Testing Hybrid Natural Inflation with BICEP2

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2. Natural inflation
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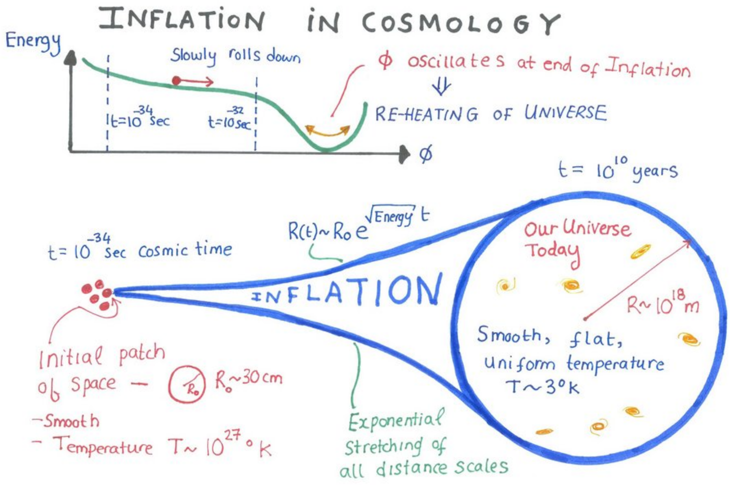
   - Hybrid natural inflation at the maximum of the tensor index

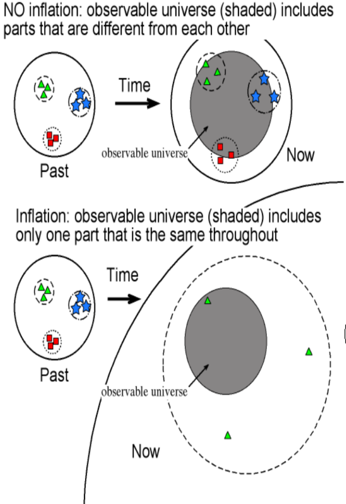

INFLATION IN COSMOLOGY
Energy
Slowly rolls down
t=10^-34 sec
t=10^-32 sec
ϕ oscillates at end of Inflation
RE-HEATING OF UNIVERSE
ϕ
t= 10^10 years
t= 10^-34 sec cosmic time
R(t)~R₀ e^√Energy t
INFLATION
Our Universe Today
R~10^18 m
Smooth, flat,
uniform temperature
T~3°K
Initial patch
of Space
R₀
R₀~30cm
-Smooth
- Temperature T~10^27 °K
Exponential
Stretching of
all distance scales

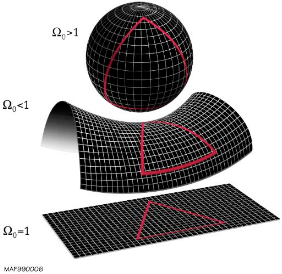
Horizon problem
Homogeneous & isotropic

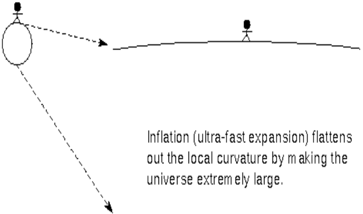
Flatness problem

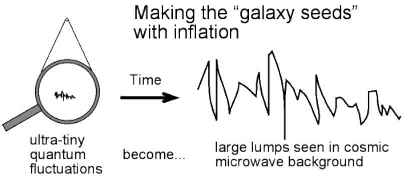
$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\, d\Omega^2\right)$$

$$(\Omega^{-1} - 1)\rho a = -3M_{Pl}k$$

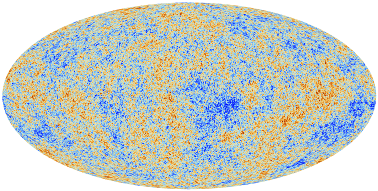




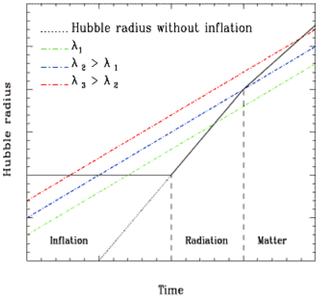




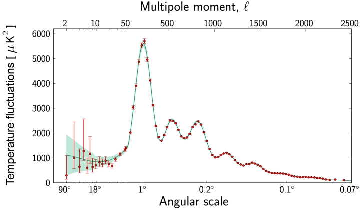
- **Primordial energy density perturbation.**
  Inflaton perturbation, Scalar curvature perturbation.
- **Primordial gravitational waves.**
  Spatial metric perturbation, transverse and traceless.



