¿Requiere la gravitación ser extendida en escalas mucho mayores a las asociadas al sistema solar?

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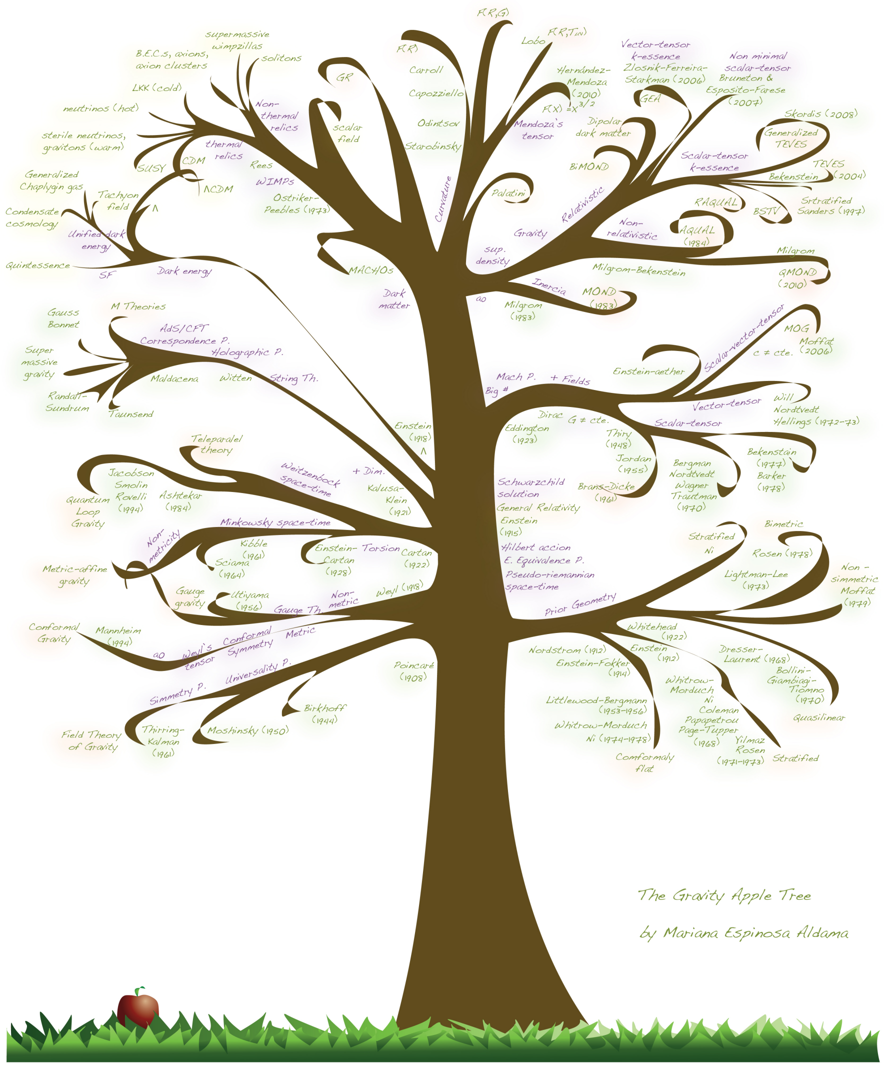
II Taller de gravitación, física de altas energías y cosmología

Instituto de ciencias físicas UNAM, Cuernavaca Morelos

Aug 06, 2014

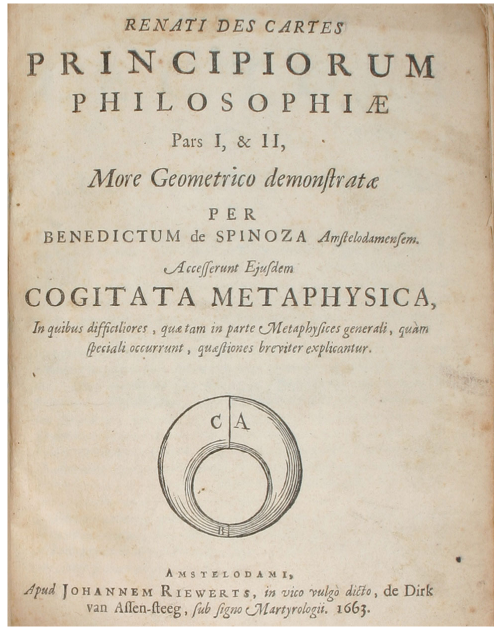
Apple gravity tree (M. Espinosa 2014)



Graphic interpretation of the generic branches of the gravitation theories developed during the 20th century until 2013. Ten main branches emerge from the main trunk, which represents, in its base, the newtonian theory (experiments in weak field), and in its middle core, the theory of general relativity (GR) of Albert Einstein. The branches emerge when different principles (mathematic, physical, philosophical or metaphysical) are followed. These principles are marked in purple, while in green are shown the names of the main proponents. Particular theories and models are noted in red and yellow. A time line can be followed both upwards and in a radial direction.

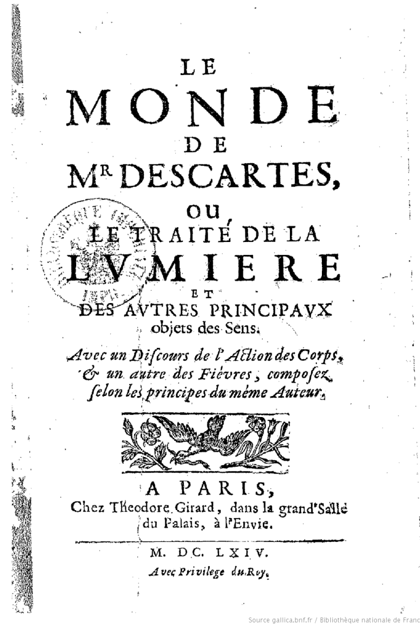
UNAM POSGRADO
Filosofía de la Ciencia

Occult fluids and Descartes' vortices.



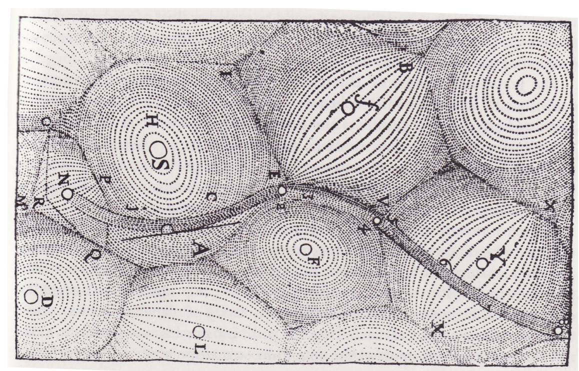
RENATI DES CARTES
PRINCIPIORUM
PHILOSOPHIÆ
Pars I, & II,
More Geometrico demonstratæ
PER
BENEDICTUM de SPINOZA *Amstelodamensem.*
Accesserunt Ejusdem
COGITATA METAPHYSICA,
In quibus difficiliores, quæ tam in parte Metaphysices generali, quàm speciali occurrunt, quæstiones breviter explicantur.

AMSTELODAMI,
Apud JOHANNEM RIEWERTS, *in vico vulgò dicto*, de Dirk van Assen-steeg, *sub signo Martyrologii.* 1663.



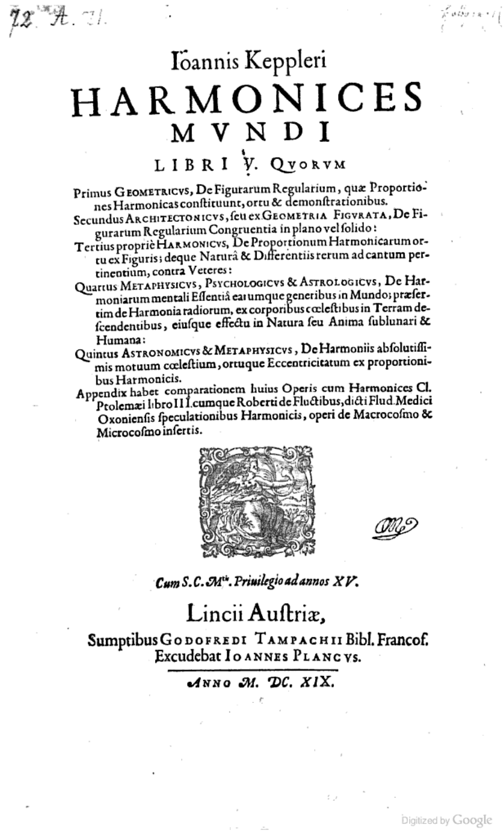
LE
MONDE
DE
M^R DESCARTES,
OU
LE TRAITÉ DE LA
LVMIERE
ET
DES AVTRES PRINCIPAVX
objets des Sens.
Avec un Discours de l'Action des Corps, & un autre des Fiévres, composez selon les principes du même Auteur.

A PARIS,
Chez Theodore Girard, dans la grand'Salle du Palais, à l'Envie.
M. DC. LXIV.
Avec Privilege du Roy.



Le Monde (1677) presents a corpuscularian cosmology in which swirling vortices made of an "*occult fluid*" explain, the creation of our solar system and the circular motion of planets about the Sun.

Kepler's third law



Ioannis Keppleri

HARMONICES MVNDI

LIBRI V. QVORVM

Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.

Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:

Tertius propriè HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Naturâ & Differentiis rerum ad cantum pertinentium, contra Veteres:

Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentiâ earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cœlestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:

Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cœlestium, ortuque Eccentricitatum ex proportionibus Harmonicis.

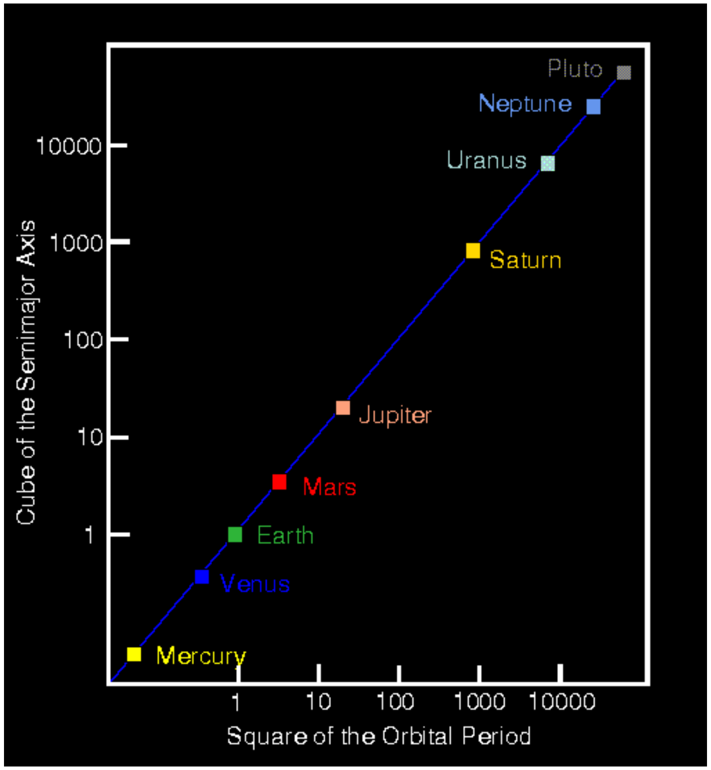
Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro III. cumque Roberti de Fluctibus, dicti Flud. Medici Oxoniensis speculationibus Harmonicis, operi de Macrocosmo & Microcosmo insertis.

Cum S.C. Mtis. Priuilegio ad annos XV.

Lincii Austriæ,

Sumptibus GODOFREDI TAMPACHII Bibl. Francof.
Excudebat IOANNES PLANCVS.

