Variational techniques in relativistic thermodynamics

Thermal dynamics in General Relativity, Proc. R. Soc. A, 8 March 2011 vol. 467 no. 2127, pp. 738-759
A consistent first-order model for relativistic heat flow, Class. Quant. Grav. 15 September 2011 **28** 195023
Variational thermodynamics of relativistic thin disks, arXiv:1306.6591

II Taller de Cosmología y altas energías · ICF-UNAM · August 2014

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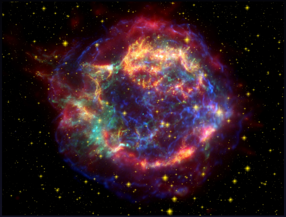
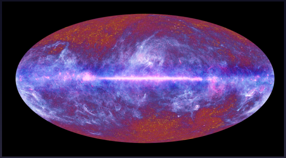
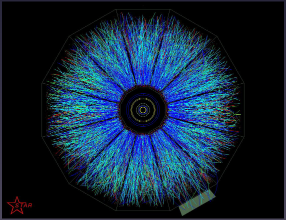
Motivation







- Neutron star physics
- Cosmology
- High energy collisions

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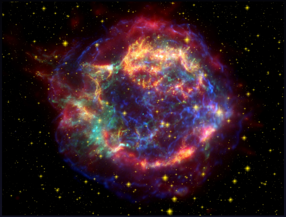
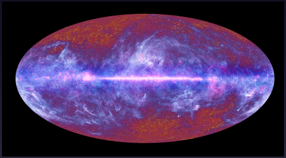
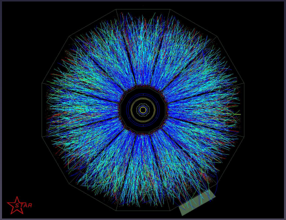
Motivation



- Neutron star physics



- Cosmology



- High energy collisions

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Multifluid thermodynamics

Multifluid thermodynamics

Covariant...

- Constrained multifluid action principle
- Fluid matter model n^a
- Introduce *entropy as another fluid* s^a.
- Allow interaction between them.
- Obtain the relativistic variational dynamics of the system.

...thermodynamics

- Understand the role of different observers.
- Interpret the readings of their measurements (Gibbs form).
- Ensure that entropy never decreases.

Covariant thermodynamics

Multifluid action principle

$$S = \int_\Omega \Lambda(n^a, s^a; g_{ab})\sqrt{-g}\,\mathrm{d}\Omega$$

- n^a Particle number density current
- s^a Entropy density current

Covariant thermodynamics

Dynamical variables

$$T_a^{\ b} = \mu_a n^b + \theta_a s^b + \Psi \delta_a^b$$

$T_{ab} = 2\frac{\partial \Lambda}{\partial g^{ab}} + \Lambda g_{ab}$ Energy-momentum tensor

$\mu_a = \frac{\partial \Lambda}{\partial n^a} = g_{ab}(B^{\mathrm{n}} n^b + A^{\mathrm{ns}} s^b)$ Conjugate momentum to n^a

$\theta_a = \frac{\partial \Lambda}{\partial s^a} = g_{ab}(B^{\mathrm{s}} s^b + A^{\mathrm{sn}} n^b)$ Conjugate momentum to s^a

$\Psi = \Lambda - \mu_a n^a - \theta_a s^a$ Generalised pressure

Covariant thermodynamics

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$$\Psi = \Lambda - \mu_a n^a - \theta_a s^a$$ Generalised pressure

Entrainment effect.

Covariant thermodynamics

Variational Identity

$$\delta S(\Omega) = \int_\Omega \left[-f^{\mathrm{n}}_a \xi^a_{\mathrm{n}} - f^{\mathrm{s}}_a \xi^a_s - T_a{}^b{}_{;b}\xi \right] \sqrt{-g}\mathrm{d}^4x$$

$$-T_a{}^b{}_{;b} = f^{\mathrm{n}}_a + f^{\mathrm{s}}_a = 0$$

$$f^{\mathrm{n}}_a = 2\mu_{[a;b]}n^b + n^b{}_{;b}\mu_a$$

$$f^{\mathrm{s}}_a = 2\theta_{[a;b]}s^b + s^b{}_{;b}\theta_a$$

Covariant thermodynamics

Variational Identity

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Particle flux is conserved

Entropy production.

