}$$

$$p_* = \frac{8\rho_*}{3-\rho_*} - 3\rho_*^2 \ , \ P_* = \frac{P}{P_c}, \ \rho_* = \frac{\rho}{\rho_c}, \quad A = \frac{3P_c}{\rho_c^2} \ , \ B = \frac{1}{3\rho_c}$$

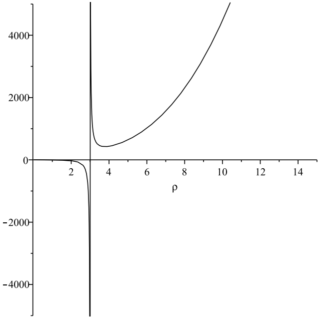


Figure: $\ddot{a}/a$ vs ρ_* with $\rho_c = 1$, $c_2/c_1 = 1/3$

Let $A = B = 0$

$$S = c_1 \ln U + c_2 \ln V \qquad R_{\mathcal{E}} = 0$$

$$P = \frac{c_2}{c_1}\frac{U}{V} = \frac{c_2}{c_1}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\rho\left(1 + \frac{3c_2}{c_1}\right)$$

$$\frac{c_2}{c_1} = \frac{1}{3} \longrightarrow \text{Radiation dominated universe}$$

$$c_2 = 0 \longrightarrow \text{Matter dominated universe}$$

$$\frac{c_2}{c_1} = -1 \longrightarrow \text{Vacuum dominated universe}$$

$$\Lambda CDM - model$$

COSMOLOGICAL FUNDAMENTAL EQUATION

$$\boxed{S = c_1 \ln\left(U + \frac{A}{V}\right) + c_2 \ln(V - B)}$$

Inflationary era:

$A \neq 0,\ B \neq 0 \Rightarrow$ (interacting fluid = "repulsive gravity")

Phase transition:

$A \to 0,\ B \to 0 \Rightarrow$ (??? fluid = "zero gravity")

ΛCDM era:

$A = 0,\ B = 0,\ \frac{c_2}{c_1} > 0 \Rightarrow$ (non-interacting fluid = "attractive gravity")

ΛCDM era:

$A = 0,\ B = 0,\ \frac{c_2}{c_1} < 0 \Rightarrow$ (non-interacting fluid = "repulsive gravity")

Conclusions

- GTD is representation invariant
- GTD is Legendre invariant
- GTD describes thermodynamic properties with geometry
- GTD in relativistic cosmology $\rightarrow$ thermodynamic history of the Universe