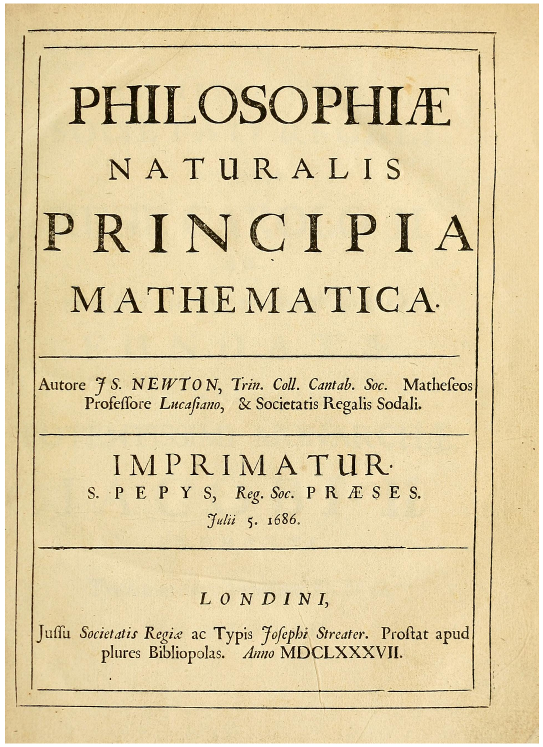
ANNO M. DC. XIX.



Kepler's 3rd law written on the Harmonices Mundi Libri V (1621): $T^2 \propto a^3/M$. For a circular orbit: $\boldsymbol{v} = v_\varphi \boldsymbol{e}_\varphi$ only and $v_\varphi = r\dot{\varphi}$. Since $\dot{\varphi} := 2\pi/T$ it follows that *the orbital velocity*:

$$v := v_\varphi \propto \sqrt{\frac{M}{r}}$$

Newton's gravity


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Newton derived his law of gravitation on his "*Philosophia Naturalis Principia Mathematica*" (1687) book using Kepler's 3rd law of planetary motion, requiring centripetal balance:

$$a = \frac{v^2}{r} = -G\frac{M}{r^2},$$

where the proportionality constant G is Newton's gravitational constant and the minus sign shows the attractive nature of gravity.

Preface to the 2nd ed. of the Principia Mathematica (1713)

arife from the particular natures of thofe bodies. But whence it is that bodies derive thofe natures they don't tell us; and therefore they tell us nothing. And being entirely employed in giving names to things, and not in fearching into things themfelves, we may fay that they have invented a philofophical way of fpeaking, but not that they have made known to us true philofophy.

Others therefore by laying afide that ufelefs heap of words, thought to employ their pains to better purpofe. Thefe fuppofed all matter homogeneous, and that the variety of forms which is feen in bodies arifes from fome very plain and fimple affections of the component particles. And by going on from fimple things to thofe which are more compounded they certainly proceed right; if they attribute no other properties to thofe primary affections of the particles than Nature has done. But when they take a liberty of imagining at pleafure unknown figures and magnitudes, and uncertain fituations and motions of the parts; and moreover of fuppofing occult fluids, freely pervading the pores of bodies, endued with an all-performing fubtilty, and agitated with occult motions; they now run out into dreams and chimera's, and neglect the true conftitution of things; which certainly is not to be expected from fallacious conjectures, when we can fcarce reach it by the moft certain obfervations. Thofe who fetch from hypothefes the foundation on which they build their fpeculations, may form indeed an ingenious romance, but a romance it will ftill be.

There is left then the third clafs, which profefs experimental philofophy. Thefe indeed derive the caufes of all things from the moft fimple principles poffible;

poffible; but then they affume nothing as a principle, that is not proved by phænomena. They frame no hypothefes, nor receive them into philofophy otherwife than as queftions whofe truth may be difputed. They proceed therefore in a twofold method, fynthetical and analytical. From fome felect phænomena they deduce by analyfis the forces of nature, and the more fimple laws of forces; and from thence by fynthefis fhew the conftitution of the reft. This is that incomparably beft way of philofophizing, which our renowned author moft juftly embraced before the reft; and thought alone worthy to be cultivated and adorned by his excellent labours. Of this he has given us a moft illuftrious example, by the explication of the Syftem of the World, moft happily deduced from the Theory of Gravity. That the virtue of gravity was found in all bodies, others fufpected, or imagined before him; but he was the only and the firft philofopher that could demonftrate it from appearances, and make it a folid foundation to the moft noble fpeculations.

I know indeed that fome perfons and thofe of great name, too much prepoffeffed with certain prejudices, are unwilling to affent to this new principle, and are ready to prefer uncertain notions to certain. It is not my intention to detract from the reputation of thefe eminent men; I fhall only lay before the reader fuch confiderations as will enable him to pafs an equitable fentence in this difpute.

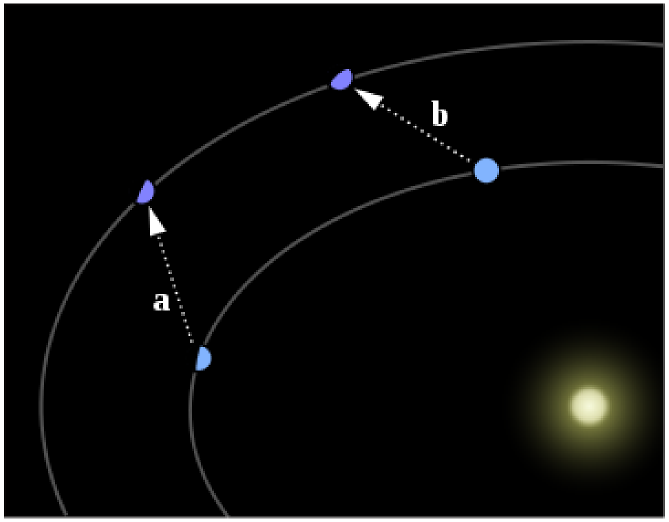
Therefore that we may begin our reafoning from what is moft fimple and neareft to us; let us confider a little what is the nature of gravity with us on Earth, that we may proceed the more fafely

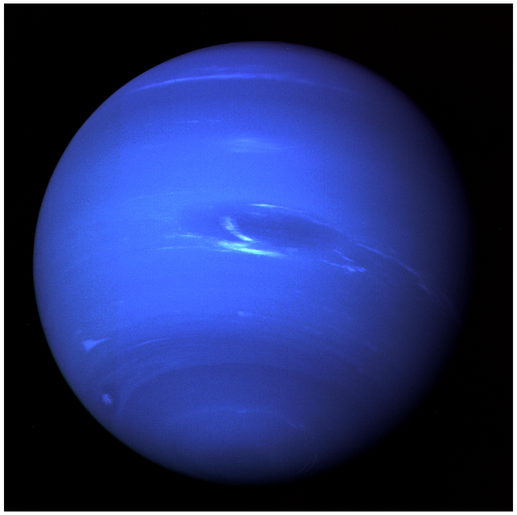
- ❄ Mas cuando se toman la libertad de imaginar a placer figuras y magnitudes desconocidas, así como situaciones y movimientos de las partes inciertos, suponiendo además fluídos ocultos que invaden libremente los poros de los cuerpos y están dotados de una sutileza que todo lo realiza, se entregan a sueños y quimeras y abandonan la verdadera constitución de las cosas que sin duda no ha de derivarse de conjeturas falaces, siendo así que difícilmente podemos descubrirla mediante las observaciones más ciertas. Quienes aceptan hipótesis como primeros principios de sus especulaciones, por más que a continuación procedan con la mayor exactitud a partir de dichos principios, construirán ciertamente una fábula ingeniosa, si bien núnca dejará de ser una fábula.
- ❄ La última clase profesa la filosofía experimental. Estos de hecho derivan las causas de todo a partir de los principios más sencillos posibles; pero luego no asumen nada como principio, que no haya sido suministrado por los fenómenos. Ellos no proponen hipótesis, ni las reciben en la filosofía sino como asuntos cuya veracidad puede ser disputada. Ellos proceden por lo tanto en un doble método, sintético y analítico. A partir de algunos fenómenos selectos ellos deducen por análisis las fuerzas de la naturaleza, y las más simples leyes de fuerzas; y de ahí por síntesis muestran la constitución del resto. Esta es incomparablemente la mejor manera de filosofar, que nuestro renombrado autor jústamente abrazó antes que el resto; y que estimó exclusivamente digna de ser cultivada y adornada por sus excelentes trabajos. De ésto él nos ha dado un muy ilustre ejemplo. Mediante la explicación del Sistema del Mundo muy afortunadamente deducida de su Teoría de la Gravedad.

- Alexis Bouvard (1821) published tables of the position of the planet Uranus. He showed that the anomalous motion of Uranus was not possible to describe using Newton's theory of gravity:

> "These irregularities or residuals, both in the planet's ecliptic longitude and in its distance from the Sun, or radius vector, might be explained by a number of hypotheses: *the effect of the Sun's gravity, at such a great distance, might differ from Newton's description*; or perhaps Uranus was being pulled, or perturbed, by an as-yet undiscovered eighth planet; or the discrepancies might simply be observational error. "

- Adams & Le Verrier (1845) independently predicted the location of **Neptune**.
- Le Verrier is remembered as *" the man who discovered a planet with the point of his pen."*

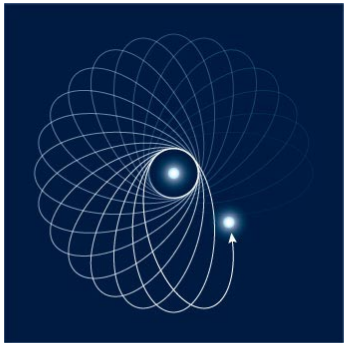


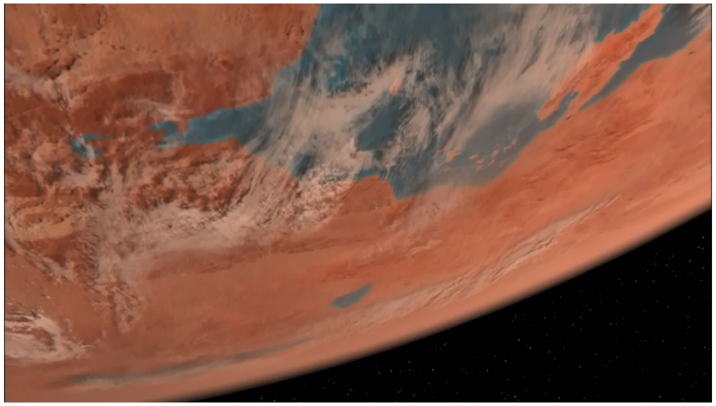


Perihelium's precession of Mercury.

- Mercury's precession took Le Verrier to postulate the existence of an unknown solar system inner planet: Vulcan.

Eddington (1914) wrote a letter to Einstein asking him whether **his relativistic theory of gravity** would be capable to reproduce the anomalous movement of Mercury. Einstein does it!