Covariant thermodynamics

Variational Identity

$$\delta S(\Omega) = \int_\Omega \left[-f^{\mathrm{n}}_a \xi^a_{\mathrm{n}} - f^{\mathrm{s}}_a \xi^a_s - T_a{}^b{}_{;b}\xi\right] \sqrt{-g}\,\mathrm{d}^4x$$

$$-T_a{}^b{}_{;b} = f^{\mathrm{n}}_a + f^{\mathrm{s}}_a = 0$$

$$f^{\mathrm{n}}_a = \left[2\mu_{[a;b]}n^b + n^b{}_{;b}\mu_a\right] n^a = 0$$

$$f^{\mathrm{s}}_a = \left[2\theta_{[a;b]}s^b + s^b{}_{;b}\theta_a\right] n^a = 0$$

Solve for $s^a{}_{;a}$ and impose the second law!

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Covariant thermodynamics

Thermodynamic construction

Frame choice

$u^a = \frac{n^a}{n}$

$-\mu_a u^a$ $\qquad$ $-\theta_a u^a$

$$\mathrm{d}\rho^{\parallel} = \mu^{\parallel}\mathrm{d}n + \theta^{\parallel}\mathrm{d}s + \sigma^a \mathrm{d}p_a$$

$T_{ab}u^a u^b$ $\qquad$ dissipative terms [III]

- Conserved particle current
- Frame choice (Eckart/matter frame)
- Measurements

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Covariant thermodynamics

Features

- Tolman-Ehrenfest effect
- Consistent first-order theory
- Effective inertia of heat
- Second sound

A quick example

A Relativistic thin disk

1. Static
2. Axial symmetry
3. Infinitesimally thin

Exact solution [Kuzmin type]

(a) $ds^2 = -e^{2\phi}dt^2 + e^{-2\beta\phi}\left[r^2 d\varphi^2 + dr^2 + dz^2\right]$

(b) $\rho(r) = 4\beta m a \bar{F}(r)$

$p(r) = \frac{(1-\beta)}{2\beta}\rho(r)$

A quick example

A Relativistic thin disk

Disk thermodynamics.

1. Static solution ⟶ Equilibrium.

2. $\Lambda(r) = -p(r)$, $\Psi = \Lambda - \mu_a n^a - \theta_a s^a = p(r)$

$$\left(\frac{d}{dr}\ln p\right)\left(\frac{d}{dr} ns\right) = \frac{(1+\beta)}{2\beta}\frac{dn}{dr}\frac{ds}{dr}$$

Solutions:

$$n = A p^{k_n}, \quad s = B p^{k_s} \quad \text{s.t.} \quad \frac{k_n + k_s}{k_n k_s} = \frac{1+\beta}{2\beta}$$

$\rightarrow p = p(n, s)$ — fundamental equation.

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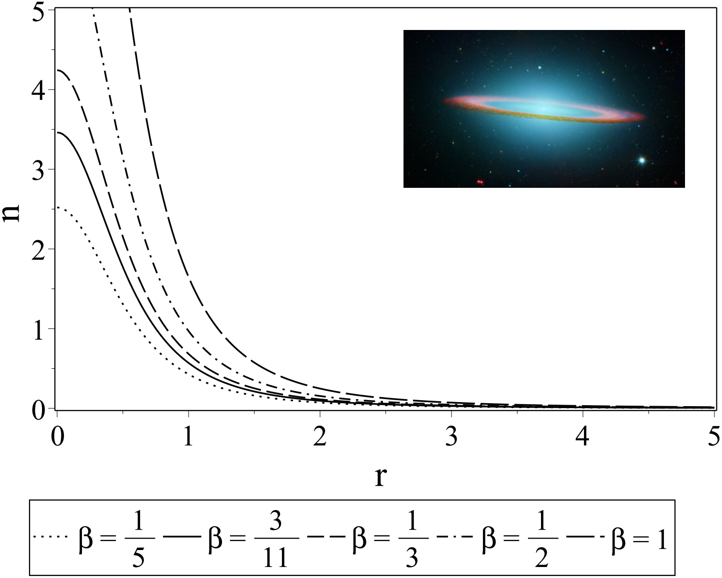
Thermal properties of the disk



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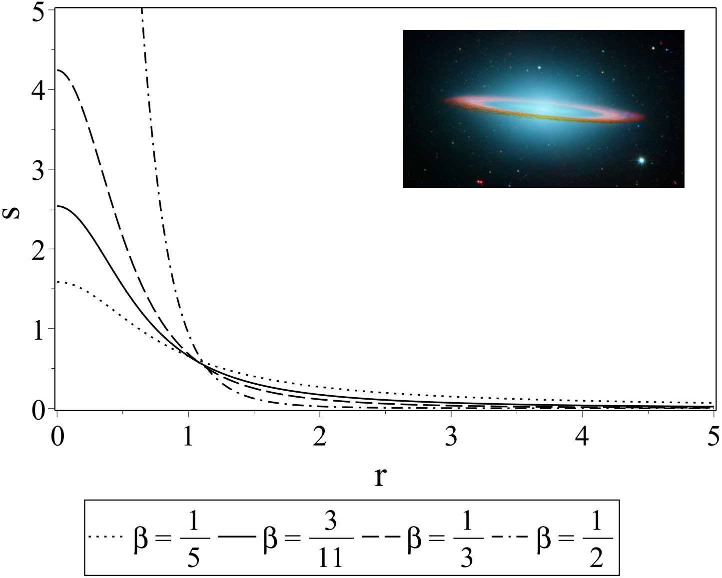