Primordial power spectra of perturbations



$$\mathcal{P}_{\mathcal{R}} = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\frac{\mathrm{d}n_s}{\mathrm{d}\ln k}\ln\frac{k}{k_*} + \cdots}$$



Primordial quantities

$$\mathcal{R}(\mathbf{k}) = \frac{1}{4}\left(\frac{a}{k}\right)^2 R^{(3)}(\mathbf{k})$$

$$h_{ij} = h_+ e^+_{ij} + h_\times e^\times_{ij}$$

$$\mathcal{P}_t = A_t \left(\frac{k}{k_*}\right)^{n_t + \frac{1}{2}\frac{\mathrm{d}n_t}{\mathrm{d}\ln k}\ln\frac{k}{k_*} + \cdots}$$

Slow-roll inflation

EVOLUTION OF INFLATON FIELD

Equation of motion $\ddot{\phi}+3H\dot{\phi}+V' = 0; \qquad H = \dot{a}/a$

Friedmann equation $H^2 = \frac{1}{3M_{Pl}^2}(\frac{1}{2}\dot{\phi}^2 + V(\phi))$

SLOW ROLL APPROXIMATION

Equation of motion $3H\dot{\phi} = -V'$

Friedmann equation $H^2 = \frac{V(\phi)}{3M_{Pl}^2}$

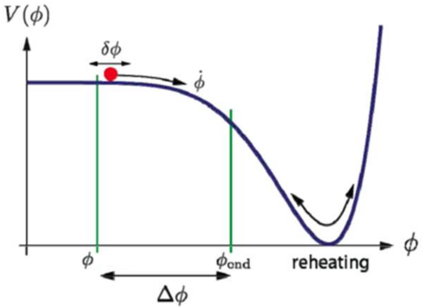
Slow-roll approximation

$$\epsilon \ll 1; \qquad |\eta| \ll 1$$

Slow-roll parameters

$$\epsilon = \frac{V'^2}{2V^2} \qquad \eta = \frac{V''}{V}$$

$$\xi^2 = \frac{V' V'''}{V^2} \qquad \omega^3 = \frac{V'^2 V''''}{V^3}$$



Observable quantities

$$\delta_H^2 = \frac{4}{25} A_s = \frac{1}{150\pi^2} \frac{V}{\epsilon_H}; \qquad r \equiv \frac{A_t}{A_{\mathcal{R}}} = 16\epsilon = -8n_t$$

$$N \equiv \int H\, \mathrm{d}t = \int_{\phi_e}^{\phi_*} \frac{V}{V'}\, \mathrm{d}\phi = \int_{\phi_e}^{\phi_*} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon}}$$

$$n_s = 1 + 2\eta - 6\epsilon$$

$$n_{sk} \equiv \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2$$

$$n_{tk} \equiv \frac{\mathrm{d}n_t}{\mathrm{d}\ln k} = 4\epsilon\eta - 8\epsilon^2$$

Constraint equation

$$\rightarrow n_{\mathrm{tk}} = \frac{r}{64}\left(r - 8\delta_{\mathrm{ns}}\right)$$

where $\delta_{\mathrm{ns}} \equiv 1 - n_s$.

Natural Inflation

Fine tuning. Flat potential requires: $\frac{\Delta V}{\Delta \phi^4} \leq \frac{height}{width^4} \leq 10^{-6}$

$$V(\phi) = \Lambda^4 (1 \pm \cos \frac{\phi}{f})$$

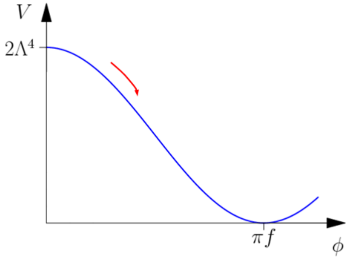
$$\epsilon = \frac{1}{2f^2} + \eta$$

$$\xi^2 = -2\frac{1}{f^2}\epsilon \qquad \omega^3 - 2\frac{1}{f^2}\epsilon\eta$$

$$f = \frac{2}{\sqrt{4\delta_{ns} - r}} \longrightarrow r \leq 4\delta_{ns} \approx 0.16$$

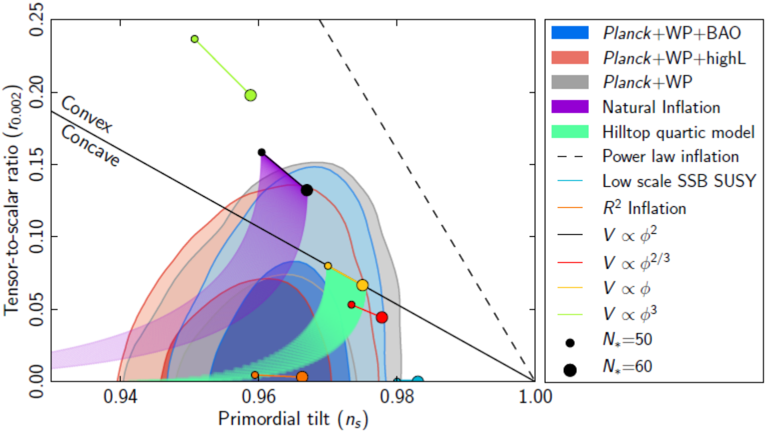
Large symmetry breaking scale!!