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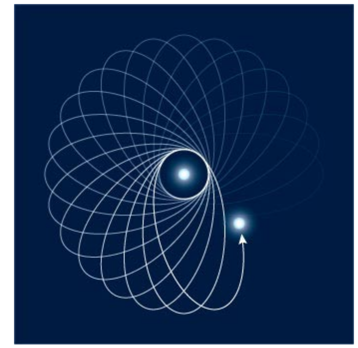
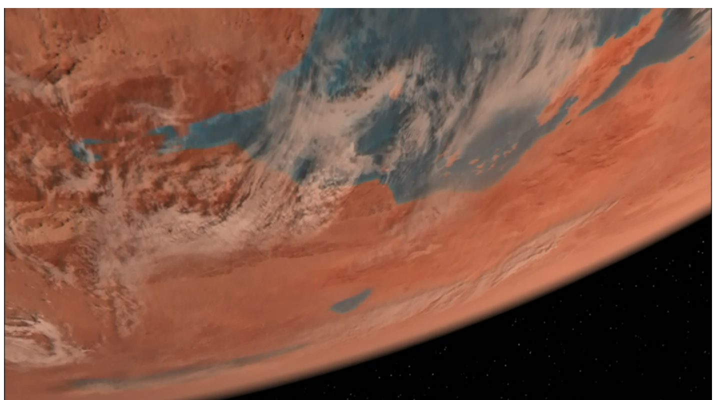
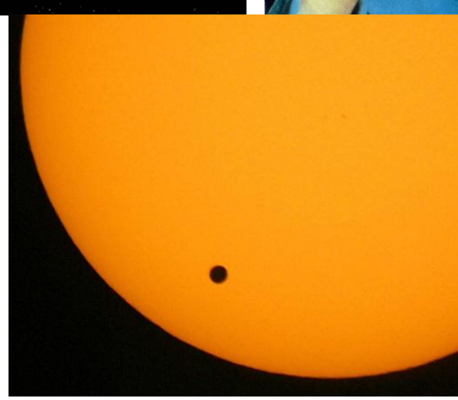




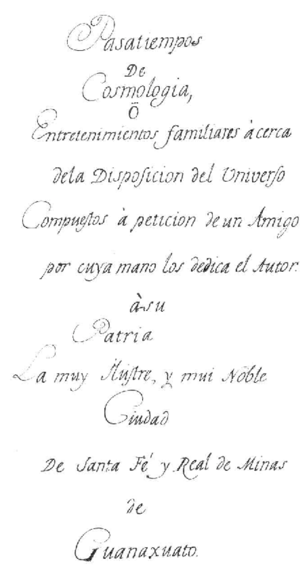
Le Verrier died in 1877 believing in the existence of Vulcan since an amateur astronomer (Lescarbault, March 26th, 1859) reported the transit of Vulcan across the Sun.



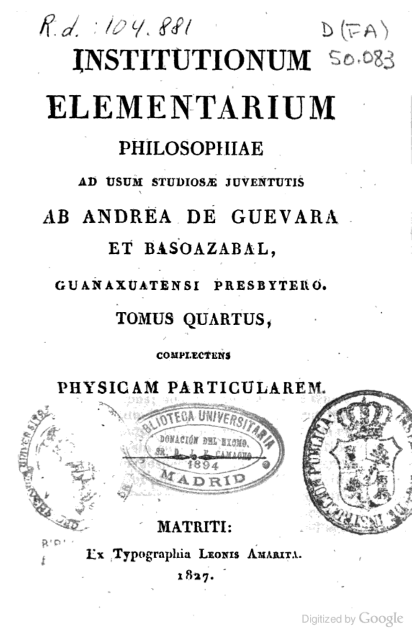
Aether, another occult fluid

- According to medieval science: "Aether is the material that fills the region of the universe above the terrestrial sphere".
- Christian Huygens: light travelled in the form of longitudinal waves via an "omnipresent, perfectly elastic medium having zero density, called aether".
- Michelson & Morley experiment and Einstein's hypothesis

En México: Andrés Guevara y Basoasabal (1748-1801).



Pasatiempos
De
Cosmologia,
ó
Entretenimientos familiares à cerca
dela Disposicion del Universo
Compuestos à peticion de un Amigo
por cuya mano los dedica el Autor
à su
Patria
La muy Ilustre, y mui Noble
Ciudad
De Santa Fé y Real de Minas
de
Guanaxuato.



INSTITUTIONUM
ELEMENTARIUM
PHILOSOPHIAE
AD USUM STUDIOSÆ JUVENTUTIS
AB ANDREA DE GUEVARA
ET BASOAZABAL,
GUANAXUATENSI PRESBYTERO.
TOMUS QUARTUS,
COMPLECTENS
PHYSICAM PARTICULAREM.

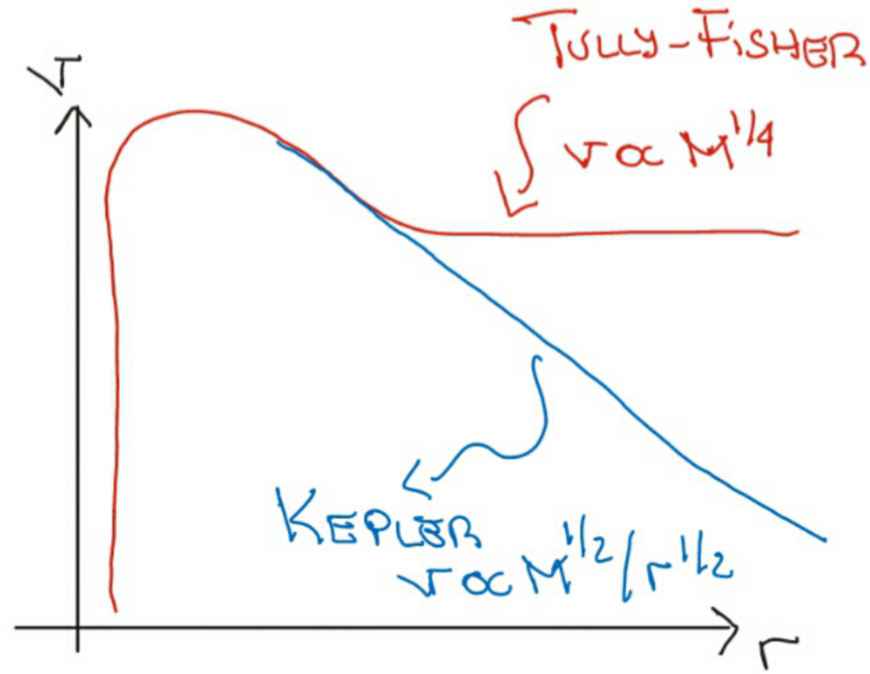
MATRITI:
Ex Typographia Leonis Amarita.
1827.

❄ Pasatiempos de cosmología o entretenimientos familiares acerca de la disposición del universo (1789)
❄ Instituonum Elementarium Philosophiæ(1796)”

Ref: S. Galindo (2012) RMF, **58**-2.

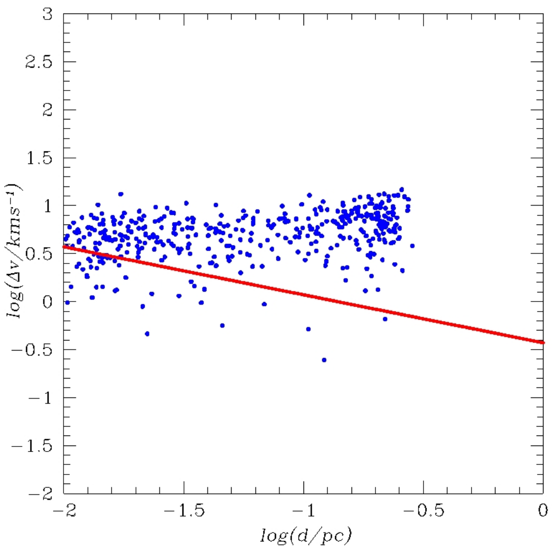
Fritz Zwicky (1933) postulates the existence of "*missing matter* on the Coma cluster due to large velocities of the internal galaxies.

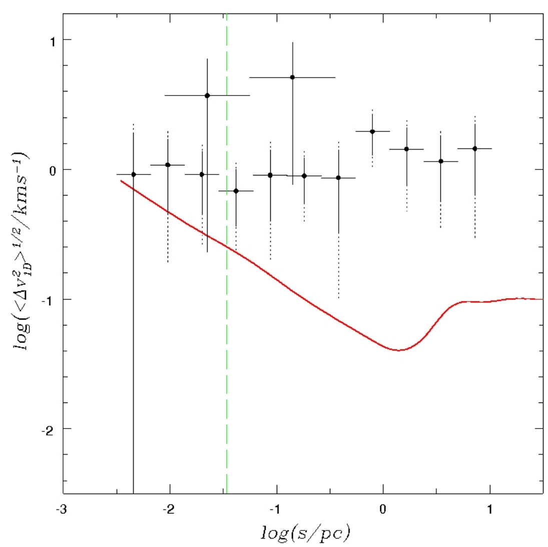
Vera Rubin (1970) concluded that the explanation for the anomalous rotation curve on spiral galaxies was due to **missing dark mattter**.



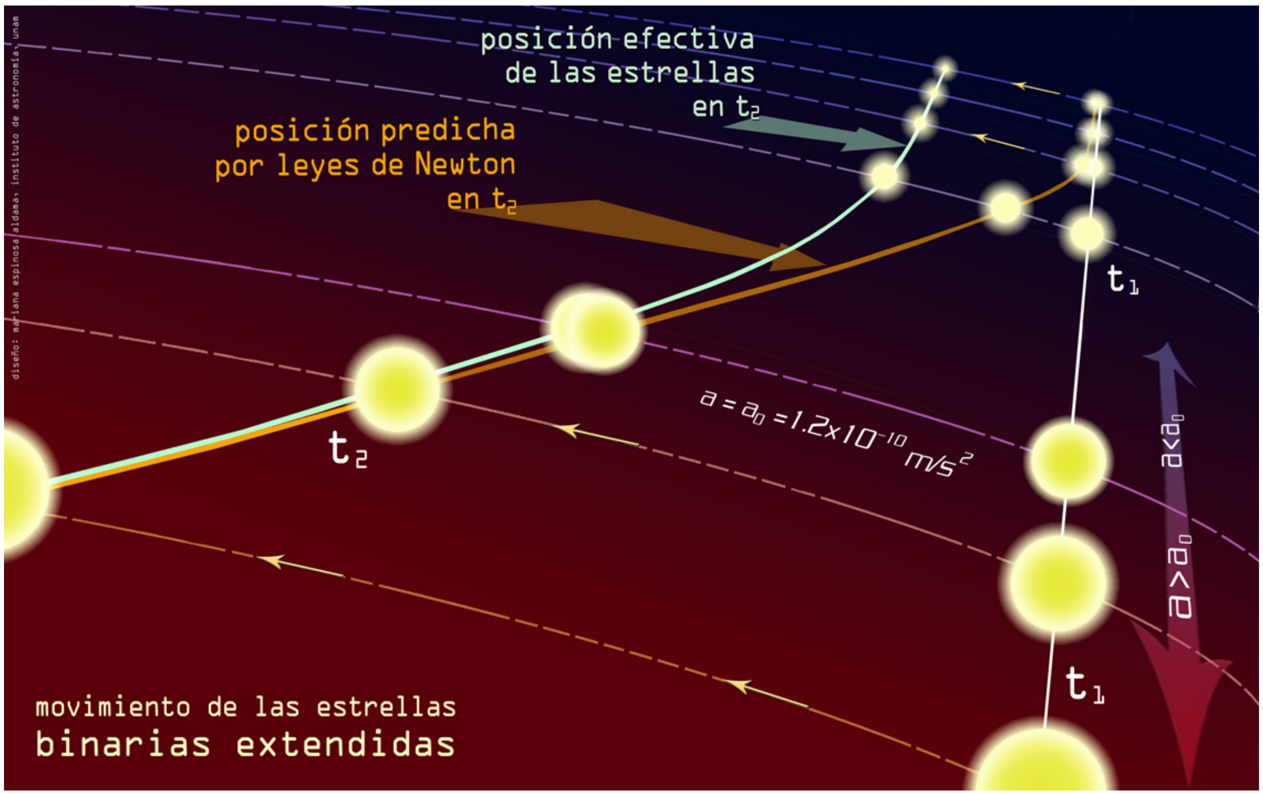
Newtonian gravity works in the inner region (where $a \gtrsim a_0 \sim 10^{-10}\mathrm{m\,s^{-2}}$). Outer region (where $a \lesssim a_0$ does not follow a Kepler's third law: $(v^2 \propto M/r)$, but a **Tully-Fisher relation**: $v^4 \propto M$.