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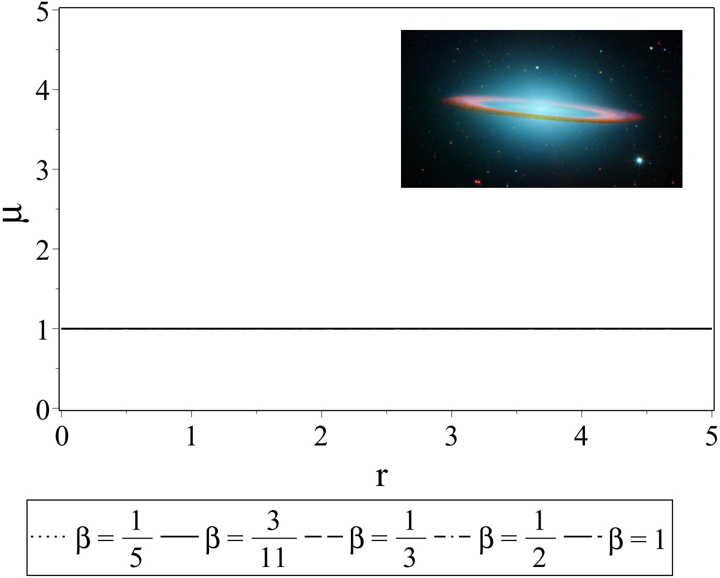

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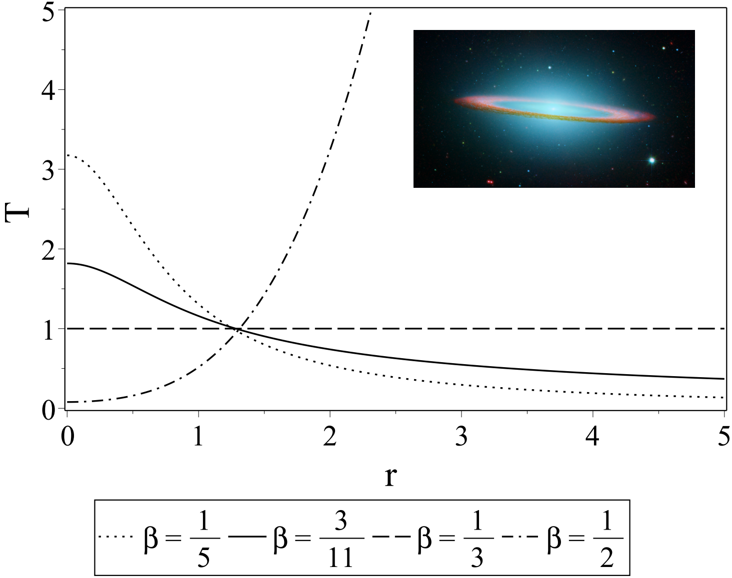
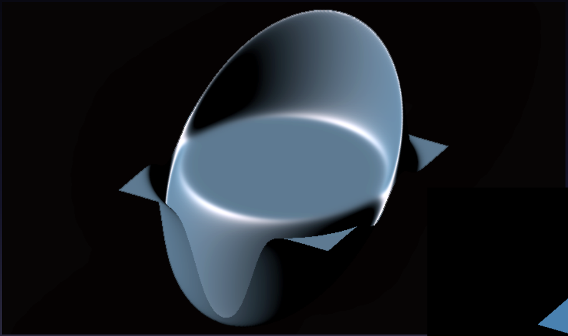
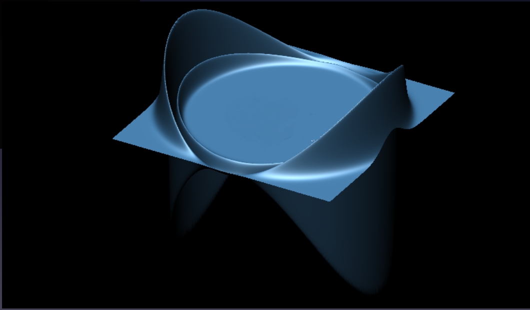