$r = 0.15 \longrightarrow f = 20 M_{Pl}$

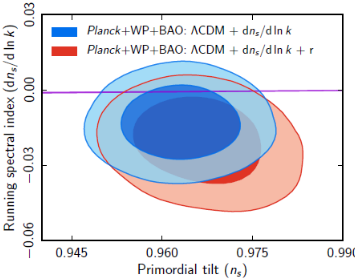


(Freese, Frieman, Olinto; 1990)

Planck collaboration



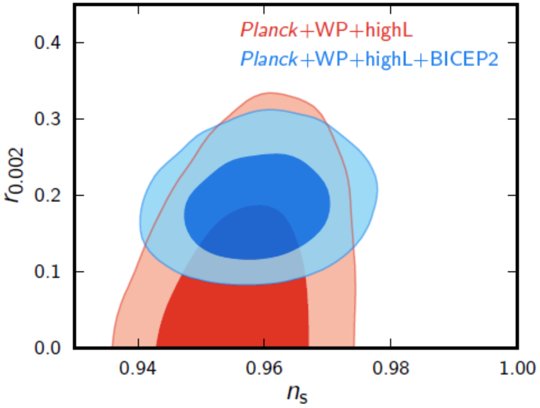
Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s $r_{0.002}$	0.9624 ± 0.0075 < 0.12	0.9653 ± 0.0069 < 0.13	0.9600 ± 0.0071 < 0.11	$0.9643 + 0.0059$ < 0.12



Model	Parameter	*Planck*+WP
ΛCDM + $\mathrm{d}n_s/\mathrm{d}\ln k$	n_s $\mathrm{d}n_s/\mathrm{d}\ln k$	0.9561 ± 0.0080 -0.0134 ± 0.0090
ΛCDM + $\mathrm{d}n_s/\mathrm{d}\ln k$ + $\mathrm{d}^2n_s/\mathrm{d}\ln k^2$	n_s $\mathrm{d}n_s/\mathrm{d}\ln k$ $\mathrm{d}^2n_s/\mathrm{d}\ln k^2$	$0.9514^{+0.087}_{-0.090}$ $0.001^{+0.016}_{-0.014}$ $0.020^{+0.016}_{-0.015}$
ΛCDM + r + $\mathrm{d}n_s/\mathrm{d}\ln k$	n_s r $\mathrm{d}n_s/\mathrm{d}\ln k$	0.9583 ± 0.0081 < 0.25 -0.021 ± 0.012

BICEP2

Detection of B-mode Polarization at degree angular scales. $r = 0.20^{+0.07}_{-0.05}$



BICEP2

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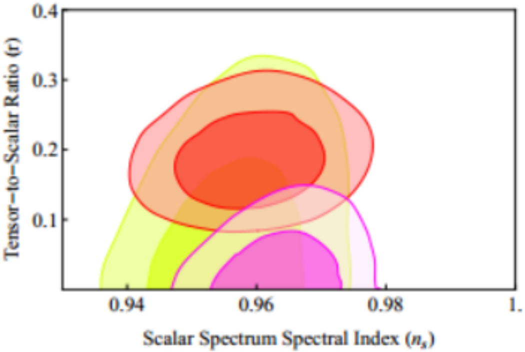


(L.A. Anchordoqui, 2014)

Hybrid natural inflation

Searching a lower symmetry breaking scale.

$$V(\phi) = V_0(1 + a\cos\frac{\phi}{f})$$

- Inflaton is a PNGB associated with spontaneous breaking of approximate continuous symmetry U(1).
- Spontaneoulsy broken global non-Abelian discrete symmetry $D_4 = Z_2 \rtimes Z_2'$.
  $\longrightarrow$ A 2nd field is responsible for ending inflation.
- Two models:
  - Non supersymmetric.
  - Supersymmetric.