¡Wide open binaries do not follow Kepler's 3rd law!

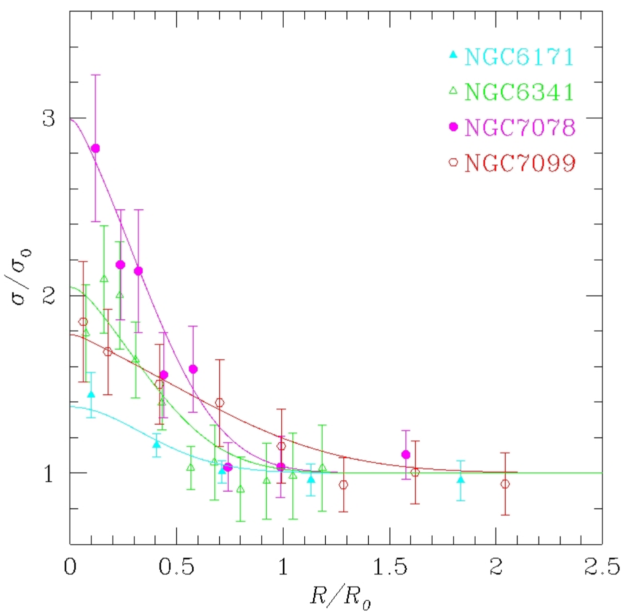


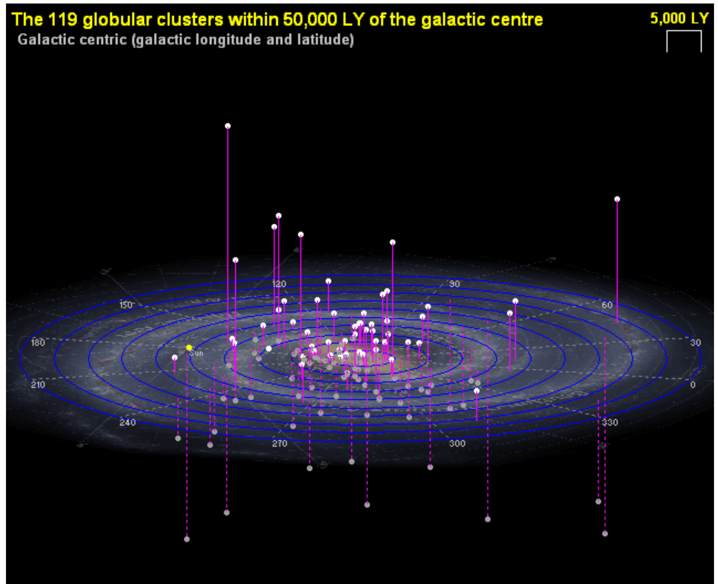


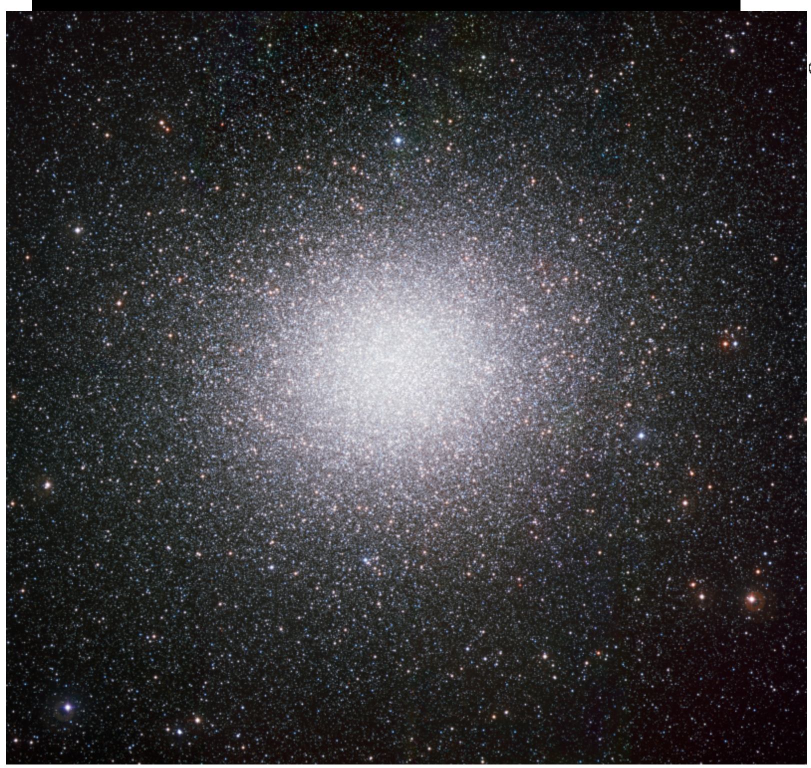
Hernandez et al. (2012)

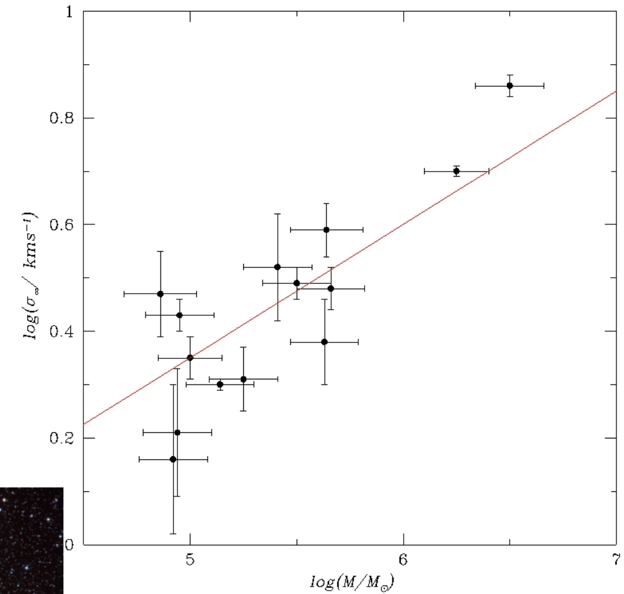


Outer regions of Globular Clusters obey the Tully-Fisher law!









Interior is **Kepler's 3rd law**: $\sigma \propto \sqrt{M/r}$

Exterior is **Tully-Fisher law** : $\sigma \propto M^{1/4}$

Hernandez et al. (2012)

Newtonian non-relativistic gravity (based on Kepler's 3rd law)	Extended non-relativistic gravity (based on Tully-Fisher's law

- Rotation curves (Kepler's third law):

$$v \propto \frac{M^{1/2}}{r^{1/2}}.$$

- Centrifugal balance $a \propto v^2/r$.
- Acceleration force is then:

$$a = -G_{\rm N} \frac{M}{r^2}.$$

- Calibrate with observations:

$$G_{\rm N} = 6.67 \times 10^{-11}\,{\rm m}^3\,{\rm s}^{-2}\,{\rm kg}^{-1}.$$

- Rotation curves (Tully-Fisher law):

$$v \propto M^{1/4}.$$

- Centrifugal balance $a \propto v^2/r$.
- Acceleration force is then:

$$a = -G_{\rm M} \frac{M^{1/2}}{r}.$$

- Calibrate with observations:

$$G_{\rm M} \approx 8.94 \times 10^{-11}\,{\rm m}^2\,{\rm s}^{-2}\,{\rm kg}^{-1/2}.$$

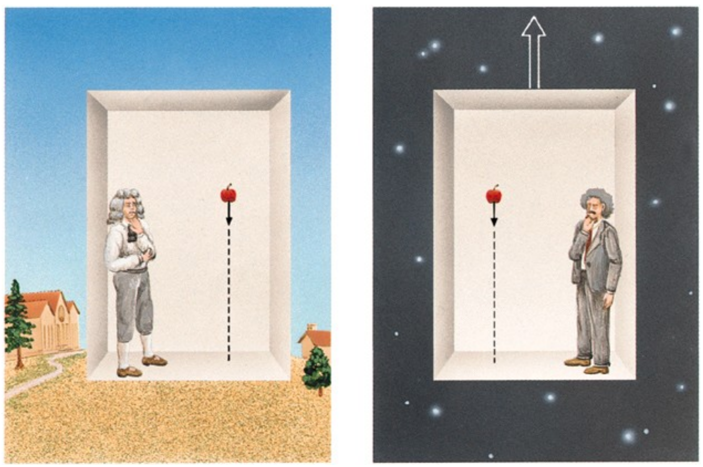
- Simplest form of MOND found since:

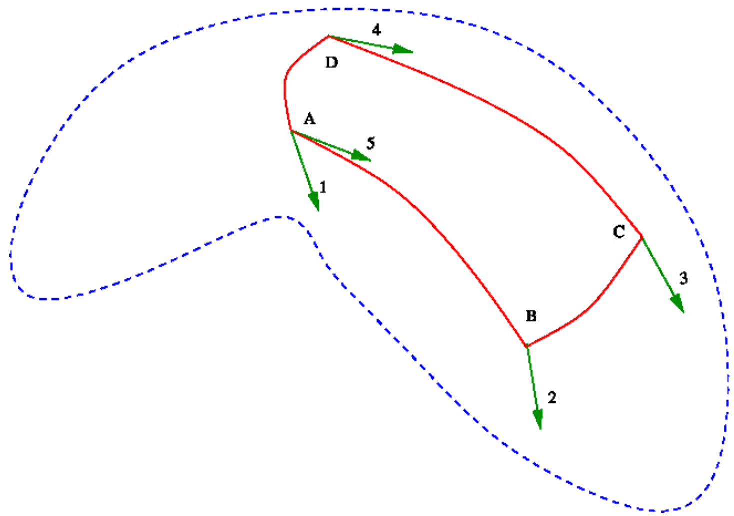
$$a_0 := \frac{G_{\rm M}^2}{G_{\rm N}}.$$

Milgrom (1983) obtained result requiring Newton's 2nd law to be modified. Since $(v^2/r) = a = GM/r^2$, then: $(v^2/r)^2 = a^2 = a_0\, GM/r^2$, i.e. MOdified Newtonian Dyn.

A new theory of gravity?

- Gravitational phenomena needs to be relativistic.
- Can space-time allow for a relativistic Tully-Fisher structure?
- Assume the Einstein Equivalence Principle to be valid.
- Allow for geodesic motion of particles.





Relativistic Kepler's 3rd law

- Weakfield geodesic motion (massive particles):

$$g_{00} = 1 + \frac{2\phi}{c^2} = 1 + \frac{2G_{\mathrm{N}}M}{rc^2}.$$

- Isotropic coordinates:

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 - \left(1 + 2\gamma\phi/rc^2\right)\delta_{kl}\mathrm{d}x^k\mathrm{d}x^l.$$

- Spherical coordinates:

$$g_{rr} = -1 - \frac{2\gamma G_{\mathrm{N}}M}{rc^2}.$$

Lensing observations imply $\gamma = 1$.
Schwarzschild solution of Einstein's field equations also imply $\gamma = 1$.

Relativistic Tully-Fisher law

- Weakfield geodesic motion (massive particles):

$$g_{00} = 1 + \frac{2\phi}{c^2} = 1 - \frac{2G_{\mathrm{M}}M^{1/2}}{c^2}\ln\left(\frac{r}{r_\star}\right).$$

- Isotropic coordinates:

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 - \left(1 + 2\gamma\phi/rc^2\right)\delta_{kl}\mathrm{d}x^k\mathrm{d}x^l.$$

- Spherical coordinates:

$$g_{rr} = -1 - \frac{2\gamma G_{\mathrm{M}}M^{1/2}}{c^2}$$

Lensing observations imply $\gamma = 1$.
Mendoza et al. (2013)
Mendoza & Olmo (2014)

- Lensing on elliptical, spiral and galaxy groups can be modelled using total matter distributions with isothermal profiles ($M_{\mathrm{T}} = v^2 r/G$ and DM profiles obey the same Tully-Fisher relation of baryonic matter of spirals: $v \propto M_{\mathrm{b}}$.
- Take GR -Schwarzschild- + DM.

$$g_{00\mathrm{S}} = 1/g_{11\mathrm{S}} = 1 - \frac{2r_{\mathrm{g}}}{r} = 1 - \frac{2GM_{\mathrm{T}}(r)}{c^2 r} = 1 - 2\left(\frac{v}{c}\right)^2 .$$

- The deflection angle $\beta_{GR} = F(g_{00\mathrm{S}}, g_{11\mathrm{S}}, r_i)$ can thus be calculated.
- This deflection angle is THE SAME for any metric theory of gravity and so $\beta_{GE} = \beta_{\mathrm{Ext}}$.
- Last relation is valid for all r_i and so, it is possible to find $g_{11\mathrm{Ext}}$ at $\mathcal{O}(2)$.