Warpdrive thermodynamics





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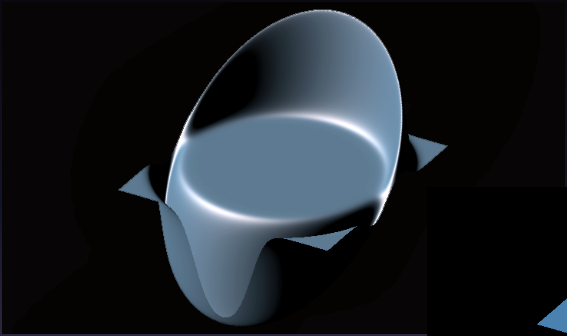
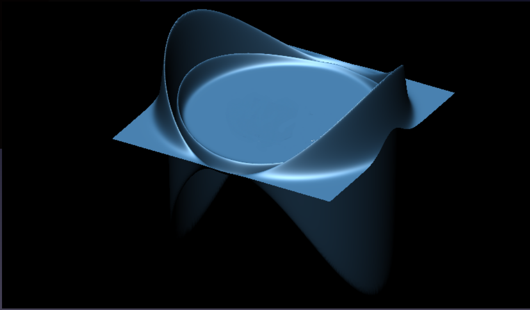
Warpdrive thermodynamics



Volume expansion

$T_{ab} k^a k^b$



Warpdrive thermodynamics

1. Obtain energy momentum tensor from g_{ab}

2. Decompose wrt an orthonormal tetrad.

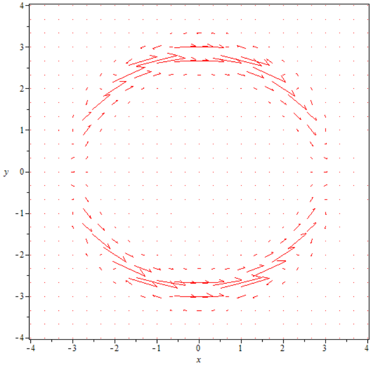
$$\rho = \overline{T}_{ab}\, \hat{e}^a_{(0)} e^b_{(0)}$$

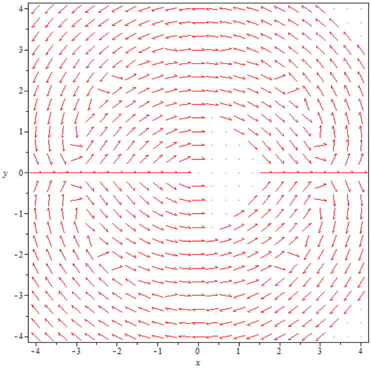
$$q^a = -h^a{}_b\, \overline{T}^b{}_c\, \hat{e}^c_{(0)}$$

] these are known.

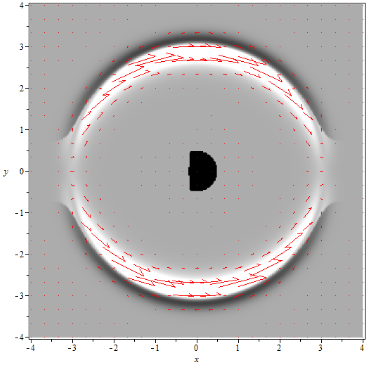
3. Identify $\overline{T}_{ab}$ with a two fluid system [not in equilibrium]

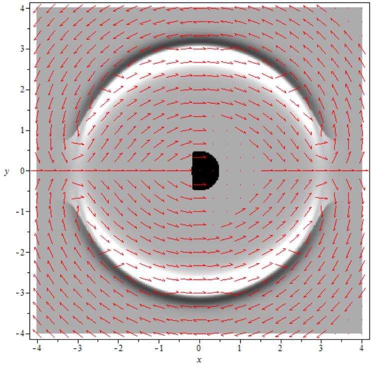
Warpdrive's heat flux



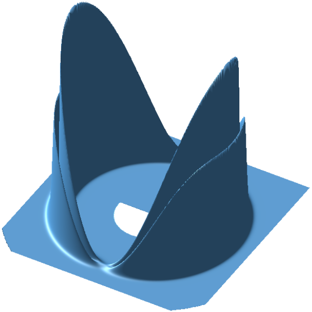


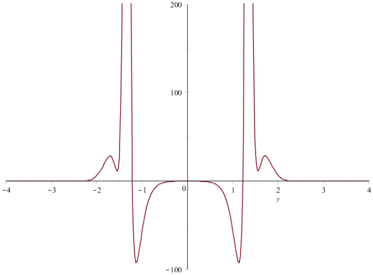
Warpdrive's temperature profile





Warpdrive’s and the 2nd Law





Summary

- Variational techniques provide us with novel ways of obtaining information of relativistic systems.
- Robust formalism (does not depend on the matter model)
- Covariant thermodynamics.
- Testing in progress
- Wide applicability in astrophysics and cosmology

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