(G. G. Ross, G. German; 2010)

Testing Hybrid Natural Inflation with BICEP2

(M. Carrillo-Gonzalez, G. German, A. Herrera-Aguilar, J. C. Hidalgo, R. Sussman; 2014)

Slow-roll parameters for hybrid natural inflation

$$\epsilon = \frac{a^2 \sin^2\left(\frac{\phi}{f}\right)}{2f^2\left(1 + a\cos\left(\frac{\phi}{f}\right)\right)^2} \qquad \xi^2 = -2\frac{1}{f^2}\epsilon$$

$$\eta = -\frac{a\cos\left(\frac{\phi}{f}\right)}{f^2\left(1 + a\cos\left(\frac{\phi}{f}\right)\right)} \qquad \omega^3 = -2\frac{1}{f^2}\epsilon\eta$$

$$\epsilon = \frac{a^2}{2f^2} + a^2\eta - \frac{(1-a^2)f^2}{2}\eta^2$$

Observables for hybrid natural inflation

$$n_{sk} = \frac{r}{32}\left(3r - 16\delta_{ns} + \frac{8}{f^2}\right)$$

$$f = \frac{\sqrt{8r}}{\sqrt{32 n_{\rm sk} - r\left(3r - 16\delta_{\rm ns}\right)}}.$$

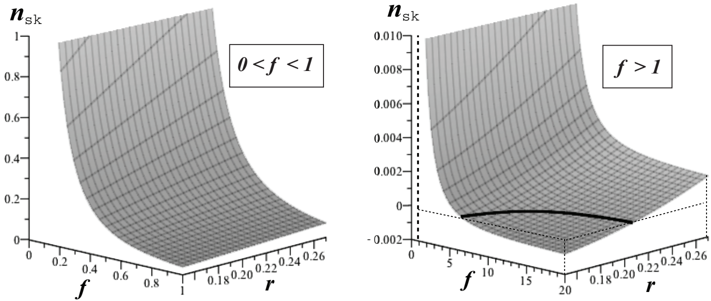
Constraints for n_{sk}

$$n_{\rm sk} \geq -9 \times 10^{-4} \quad \mathit{for} \quad r \geq 0.15$$

$$n_{\rm sk} > 0 \quad \mathit{for} \quad r > \frac{16}{3}\delta_{\rm ns} \approx 0.21$$

$$N_* = \frac{f^2}{2a}\left((1+a)\ln\left(\frac{1-c_e}{1-c_*}\right) - (1-a)\ln\left(\frac{1+c_e}{1+c_*}\right)\right)$$

Running of the spectral index



Natural Inflation vs Hybrid Natural Inflation

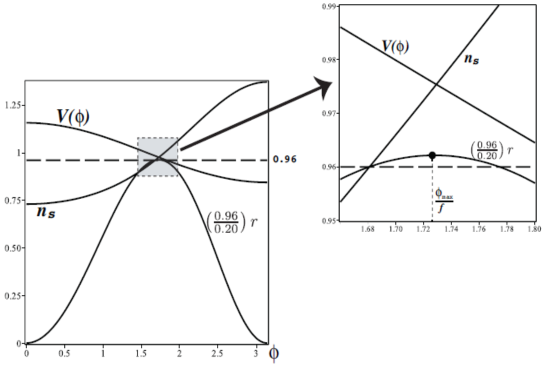
	f	a	n_s	r	n_{sk}	n_{skk}	n_{tk}	Λ (GeV)	N
NI	8.9	–	0.96	0.11	-7.2×10^{-4}	-2.9×10^{-5}	-3.6×10^{-4}	1.88×10^{16}	51
NI	20	–	0.96	0.15	-8.0×10^{-4}	-3.2×10^{-5}	-4.0×10^{-4}	2.04×10^{16}	50
NI	∞	–	0.96	0.16	–	–	–	–	–
HNI	1	0.117	0.96	0.11	2.6×10^{-2}	2.2×10^{-3}	-3.6×10^{-4}	1.88×10^{16}	N_χ
HNI	1	0.136	0.96	0.15	3.7×10^{-2}	3.6×10^{-3}	-4.0×10^{-4}	2.04×10^{16}	N_χ
HNI	1	0.140	0.96	0.16	3.9×10^{-2}	4.0×10^{-3}	-4.0×10^{-4}	2.07×10^{16}	N_χ
HNI	1	0.156	0.96	0.20	5.0×10^{-2}	5.8×10^{-3}	-3.8×10^{-4}	2.19×10^{16}	N_χ
HNI	1	0.181	0.96	0.27	6.9×10^{-2}	9.8×10^{-3}	-2.1×10^{-4}	2.36×10^{16}	N_χ

Maximum of the tensor index

$$\cos\frac{\phi_{max}}{f} = -a$$

Universality

$$r_{\text{critical}} = 8\delta_{ns}; \qquad \epsilon_{\text{critical}} = \frac{1}{2}\eta_{\text{critical}}$$



If $\phi_H = \phi_{max} \rightarrow r_{critical} = 0.32$

Conclusions

Natural Hybrid Inflation

- Lower symmetry breaking scale f.
- Allows the BICEP2 reported values for r.
- Predicts a “large” value for the running n_{sk}:
  - could be observable in the near future.
  - possible resolution to the apparent tension between high values of r and previous limits.

Observable inflation doesn't seem to occur at the maximum of the tensor index.