In short, Tully-Fisher law + lensing observations, at $\mathcal{O}(2)$ yield:

$$g_{00} = 1 - \frac{2G_{\mathrm{M}}M^{1/2}}{c^2} \ln\left(\frac{r}{r_\star}\right), \qquad g_{rr} = -1 - \frac{2G_{\mathrm{M}}M^{1/2}}{c^2}.$$

Hence: $\gamma = 1$ as in relativistic Kepler's 3rd law

Mendoza et al. (2013), Mendoza & Olmo (2014)

Extended Newtonian gravity (Mendoza et al. 2011)

- Assume that a_0 is a fundamental constant of nature.
- Gravitational acceleration a experienced by a test particle is characterised by M, r, G, a_0.
- Since there are three independent dimensions m, l, t Buckingham's Π theorem of dimensional analysis implies that:

$$a = a_0 \, f(x),$$

where

$$x := l_{\mathrm{M}}/r, \tag{1}$$

and the *mass-length* l_M given by

$$l_{\mathrm{M}} := \left(\frac{GM}{a_0} \right)^{1/2}. \tag{2}$$

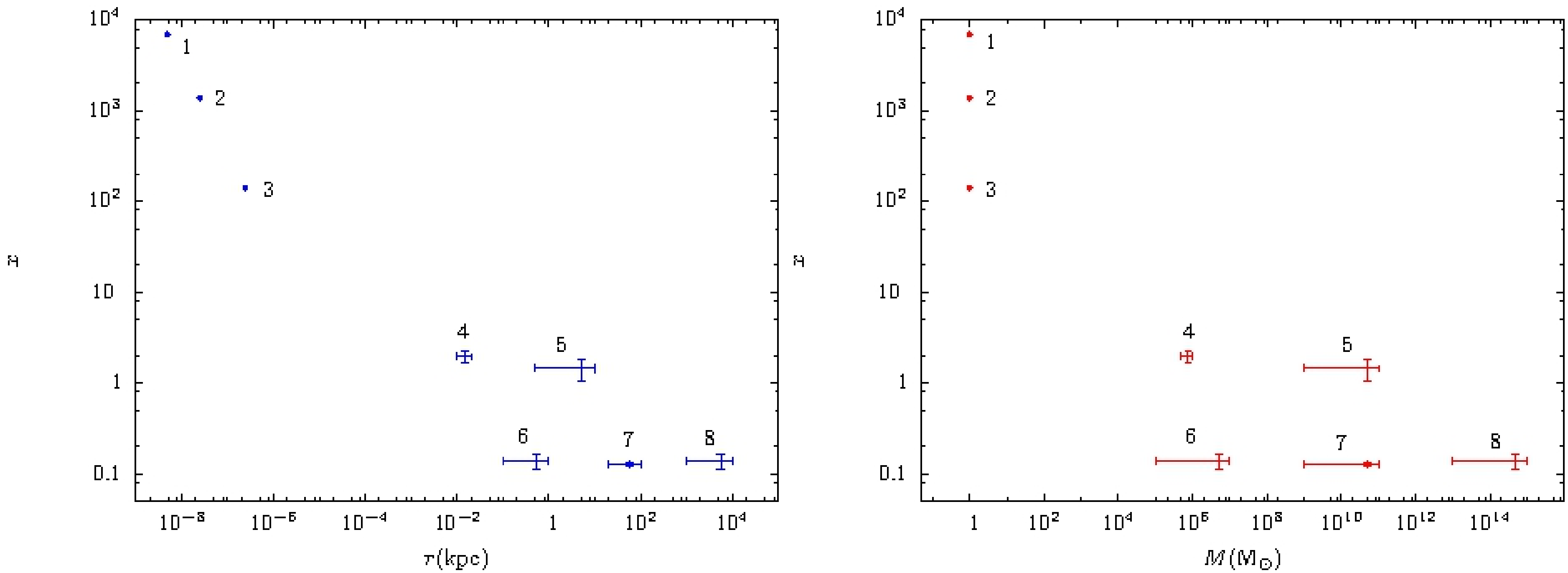
- E.g. $f(x) = x^\alpha$, a power law.

$$f(x) = \begin{cases} x^2, & \text{Newton} \\ x, & \text{MOND} \\ x + x^2, & \text{Bekenstein (2004)} \end{cases}$$

- Hernandez, Mendoza et al. (2010) showed that Bekenstein's formula is useful for dwarf galaxies, but fails at large distances in the solar system. Famaey & Binney (2009) showed that it is also not useful for our galaxy.
- The natural limits of the dimensionless function f are:

$$f(x) \longrightarrow \begin{cases} x^2, & \text{when } x \longrightarrow \infty, \\ x, & \text{when } x \longrightarrow 0. \end{cases}$$

- An object with a size much grater than l_M behaves MONDianly ($x \ll 1$) and an object with size much smaller than l_M is in the Newtonian regime of gravity.

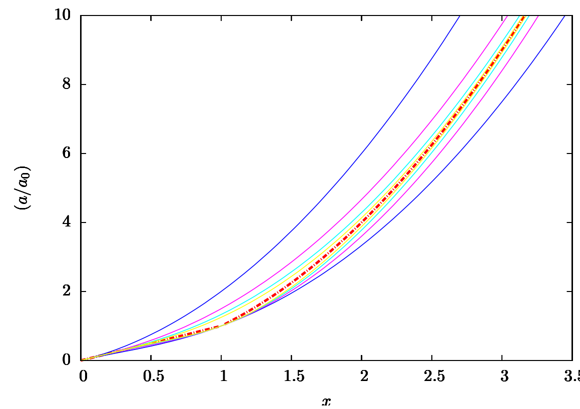


$x \propto M^{1/2}/r$: (1) Solar system at Earth's orbit. (2) Solar system at Jupiter's orbit. (3) Solar system at the Kuiper belt radius. (4) Globular clusters. (5) Elliptical galaxies and bulges of spirals. (6) Dwarf spheroidal galaxies. (7) Outer regions of spiral galaxies. (8) Galaxy clusters

Transition function $f(x)$

❄ Mathematical approximations about MONDian and Newtonian limits lead to a good Postulate:

$$\left(\frac{a}{a_0}\right)_{\pm} := f(x) = x\,\frac{1 \pm x^{n+1}}{1 \pm x^{n}}.$$



❄ Extreme case occurs when $n = \infty$ (different astronomical fits imply $n \gtrsim 8 - 10$).

$$\left(\frac{a}{a_0}\right)_{\mathrm{e}} = \begin{cases} x, & \text{when } 0 \le x \le 1 \quad \text{(MONDian).} \\ x^2, & \text{when } x \ge 1 \quad \text{(Newtonian).} \end{cases}$$

Relativistic approach to an extended theory of gravity: dimensionality

- Extended relativistic gravity means that the following parameters have to be introduced into the theory:
  - M. Mass of the object producing gravitational field
  - G. Newton's gravitational constant
  - c. Speed of light
  - a_0. Milgrom's acceleration constant.

  $\Longrightarrow$ possible to build two characteristic lengths:

$$\text{Mass length scale } l_M := \left(\frac{GM}{a_0} \right)^{1/2},$$

$$\text{Gravitational radius } r_{\mathrm{g}} := \frac{GM}{c^2}.$$

(Bernal, Capozziello, Hidalgo & Mendoza 2011)

Action and field equations

- Hilbert's action for the gravitational field:

$$S_{\mathrm{H}} = -\frac{c^3}{16\pi G}\int \frac{f(\chi)}{L_M^2}\sqrt{-g}\,\mathrm{d}^4x,$$

where:

$$\chi := L_M^2 R,$$

and L_M is a "characteristic length" of the theory.

- Matter action has its usual form:

$$S_{\mathrm{m}} = -\frac{1}{2c}\int \mathcal{L}_{\mathrm{m}}\sqrt{-g}\,\mathrm{d}^4x,$$

❄ Null variations of complete action (i.e. $\delta\left(S_{\mathrm{H}}+S_{\mathrm{m}}\right)=0$) give field equations:

$$f'(\chi)\,\chi_{\mu\nu}-\frac{1}{2}f(\chi)g_{\mu\nu}-L_M^2\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Delta\right)f'(\chi)=\frac{8\pi G L_M^2}{c^4}T_{\mu\nu},$$

❄ Trace of field equations:

$$f'(\chi)\,\chi-2f(\chi)+3L_M^2\,\Delta f'(\chi)=\frac{8\pi G L_M^2}{c^4}T.$$

Einstein's general relativity

- Trace of Hilbert-Einstein equations for dust:

$$R = -\frac{8\pi G}{c^4} T = -\frac{8\pi G \rho c^2}{c^4}.$$

- Since ${}^{(2)}R = -(2/c^2)\nabla^2\phi$, so Poisson's equation is obtained:

$$\nabla^2\phi = 4\pi G\rho.$$

- At order of magnitude $\nabla^2\phi \approx a/r$ and so, since $\rho \approx M/r^3$ then

$$a \approx GM/r^2.$$

Newton's gravity is the weak field limit of General relativity

Extended relativistic gravity

- Assume $f(\chi) = \chi^b$ To order of magnitude, trace equation is:

$$\chi^b\,(b-2) + 3bL_M^2 \frac{\chi^{(b-1)}}{r^2} \approx \frac{8\pi G M L_M^2}{c^2 r^3}.$$

- At ${}^{(2)}R = -(2/c^2)\nabla^2\phi$, flat rotation curves (i.e. $a \propto 1/r$) when $b = 3/2$
- The choice $L_M^2 \propto l_M r_{\mathrm{g}}$ ensures c does not appear on the resulting equation:

$$a \approx \frac{(a_0 G M)^{1/2}}{r}.$$

MOND's acceleration is the weak field limit of $f(\chi) = \chi^{3/2}$

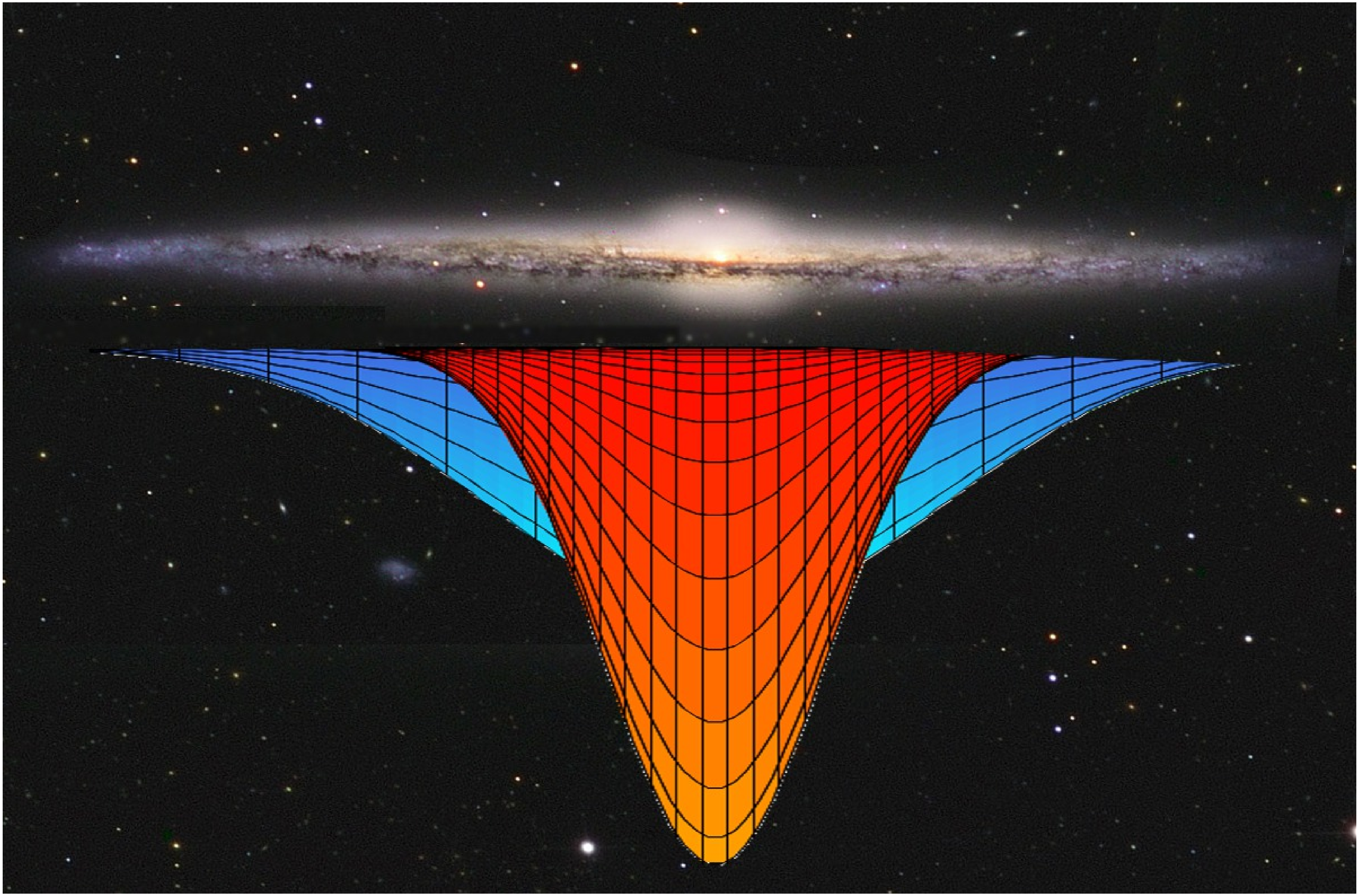
"*..one should not be surprised if some of the commonly accepted notions, even at the fundamental level of the action require generalisations and rethinking.* (Sobouti 2007, Mendoza & Rosas-Guevara 2007, Bernal et al. 2011)"

$$S_{\mathrm{H}} = -\frac{c^3}{16\pi G} \int \frac{f(\chi)}{L_M^2} \sqrt{-g}\, \mathrm{d}^4 x, \qquad \chi := L_M^2 R,$$

$$L_M = \frac{2\sqrt{2}}{9} \left(r_{\mathrm{g}}\, l_M \right)^{1/2}, \qquad r_{\mathrm{g}} := \frac{GM}{c^2}, \qquad l_M := \left(\frac{GM}{a_0} \right)^{1/2}.$$

$$f(\chi) = \begin{cases} \chi & \Longrightarrow \quad \text{general relativity} \\ \chi^{3/2} & \Longrightarrow \quad \text{extended relativistic MOND} \end{cases}$$

$\Longrightarrow$ Field's action depends on the mass/energy.



$F(R,T)$ gravity connection

- Harko et al. (2011) have developed an $F(R,T)$ gravity theory, with $T := T^{\alpha}_{\alpha}$ and so:

$$F(R,T) := f(\chi)/L_M^2, \qquad \text{and “mass-energy”} \qquad M := \frac{4\pi}{c^2}\int T\, r^2 \mathrm{d}r$$

- $F_T(R,T) = 0$ implies Euler's relativistic equation of motion (which is derived from $\nabla_\mu T^{\mu\nu} = 0$), i.e. geodesic motion.
- When $p \to 0$ then equations are geodesic.
- In any case one must search for solutions that follow geodesic motions.
- Note that the function

$$f(\chi) = \chi^{3/2}\,\frac{1 \pm \chi^{b+1}}{1 \pm \chi^{3/2+b}} \longrightarrow \begin{cases} \chi^{3/2} & \text{when } \chi \ll 1 \quad \text{(Ext. Rel. MOND)} \\ \chi & \text{when } \chi \gg 1 \quad \text{(Gen. Rel.)} \end{cases}$$

for $b \geq -1$.

Cosmological dust model (Carranza, Mendoza & Torres 2013)

- Take a FLRW universe with no cosmological constant and with no dark matter components, but assume that $F(R,T) := f(\chi)/L_M^2$ and the "observable (causally connected with us) mass-energy" is:

$$M := 4\pi \int_0^{r_\mathrm{H}} \rho \, r^2 \, \mathrm{d}r$$

Evaluation of Friedmann's equation today for $f(\chi) = \chi^b$ gives:

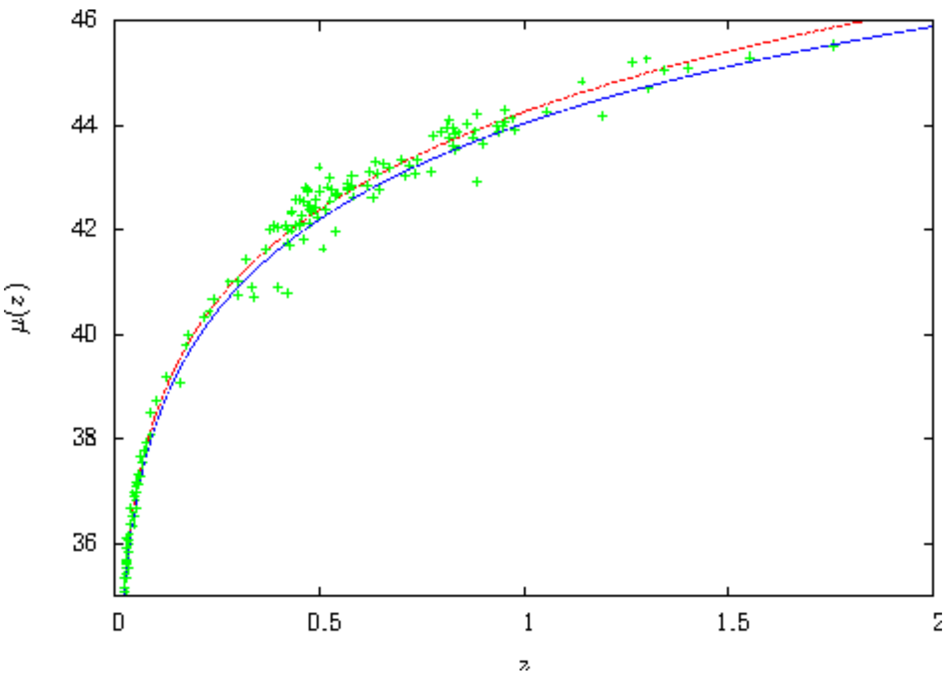
$$\mathfrak{a}_0 = \frac{9}{2}\zeta^4 \left(1 - q_0\right)^2 \left(bZ\right)^{\frac{2}{b-1}} \left(\Omega_m^{(0)}\right)^{\frac{3b-5}{b-1}} cH_0 \approx cH_0$$

(in agreement with Bernal et al. 2012) where,

$$Z = Z(q_0, j_0, \alpha) \approx 1$$

$$a\left(t\right) = a_0 \left(\frac{t}{t_0}\right)^{\alpha}, \quad \rho\left(a\right) = \rho_0 \left(\frac{a}{a_0}\right)^{\beta}$$

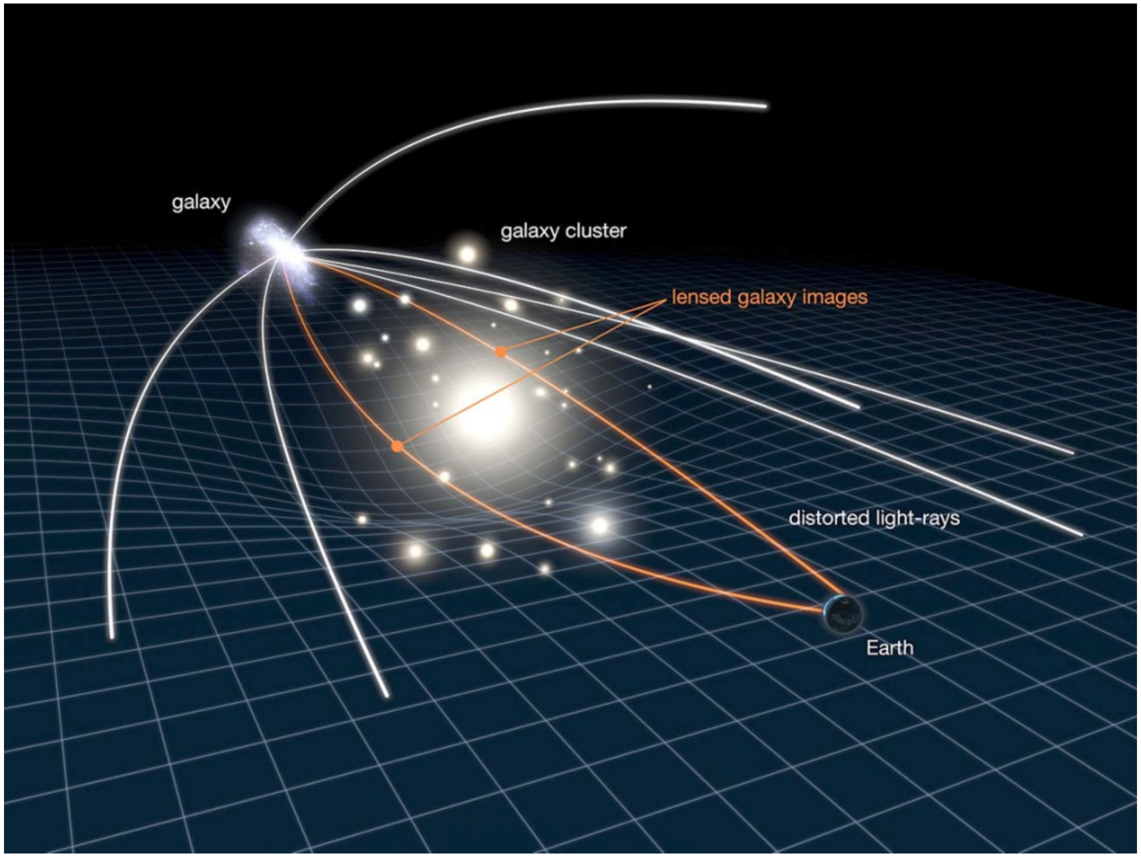
- Take SNIa cosmological data and find the best fit to the above to to find: $\boxed{b = 1.57 \approx 3/2}$.
- This is coherent since $l_M/r \approx$ "a few" for the universe and it must be in the relativistic extended regime.
- $f(\chi) = \chi^{3/2} \Rightarrow h_0 = 0.6409,\ q_0 = -0.2642\ ,\ j = -0.1246,\ s = 0.15.$

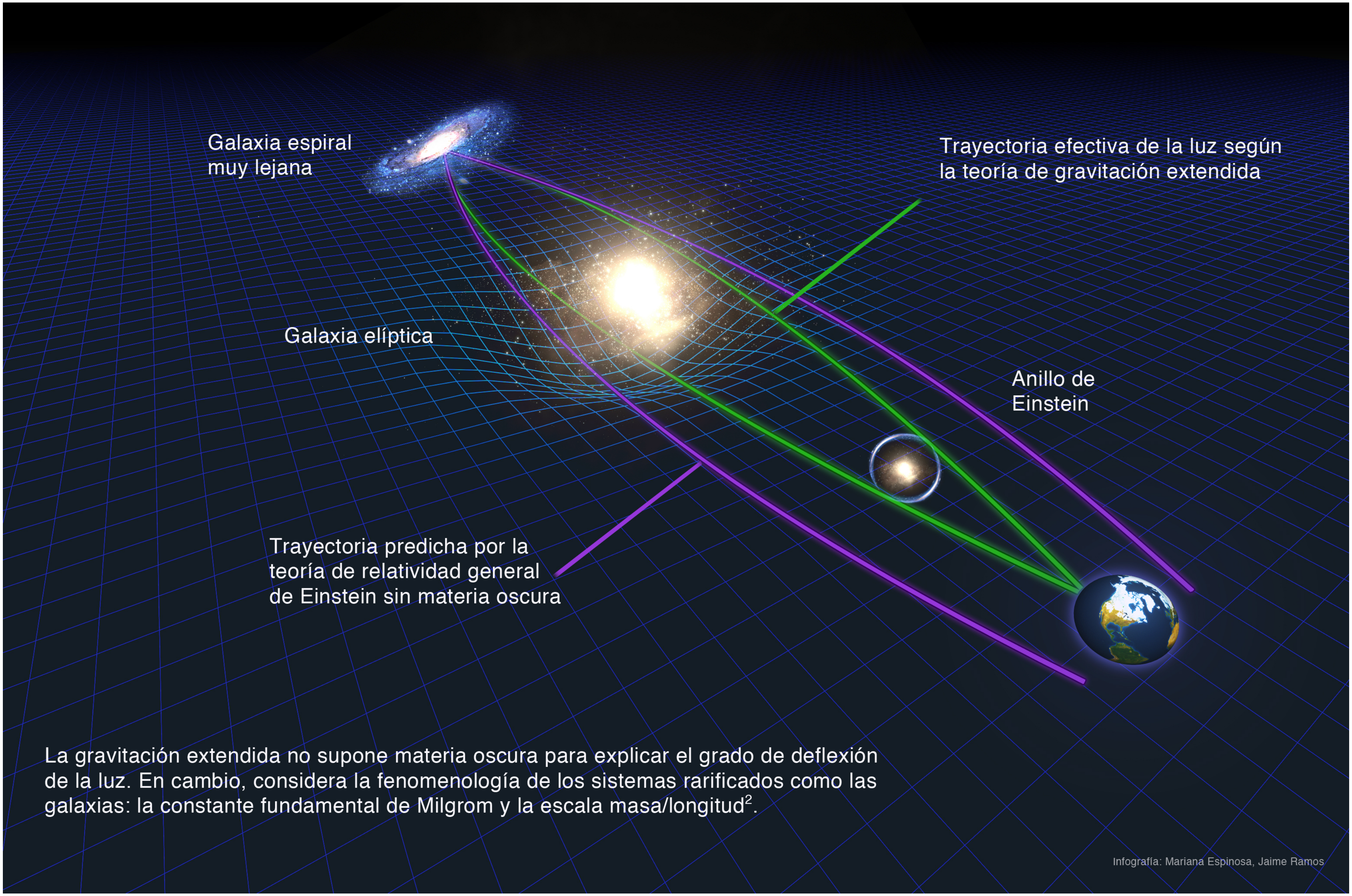


Redshift-magnitude plot. Blue line is concordance ΛCDM, red is best $\chi^{3/2}$ fit.

Gravitational lensing (Mendoza et al. 2013)

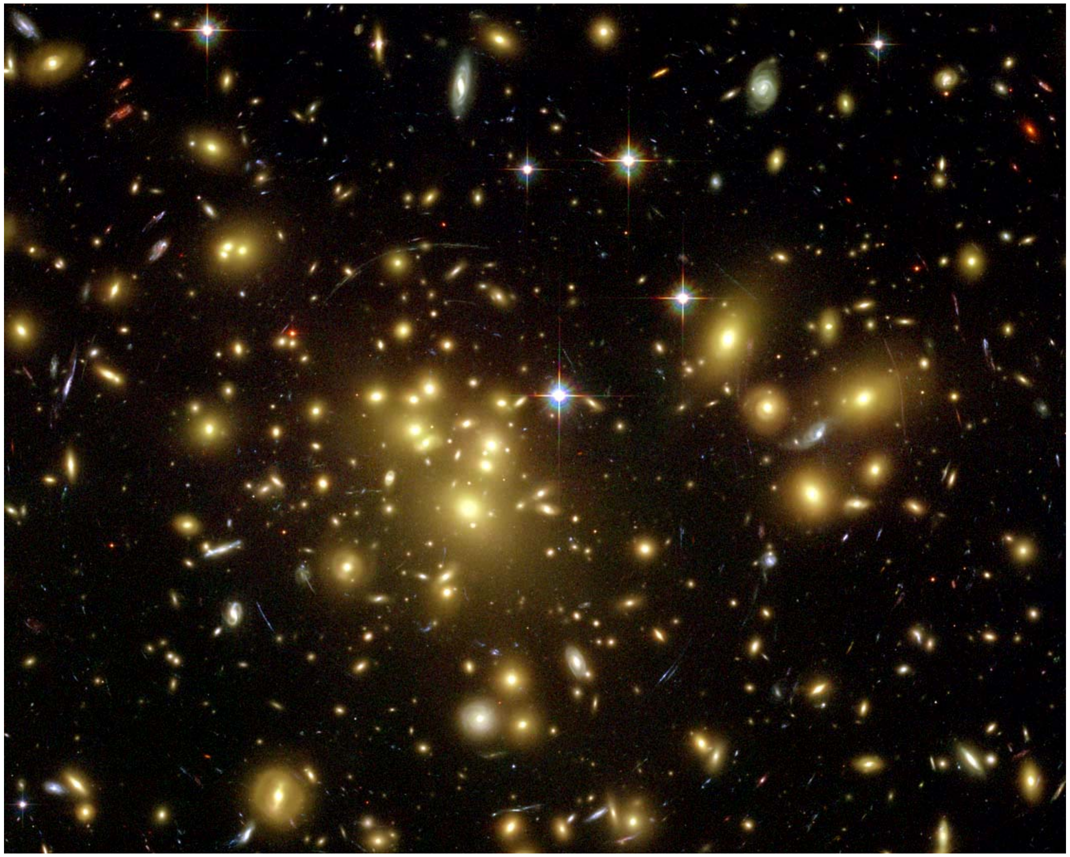
- For this we developed a free (GPL'd) Maxima code: Metric EXtended-gravity Incorporated through a Computer Algebra System (MEXICAS) www.mendozza.org/sergio/mexicas,




Galaxia espiral
muy lejana
Trayectoria efectiva de la luz según
la teoría de gravitación extendida
Galaxia elíptica
Anillo de
Einstein
Trayectoria predicha por la
teoría de relatividad general
de Einstein sin materia oscura
La gravitación extendida no supone materia oscura para explicar el grado de deflexión
de la luz. En cambio, considera la fenomenología de los sistemas rarificados como las
galaxias: la constante fundamental de Milgrom y la escala masa/longitud^2.
Infografía: Mariana Espinosa, Jaime Ramos

Clusters of galaxies (Bernal, Lopez-Corona & Mendoza 2014)

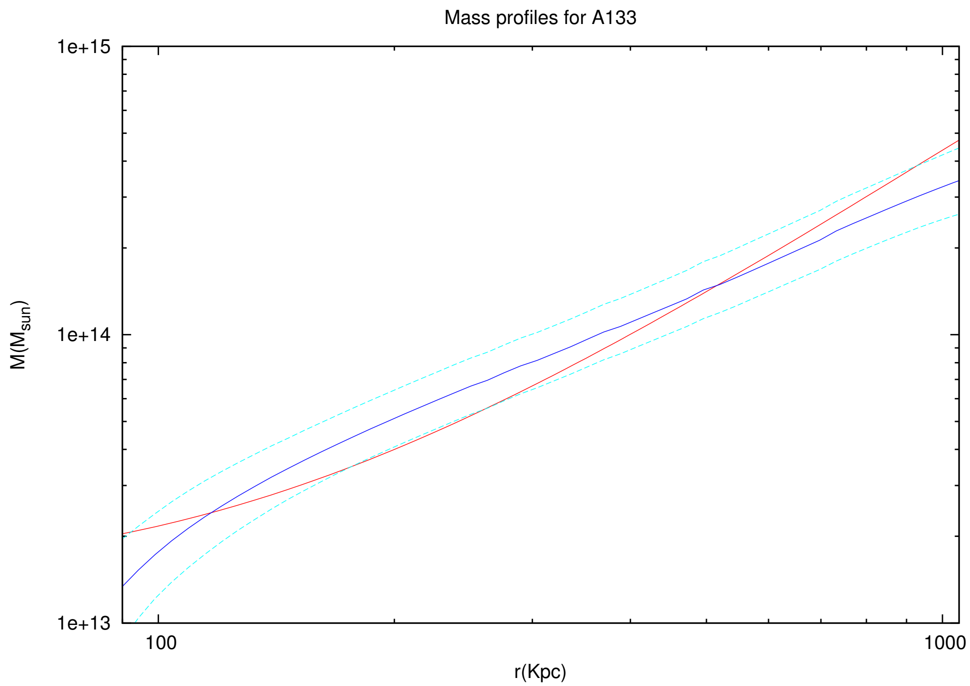
- Mercury's anomaly explained (mainly) by relativistic corrections using $v \sim 48\text{km/s} \Rightarrow \Gamma \sim 1.4 \times 10^{-4}$. For a cluster of galaxies $v \sim 1000\text{km/s} \Rightarrow \Gamma \sim 3.3 \times 10^{-3}$.



Acceleration from the geodesic equation at $O(4)$:

$$a = -\frac{c^2}{2}\left[g^{(2)}_{00,r} + g^{(2)}_{11}\, g^{(2)}_{00,r} + g^{(4)}_{00,r}\right];$$

$$a = -\frac{[GM_b(r)a_0]^{1/2}}{r} + \frac{23GM_b(r)}{c^2 r} - \frac{2GM_b(r)a_0}{c^2 r}\ln\left(\frac{r}{r_\star}\right) + \frac{Ar^2}{c^2 G} + \frac{B}{c^2 G},$$



Blue line is the observed mass, the dashed lines are the uncertainties and the red line is our theoretical mass model. The constants are $A = 10^{-30}c^2$, $B = 0.6 \times 10^{-26}c^2$.

Hilbert-Einstein

$$S = -\int \left(\frac{c^3}{16\pi G} R + \frac{1}{2c} L_{\rm m} \right) \sqrt{-g} \mathrm{d}^4 x$$

Non-relativistic limit:

$$S = -\int \left(-\frac{c^2}{8\pi G} \nabla^2 \phi + \rho\phi \right) \sqrt{-g}\, \mathrm{d}t\, \mathrm{d}^3 x$$

For the first term use $-g = 1 + 2\phi/c^2$ & $\nabla \cdot (\phi \nabla \phi) = \phi \nabla^2 \phi + |\nabla \phi|^2$ to get:

$$S = -\int \left(\frac{|\nabla \phi|^2}{8\pi G} + \rho\phi \right) \mathrm{d}t\, \mathrm{d}^3 x$$

Variations of this yield Poisson equation and so:

$$a = -G \frac{M}{r^2}. \tag{3}$$

$f(R)$

$$S = -\int \left(\frac{c^3}{16\pi G \kappa} f(R) + \frac{1}{2c} L_{\rm m} \right) \sqrt{-g} \mathrm{d}^4 x$$

Non-relativistic limit:

$$S = -\int \left(\frac{c^4}{16\pi G \kappa} f\left(\frac{\nabla^2 \phi}{2c^2} \right) + \rho\phi \right) \mathrm{d}t\, \mathrm{d}^3 x$$

$\Longrightarrow$ Nothing, even when $f(R) = R^b$.

AQUAL

$$S = -\int \left(\frac{a_0^2}{8\pi G} \frac{f\left(|\nabla \phi|^2\right)}{a_0^2} + \rho\phi \right) \mathrm{d}t\, \mathrm{d}^3 x$$

Variations of this yield AQUAL's field equations and the choice $f(y) = y^{3/2}$ gives MONDian regime of gravity:

$$a = -\frac{(G M a_0)^{1/2}}{r}.$$

It is not possible to use a full $f(R)$ that can converge to MOND's acceleration regime **unless** the energy-momentum tensor appears on the gravitational action. Simplest form use the $f(R,T)$ theories by Harko et al. (2012)

- Possible to find MONDian solutions when ($T = \rho c^2$ -dust)

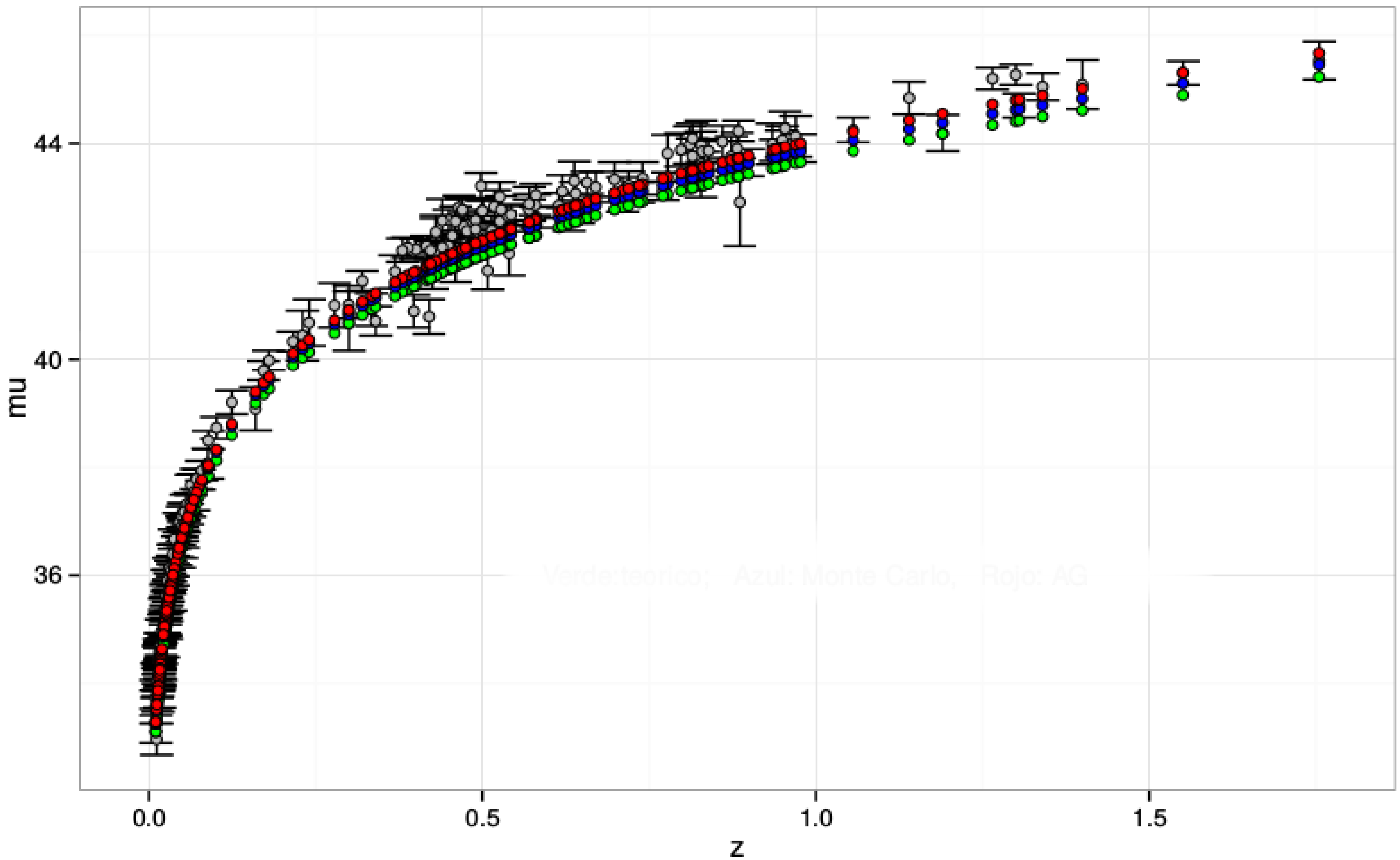
$$F(R,T) = \frac{f(\chi)}{L_M^2}, \qquad \text{with} \qquad f(\chi) = \chi^{-3}$$

$$\text{and} \qquad L_{\rm M} \propto r_{\rm g}^{5/4} l_{\rm M}^{-1/4} \qquad \propto a_0^{-1/4} G^{-3/8} c^{5/4} \rho^{-3/8}$$

$$r_{\rm g} := \frac{c}{\sqrt{G\rho}}, \qquad l_{\rm M} := \frac{a_0}{G\rho}.$$

- SNIa best fit yields: $f(\chi) = \chi^{-3}$, $L_{\rm M} \propto \rho^{-0.37} \approx \rho^{-3/8}$ and $h = 0.75$.
- Currently developing gravitational lensing and dynamics of clusters of galaxies with this approach.

Mendoza (2014), Carranza, Lopez-Corona & Mendoza (2014) -In prep.


mu
44
40
36
0.0
0.5
1.0
1.5
z
Verde:teorico; Azul: Monte Carlo, Rojo: AG

$$S = -\frac{c^3}{16\pi G}\int F(R,T)\sqrt{-g}\,\mathrm{d}^4x - \frac{1}{2c}\int \mathcal{L}_{\mathrm{m}}\sqrt{-g}\,\mathrm{d}^4x = S_{\mathrm{g}} + S_{\mathrm{m}},$$

$$\text{with:}\quad \delta S_{\mathrm{m}} = -\frac{1}{2c}T_{\alpha\beta}\,\delta g^{\alpha\beta},\ \text{and}\quad T := T^{\alpha}_{\ \alpha}.$$

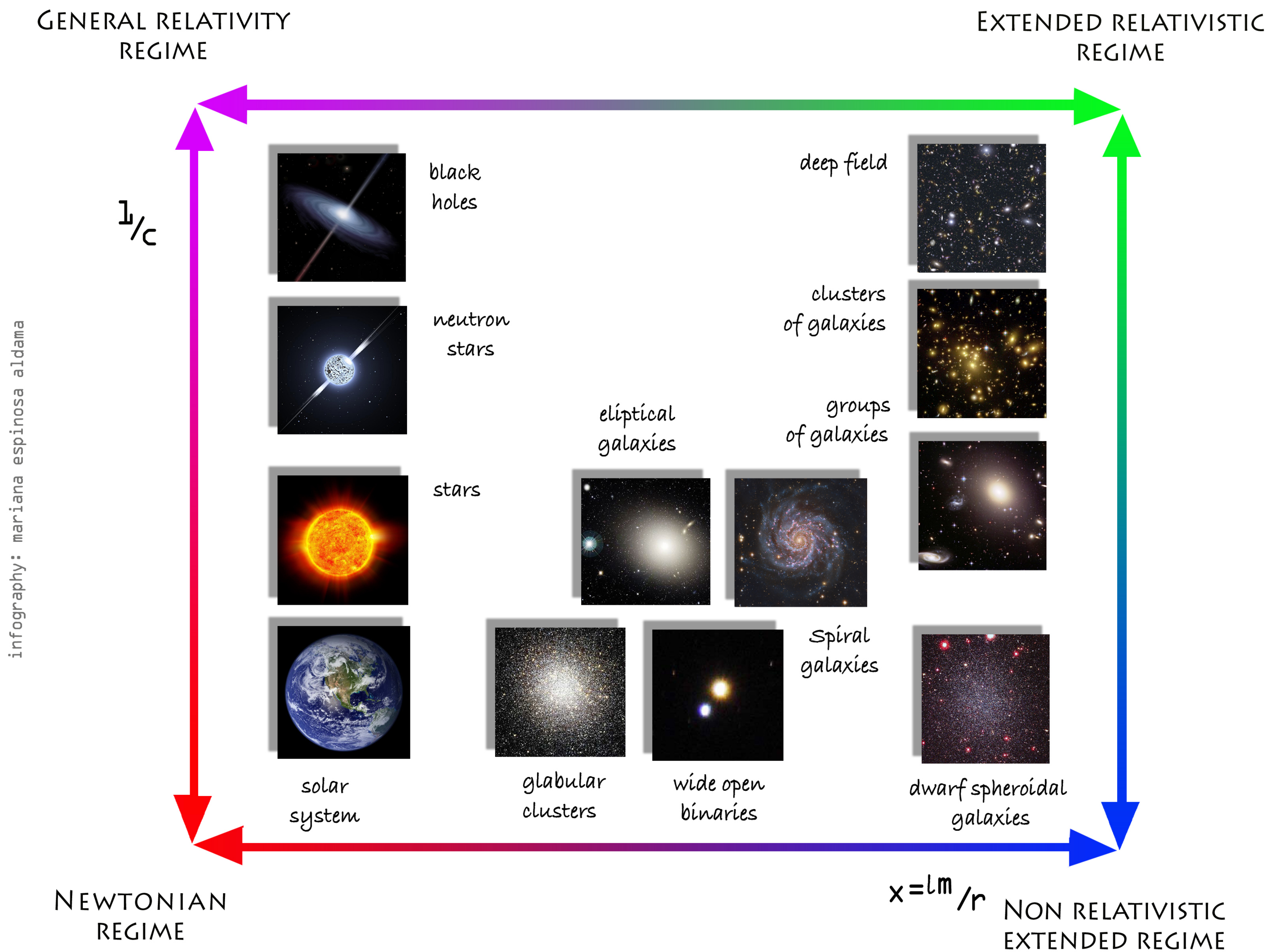
$$F(R,T) = \frac{f(\chi)}{L_M^2},\qquad \text{with}\qquad f(\chi) = \chi^{-3}$$

$$\text{and}\qquad L_{\mathrm{M}} = \frac{3}{4}r_{\mathrm{g}}^{5/4}l_{\mathrm{M}}^{-1/4} \qquad = \frac{3}{4}a_0^{-1/4}G^{-3/8}c^2T^{-3/8}$$

$$r_{\mathrm{g}} := \frac{c^2}{\sqrt{GT}},\qquad l_{\mathrm{M}} := \frac{a_0c^2}{GT}.$$

$$f(\chi) = \chi^{-3}\,\frac{1\pm\chi^{b+1}}{1\pm\chi^{-3+b}} \longrightarrow \begin{cases}\chi^{-3} & \text{when } \chi \ll 1 \quad \text{(Ext. Rel. MOND)}\\ \chi & \text{when } \chi \gg 1 \quad \text{(Gen. Rel.)}\end{cases}$$

for $b \geq 3$.


General relativity regime
Extended relativistic regime
Newtonian regime
Non relativistic extended regime
$1/c$
$x = \sqrt{Gm}/r$
black holes
neutron stars
stars
solar system
deep field
clusters of galaxies
groups of galaxies
eliptical galaxies
spiral galaxies
glabular clusters
wide open binaries
dwarf spheroidal galaxies
infography: mariana espinosa aldama